Linear and nonlinear programming to optimize the nutrient density of a population's diet: an example based on diets of preschool children in rural Malawi[1,2]

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ABSTRACT

Background: Food consumption surveys are often used to detect inadequate nutrient intakes but not to determine whether inadequate nutrient intakes are due to suboptimal use of locally available foods or to insufficient availability of nutrient-dense foods.
Objectives: The objectives were to describe the use of linear programming as a method to design nutrient-adequate diets of optimal nutrient density and to identify the most stringent constraints in nutritional recommendations and food consumption patterns in a population's diet.
Design: This analysis was conducted with the use of food consumption data collected during 2 seasons from rural Malawian children aged 3–6 y. Linear programming was used to select diets based on local foods that satisfied a set of nutritional constraints while minimizing the total energy content of the diet. Additional constraints on daily intakes of foods and food groups were also introduced to ensure that the diets were compatible with local food patterns. The strength of the constraints was assessed by analyzing nonlinear programming sensitivity.
Results: In the harvest season, it was possible to satisfy nutritional recommendations with little departure from the local diet. In the nonharvest season, nutritional adequacy was impaired by the low availability of riboflavin- and zinc-rich animal or vegetable foods and by the high phytate content of other foods.
Conclusions: This analysis suggests that nutrition education may help improve the diets of children in the harvest season, whereas changes in the range of available foods might be needed in the nonharvest season. Linear and nonlinear programming can be used to formulate recommendations with the use of data from local food consumption surveys. *Am J Clin Nutr* 2002;75:245–53.

KEY WORDS Linear programming, recommended nutrient intakes, food consumption patterns, nutrient density, children, season, Malawi, Africa

INTRODUCTION

The diets of children in poor countries are frequently deficient in key nutrients such as iron, zinc, calcium, riboflavin, vitamin A, and vitamin C (1–4). This deficiency can be explained by either a shortage of micronutrient-dense foods (ie, foods with a high concentration of nutrients in relation to energy) or an inappropriate selection of local foods. These 2 possibilities have quite different programmatic implications. The first possibility suggests that the deficiency can only be improved by increasing the availability of nutrient-rich foods, either through food fortification or agricultural programs that introduce new crop varieties. The second possibility suggests that nutrition education programs that focus on the best use of locally available foods should be given priority.

Little attention has been given to the questions underlying these alternative possibilities, namely: *1*) Is it possible to design a diet that fulfills all nutritional recommendations for children through the use of locally available foods? and *2*) If such a diet is possible, what is the best combination of these foods to achieve a nutrient-dense diet? Answering these questions requires an assessment of all combinations of locally available foods to determine which combination provides the most nutrient-dense diet while concurrently meeting nutrient intake recommendations. This optimization problem can now be examined on most personal computers with the use of optimization functions based on linear and nonlinear programming. These techniques are classic tools used to formulate animal diets (5), and the possibilities these techniques offer in human nutrition at the population level were described in detail as early as 1959 (6). Yet, this seminal work received little attention, perhaps because of the difficulties in applying these techniques at a time when computers were not widely available. Specific applications of linear programming, such as the prescription of personalized diets in clinical (7–9) or institutional (10) practice, led to the development of widely used computer software programs such as MICRODIET (1990; Salford University, Salford, United Kingdom). Linear programming has also been used to formulate low-cost nutritious diets (11, 12) and to analyze the economic constraints on human diets (13, 14).

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Received November 30, 2000.
Accepted for publication March 6, 2001.

TABLE 1
Three sets of nutritional constraints used in the different analyses[1]

	UK RNIs (17)	Published FAO/WHO recommendations (18–21)	Preliminary FAO/WHO recommendations (22)
		Minimum imposed	
Protein (g/d)	19.7	20	20
Calcium (mg/d)	450	400	600
Iron (mg/d)	6.1	7	5.5
Folate (μg/d)	100	50	200
Vitamin B-12 (μg/d)	0.8	0.5	1.2
Vitamin C (mg/d)	30	20	30
Thiamine (mg/d)	0.7	0.7	0.6
Riboflavin (mg/d)	0.8	1.1	0.6
Niacin (mg/d)	11	12.1	8
Vitamin A (μg/d)	500	400	450
Zinc (mg/d)	6.5	6.5	4.8
Copper (mg/d)	0.6	0.57	Unspecified
Molar ratio of phytate to zinc[2]	15	15	15

[1] RNI, reference nutrient intakes; FAO, Food and Agriculture Organization; WHO, World Health Organization.

[2] Maximum imposed.

In the present study, our goal was to demonstrate the merits of linear and nonlinear programming for planners of nutrition intervention programs in sufficient detail to facilitate its application. Food consumption data collected over 2 seasons from preschool children living in rural Malawi (East Africa) were used for this purpose.

SUBJECTS AND METHODS

Description of optimization models

An optimization model is defined by an objective function dependent on a set of variables (ie, decision variables) restricted by various constraints. The goal of optimization is to find a set of decision variables that generates the optimal value for the objective function while satisfying all the imposed constraints. In the present study, optimization models were used to design a nutrient-dense diet, ie, a diet meeting specific nutrient intake recommendations at the lowest energy content achievable. The diet identified was represented by a set of food weights, subsequently called food variables, each representing a decision variable for the models. To obtain the optimal nutrient density in the diet, the total energy content of the diet was chosen as the objective function, and this function was minimized. The value obtained after optimization was the minimal amount of energy required to satisfy all the constraints with use of foods habitually consumed by the population of interest. The foods selected in the optimized solution by linear programming should be considered as an average daily food basket to be eaten over several days and not necessarily every day.

The constraints used in the models were divided into 2 categories: *1*) nutritional constraints and *2*) food consumption (ie, foods and food groups) constraints. The former constraint ensured the nutritional quality of the diet, and the latter constraint ensured the palatability and social acceptability of the designed diets. The use of linear and nonlinear programming techniques to optimize the nutrient density of a diet was tested by using dietary data collected with 3-d weighed food records over 2 seasons (the harvest season: March through April; the nonharvest season: July) in 1986 from 65 randomly selected children living in one rural village in southern Malawi. Fifty-seven percent of the children had a height-for-age *z* score < -2 SD of the National Center for Health Statistics reference (15). The dietary assessment procedures, the food-composition database, and the sociodemographic characteristics of this African population are detailed elsewhere (15, 16). The nutritional and food consumption constraints introduced into different models are described in detail below, followed by a description of the linear and nonlinear programming procedures. Ethical approval was granted by the Human Ethics Committee, University of Guelph, Canada, and from the Center for Social Research, University of Malawi.

Nutritional constraints

The nutritional constraints, based on the reference nutrient intakes (RNIs) defined in the United Kingdom (17), were used in all models to ensure that nutritionally adequate diets were selected. Select results were then compared with results generated with use of recommendations published by the WHO (18–21) and the more recent preliminary recommendations of the WHO available on the Internet (22) to assess the sensitivity of results to the nutritional constraints selected (**Table 1**). For zinc and iron, moderate bioavailability was assumed. To ensure that this assumption was satisfied, an additional constraint was introduced that limited the dietary molar ratio of phytate to zinc (P:Z) to 15 (19). The amounts of absorbable iron and absorbable zinc were calculated for each diet, with the use of published formulas (4, 23), to test the consistency of the results. These formulas were not, however, included among the constraints.

Food consumption constraints

In the first stage of the analysis, only nutritional constraints were included in the optimization models. In the second stage, constraints limiting the proportion of energy coming from different food groups to between the 25th and 75th percentiles of those observed in the whole population were introduced and then retained for all subsequent analyses. The food group constraints are shown in **Table 2**. These constraints varied by season, especially those for plant-based products. In the nonharvest season, constraints on vegetables, legumes, and fruit were particularly severe because these foods were consumed infrequently and less frequently than in the harvest season. Constraints on animal products were less affected by seasonality.

In the third stage, limits were also placed on each food variable to limit the daily portions (g/d) of each food selected to avoid exceeding quantities actually eaten by children in the community. The limits chosen were based on the distribution of portion sizes actually consumed by the children who consumed the food (ie, consumer intake distributions). Two sets of limits were assessed in different models, which corresponded to the 90th percentiles and then, more severely, the 75th percentiles of the consumer intake distribution. In the final analysis, the third-stage analysis was repeated excluding rarely consumed foods, which were defined as those foods consumed by <10%, and in other models, <25% of the entire population. Application of these final constraints reduced the number of eligible foods in the analysis from 63 to 35 foods (10% of the population) and to 19 or 18 foods (25% of the population in the harvest and nonharvest seasons, respectively).

TABLE 2

Food group constraints used in different analyses during the harvest and nonharvest seasons[1]

	Harvest season		Nonharvest season	
	Minimum (25th)	Maximum (75th)	Minimum (25th)	Maximum (75th)
	% of total energy			
Cereals	43	60	63	77
Fruit	1	5	0	3
Legumes	8	30	3	10
Meat, fish, and eggs	2	7	2	6
Roots	0	9	4	16
Vegetables	3	19	1	3

[1] The lower and upper limits for the constraint are the 25th and 75th percentiles of the percentage of energy contributed by a food group in the population.

Linear programming

A linear function of food variables can be expressed in the following form:

$$a_0 + a_1W_1 + a_2W_2 + \ldots + a_nW_n \tag{1}$$

where a_0, a_1, $a_2 \ldots a_n$ are constant coefficients. The energy content of the diet is a linear function of food variables, where $a_0 = 0$ and a_1, $a_2,\ldots a_n$ are equivalent to the energy content per unit weight for foods F_1 to F_n, and W_1, $W_2 \ldots W_n$ are equivalent to the weight (in g) of foods F_1, $F_2 \ldots F_n$.

A linear constraint is expressed by an inequality in which the first term is a linear function of the food variables. Constraints setting minimal nutrient intakes in the diets are linear constraints and are expressed as $N_i \geq \text{RNI}_i$, where N_i is equivalent to the total amount of each nutrient provided by the diet, and RNI_i equals the selected nutrient recommendation for that nutrient (eg, the UK RNI). In contrast, constraints expressed by a ratio of ≥2 food variables are nonlinear. A model is linear when all constraints are linear and is nonlinear when some constraints are nonlinear. In a linear programming optimization model, the optimum found is the best definitive value that can be attained (global optimum). In contrast, models including nonlinear constraints may have several solutions, depending on the initial values. These solutions do not always reach the best value (local optimums).

To guarantee the global optimum per analysis, each model was first analyzed by linear programming. The constraints expressed as ratios, namely that on the P:Z and the food groups constraints, were transformed into an equivalent linear constraint with the use of appropriate mathematical transformations.

For example, the constraint on the P:Z was nonlinear when expressed as follows:

$$[(W_1P_1 + W_2P_2 \ldots + W_nP_n)/(W_1Z_1 + W_2Z_2 \ldots + W_nZ_n)] \leq 15 \tag{2}$$

where P_n and Z_n are the phytate and zinc contents (in mmol) in W_n (in g) per food F_n.

For the linear models, the constraint on P:Z was expressed as a linear function of the weight of different foods. In other words, Equation *2* is equivalent to

$$W_1(P_1 - 15\,Z_1) + W_2(P_2 - 15\,Z_2) \ldots W_n(P_n - 15\,Z_n) \leq 0 \tag{3}$$

Likewise, the food group constraints (ie, the percentage of energy provided per food group) were transformed by limiting the energy (in MJ) provided by each food group in the optimized diet by a lower and an upper limit corresponding to the 25th and 75th percentiles observed in the population distribution for that food group, respectively.

The global optimum of each model was determined by linear programming with use of these linear inequalities for the constraint on P:Z (Equation *3*) and the food group constraints.

Sensitivity analysis by nonlinear programming

After the global optimum for each model was achieved by linear programming, the strength of the different constraints was examined by sensitivity analysis in nonlinear programming. Nonlinear programming was preferred because it allowed a direct interpretation of the constraints usually expressed in the form of a ratio, such as the constraint on P:Z and the food group constraints.

Sensitivity analysis was used to examine how variations in each constraint would affect the minimal amount of energy required to satisfy all the constraints. A sensitivity report, providing Lagrange multipliers for nutritional and food consumption constraints, was generated after each optimization process. A Lagrange multiplier on a constraint measures the strength of the related constraint. A zero Lagrange multiplier indicates that this particular constraint will be automatically satisfied when all the other constraints are met. A nonzero Lagrange multiplier indicates that a one-unit change in the constraint limit will lead to an improvement in the objective function, equivalent to the value attained by the Lagrange multiplier. The direction of the unit change required to produce the expected improvement depends on whether the constraint is minimal or maximal (ie, a decrease in the limit for a minimal constraint or an increase for a maximal one). The interpretation of a Lagrange multiplier of 0.10 or −0.10, for example, on a food group constraint indicates that allowing a decrease or increase of 1% in the energetic contribution of the related food group constraint will decrease the minimum energy required to satisfy the constraint by 0.10 MJ.

To compare the relative strength of different nutritional constraints, Lagrange values must be standardized. This was achieved by standardizing the units of the nutritional constraints. To do this, the nutrient contents of the diet and the P:Z were expressed in tenths of the recommended daily intakes (or ratio). The lower limit for the whole diet and the upper limit for the P:Z was then set to 10 for each nutrient or for the P:Z. For example, for the P:Z, the following nonlinear inequality (Equation *4*) was used to estimate the strength of the constraint in nonlinear programming:

$$(10/15) \times [(W_1\,P_1 + W_2\,P_2 \ldots + W_n\,P_n)/(W_1\,Z_1 + W_2\,Z_2 \ldots + W_n\,Z_n)] \leq 10 \tag{4}$$

After standardization, a Lagrange multiplier of 0.10 or −0.10 on a nutritional constraint indicates that a decrease or increase in the constraint by 10% will decrease the minimum energy required to satisfy the constraints by 0.10 MJ. For example, a Lagrange multiplier of −0.10 on the constraint for P:Z indicates that an increase in the upper limit of the constraint from ≤15 to ≤16.5 will decrease the minimum energy required by 0.10 MJ.

Nonzero Lagrange values were used to define limiting nutrients and limiting food groups in the diet. Among the limiting nutrients, the one with the highest Lagrange value (in absolute value) is defined as the first limiting nutrient of the diet. Similarly,

TABLE 3

Minimum energy required to satisfy the same set of nutritional constraints in 2 seasons with different food consumption constraints[1]

	Food consumption constraints			Minimum energy required	
Model	Food group constraints	Foods eligible for analysis	Maximum weight of foods	Harvest season	Nonharvest season
				MJ	
1	No constraint	No constraint	No constraint	1.49	1.49
2	25th–75th percentiles[2]	No constraint	No constraint	2.09	2.80
3	25th–75th percentiles[2]	No constraint	90th percentile[3]	2.21	4.39
4	25th–75th percentiles[2]	No constraint	75th percentile[3]	2.33	4.54
5	25th–75th percentiles[2]	90th percentile[4] ≠ 0	90th percentile[3]	2.32	5.91
6	25th–75th percentiles[2]	90th percentile[4] ≠ 0	75th percentile[3]	2.51	6.42
7	25th–75th percentiles[2]	75th percentile[4] ≠ 0	90th percentile[3]	2.39	7.40
8	25th–75th percentiles[2]	75th percentile[4] ≠ 0	75th percentile[3]	2.62	Infeasible

[1] The nutritional constraints were the UK reference nutrient intakes (17) and the molar ratio of phytate to zinc in the diet.

[2] Percentiles of the percentage of energy contributed by a food group in the population.

[3] Percentiles of the consumer intake distribution (ie, distribution of quantities consumed by the children who consumed the food).

[4] Percentiles of the population intake distribution (ie, distribution of quantities consumed in the entire population).

the first limiting food group is the food group with the highest Lagrange value (in absolute value).

Effect of varying food group constraints

The strength of a constraint can also be displayed graphically by curves showing how variations in this constraint will modify the minimal energy required to satisfy all other constraints. This was done for each food group by changing its constraint from a range constraint (eg, percentage of energy from fruit $< 5\%$) to an equality constraint (eg, percentage of energy from fruit $= 5\%$). Different models were then generated in which only the equality constraint was changed. The effect of varying the food group constraint on the minimum energy required to meet all nutritional and food consumption constraints was then graphically displayed by plotting the results from each sequence of models per food group (ie, the minimal amount of energy required against the percentage of energy coming from this food group fixed at varying values). The nutritional constraints used in this analysis were the RNIs (17) and the P:Z. Only foods eaten by $\geq 10\%$ of the population were included in the analysis, and the maximum daily portion corresponded to the 90th percentile observed among consumers.

All optimization models were developed with the use of standard Microsoft EXCEL SOLVER both for linear and nonlinear programming (Frontline Systems, Inc, Incline Village, NV). The constraints used in each model are summarized in **Table 3**.

RESULTS

Minimum energy required to satisfy nutritional and food consumption constraints

When food consumption constraints were not included in the model (model 1), an unrealistic diet was obtained. The minimum energy required to satisfy only the nutritional constraints (ie, RNIs and P:Z) was only 1.49 MJ (356 kcal) (Table 3). The resultant diet was based on 5 foods selected from 3 of 6 food groups specified in the database: vegetables, legumes, and animal products (meat, fish, and eggs). In this highly nutrient-dense diet, the vegetable food group (172 g cassava leaf, 172 g Chinese cabbage, and 652 g tomato) was the major contributor of both total energy (76.8%) and total weight (97.9%) and contributed $> 100\%$ of the calcium, iron, folate, vitamin C, and copper needs. Legumes (14 g groundnut flour) and fish (7 g dry usipa) contributed 16.7% and 6.5% of the total energy, respectively.

The effect of strengthening the food consumption constraints on the minimum energy required to satisfy the nutritional recommendations is also shown in Table 3. These results showed that adding constraints to the proportion of energy provided by different food groups (ie, the 25th and 75th percentiles of intakes; model 2) increased the minimum energy required to satisfy the nutritional constraints from 1.49 to 2.09 MJ in the harvest season and to 2.8 MJ in the nonharvest season. Limiting the maximum weight of foods to the 90th (model 3) and the 75th (model 4) percentiles of the consumers' intake distribution further increased the minimum energy required to satisfy the RNIs and the P:Z, especially in the nonharvest season. Exclusion of foods consumed by $< 10\%$ (models 5 and 6) or 25% (models 7 and 8) of the entire population had little effect on the minimum energy required in the harvest season, but resulted in an important increase in the nonharvest season. In the nonharvest season, excluding foods consumed by $< 25\%$ of the population and limiting their weights to the 90th percentiles of the consumers' intake distribution (model 7) raised the minimal energy required to 7.40 MJ (1769 kcal), ie, more than the 6.80 MJ recommended for 4–6-y-old children (17). A solution was not achievable when the weights were further limited to the 75th percentiles of the consumers' intake distribution (model 8) in the nonharvest season. This finding indicated an incompatibility between the nutritional constraints and the food consumption constraints included in model 8.

For the same set of food consumption constraints, the minimum energy required to satisfy the nutritional constraints was always greater in the nonharvest than in the harvest season. This finding reflected the lower availability of some nutrient-dense foods, the different percentages of energy obtained from some food groups in the nonharvest than in the harvest season, or both (Table 2).

Composition of a diet satisfying both nutritional and food consumption constraints

The foods selected for the optimized diet with use of model 5 in the nonharvest season are shown in **Table 4**. The model included nutritional and food consumption constraints and

TABLE 4

Composition of an optimized diet satisfying nutritional and food consumption constraints in the nonharvest season[1]

Food group	Contribution of energy	Food	Weight
	%		*g*
Cereals	63.0[2]	Baked African cake[3]	110[4]
		Uncooked 95% extraction maize flour	98
		Uncooked rice	32
		Soaked uncooked sorghum	70[4]
Fruit	3.0[4]	Tangerine	77
Legumes	9.0	Boiled pigeon peas	21
		Boiled Bengal beans (kalangonda)	80[4]
Meat, fish, and eggs	6.0[4]	Uncooked dry matemba (small fish)[5]	2
		Uncooked fresh matemba	90[4]
Roots	16.0[4]	Roast potato	217
Vegetables	3.0[4]	Uncooked Chinese cabbage	160[4]
		Uncooked okra leaf	34

[1] The nutritional constraints were the UK reference nutrient intakes (17) and the molar ratio of phytate to zinc in the diet (<15). The food consumption constraints were based on model 5 (*see* Table 3): energetic contribution of food groups between the 25th and 75th percentiles of intakes in the population, only foods consumed by ≥10% of the population, and the maximum weight of foods was equal to the 90th percentile in consumers. The minimum energy required to satisfy these constraints was 5.91 MJ.

[2] Food group introduced at the imposed minimal limit.

[3] Maize flour, sugar, and water.

[4] Foods and food groups introduced at the imposed maximal limit.

[5] Small fish *Barbus paludinosus*.

provided 5.91 MJ (1412 kcal), which is close to the mean intake of energy actually consumed by these Malawian children, ie, 5.42 MJ (15). In this diet, the percentage of energy provided by cereals was at the minimum level imposed (ie, the 25th percentile of the cereal's energetic contribution in the children's diet). In contrast, fruit, roots, vegetables, and animal products (meat, fish, and eggs) were introduced at the maximum level (ie, the 75th percentile of the childrens' energetic contribution in the population). In each food group, some foods were introduced at the maximal weight allowed (ie, the 90th percentile of the consumers' distribution), including African cake, fresh matemba, Chinese cabbage, and boiled Bengal beans. Foods such as meat and eggs were excluded from the model because they were consumed by <10% of the population in the nonharvest season. The percentages of energy provided by protein, lipids, and carbohydrate in this diet were 15.2%, 8.6%, and 76.2%, respectively. In the actual diets of these Malawian children in the nonharvest season, the average percentage of energy provided by lipids was also low (ie, 7.8%). This optimized diet also contained an acceptable amount of absorbable zinc (2.5 mg) and absorbable iron (2.4 mg); it also had a high dietary fiber content (32.5 g).

Comparison of the effect of different nutritional recommendations on the minimum energy required

The minimum energy required and the strength of the individual constraints were compared between the 2 seasons for the 3 sets of recommendations with the use of model 5. As shown in **Table 5**, the minimum energy required to satisfy the UK RNIs was lower than that required to satisfy the published FAO/WHO recommendations and higher than that required to satisfy the preliminary FAO/WHO recommendations in both seasons.

In most of the optimized diets shown in Table 5, riboflavin had the highest Lagrange multiplier in absolute value, indicating that riboflavin was the first limiting nutrient irrespective of the set of recommendations or the season studied. The sole exception was the diet optimized in the harvest season with the preliminary FAO/WHO recommendations. For this diet, vitamin B-12 was the only limiting nutrient. Zinc, niacin, and the P:Z were the second limiting nutritional factors, depending on the set of recommendations used and the season studied. In the nonharvest season, however, the second limiting nutritional factor was always the P:Z, indicating that zinc was a possible second limiting nutrient.

The ranking of limiting food groups was different between the 2 seasons. In the harvest season, animal products (meat, fish, and eggs) was the first limiting food group, followed by vegetables or roots. In the nonharvest season, vegetables was the first limiting food group, followed by animal products. For the same set of recommendations and food consumption constraints, Lagrange values observed for nutrients or food groups were greater in the

TABLE 5

Limiting nutritional constraints, limiting food groups, and the minimum energy required to satisfy the same food consumption constraints with 3 different sets of nutritional constraints during 2 seasons[1]

	UK RNIs (17)		Published FAO/WHO recommendations (18–21)		Preliminary FAO/WHO recommendations (22)	
	Harvest season	Nonharvest season	Harvest season	Nonharvest season	Harvest season	Nonharvest season
Minimum energy required (MJ)	2.32	5.91	2.94	8.80	2.19	4.17
First limiting nutritional constraint	Riboflavin	Riboflavin	Riboflavin	Riboflavin	B12	Riboflavin
Lagrange value (MJ)	0.12	0.75	0.28	1.06	0.22	0.41
Second limiting nutritional constraint	Zinc	P:Z	Niacin	P:Z	None	P:Z
Lagrange value (MJ)	0.11	−0.60	0.02	−0.93	—	−0.16
First limiting food group	Meat, fish, and eggs	Vegetables	Meat, fish, and eggs	Vegetables	Meat, fish, and eggs	Vegetables
Lagrange value (MJ)	−0.02	−0.52	−0.06	−0.78	−0.31	−0.27
Second limiting food group	Vegetables	Meat, fish, and eggs	Root	Meat, fish, and eggs	None	Meat, fish, and eggs
Lagrange value (MJ)	−0.01	−0.30	−0.04	−0.46	—	−0.12

[1] The nutritional constraint limiting the molar ratio of phytate to zinc (P:Z) in the diet to 15 was used in all the analyses. The food consumption constraints were based on model 5 (*see* Table 3): the energetic contribution of food groups between the 25th and the 75th percentiles of intakes in the population, only foods consumed by ≥10% of the population, and the maximum weight of foods was equivalent to the 90th percentile in the consumer intake distribution. RNI, reference nutrient intakes; FAO, Food and Agriculture Organization; WHO, World Health Organization.

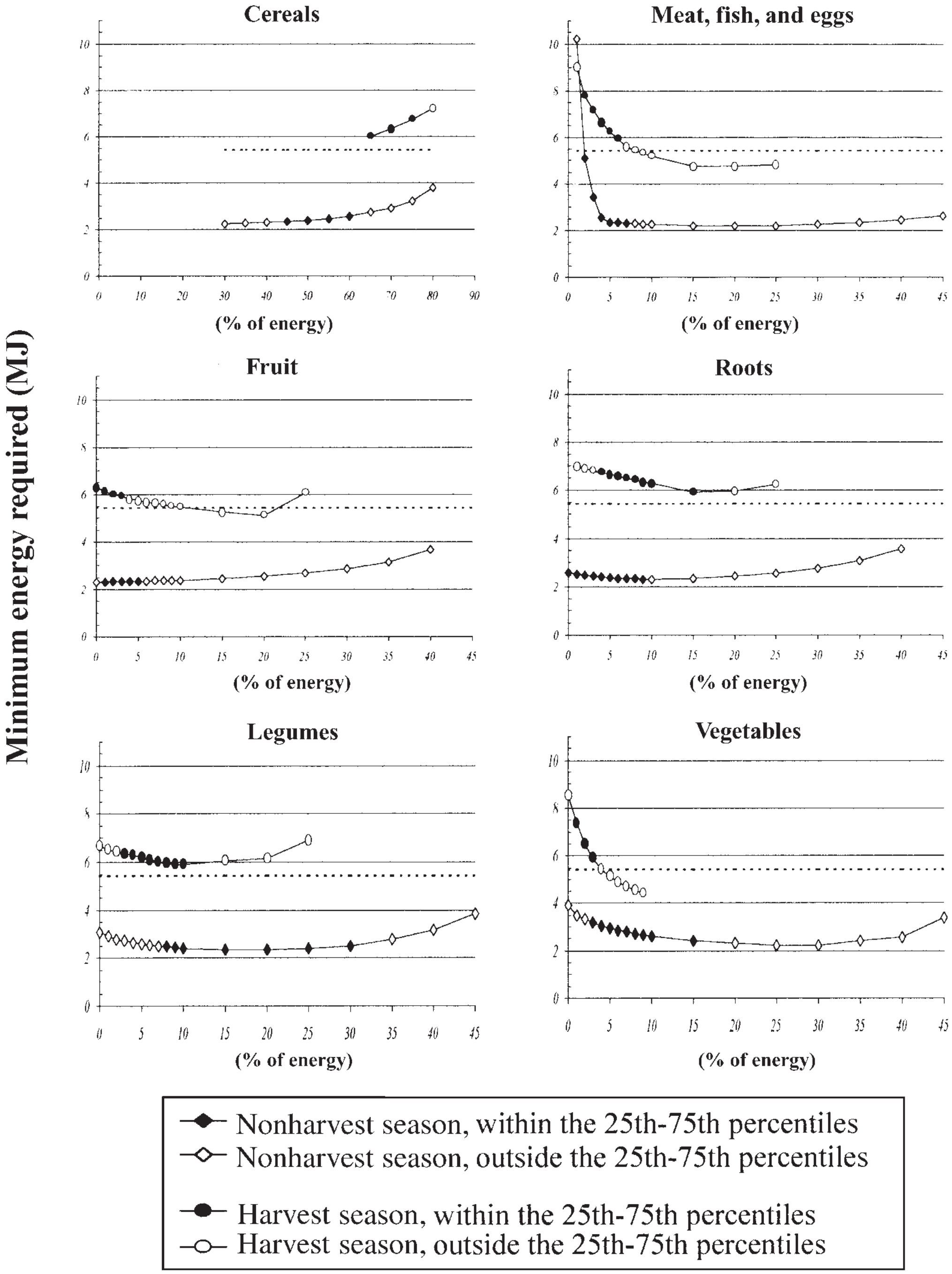

FIGURE 1. The minimum energy required to satisfy the nutritional constraints [reference nutrient intakes (17) and the dietary ratio of phytate to zinc] at different percentages of energy from specific food groups. The energy contribution of all other food groups was kept between the observed 25th and 75th percentiles. Only foods eaten by ≥10% of the population were included in the analysis and had a maximal weight corresponding to the 90th percentile observed among consumers. The dotted line represents the average energy intake actually consumed by the Malawian children (5.4 MJ) (15). The absence of points indicates that an optimization solution was not possible at that percentage of energy contributed by that food group.

nonharvest than in the harvest season. This finding indicated that constraints were more severe in the nonharvest season.

Differential effect of food groups on the minimum energy required to satisfy constraints

The effect of varying a specific food group constraint on the minimum energy level required to satisfy the nutritional and food consumption constraints of model 5 is shown for each food group by season in **Figure 1**. Each curve was compared with the dotted line, representing the average energy intake actually consumed by Malawian children (5.4 MJ) (15). For example, when the energy contribution of vegetables was fixed to 4% in the nonharvest season (ie, above the 75th percentile observed in the population), the energy required to fulfill all constraints was just equal to this expected energy intake. Overall, the minimum energy intake required to meet all constraints in the harvest season was consistently below the average energy intake observed in the population when ≥2% of energy was provided by meat, fish, and eggs. In contrast, the minimum energy required was consistently >5.4 MJ in the nonharvest season unless the energy contributed by vegetables or meat, fish, and eggs was above the 75th percentile observed in the population. For other food groups, unlike vegetables or meat, fish, and eggs, the graphic presentation shows that making the upper constraint less strict (ie, >75th percentile) did not result in an improvement in the optimized diet (ie, lower energy intake). Therefore, as already suggested by the Lagrange multipliers, this graphic analysis showed that animal products (meat, fish, and eggs) and vegetables were the food groups that had the greatest effect on the minimal energy required to satisfy the constraints in this population.

DISCUSSION

The results of this study showed that linear and nonlinear programming can be used to assist in the formulation of nutritional recommendations with the use of data from local food consumption surveys. The findings in Malawi confirmed that low riboflavin (24), low zinc, and high phytate intakes (15) are common problems in poor African communities. More importantly, the findings indicate that an increase in the consumption of vegetables and foods of animal origin greatly improves the nutrient density of the diet during both the harvest and nonharvest seasons. It is apparent that nutrition education can improve the diets of children in Malawi during the harvest season, provided that nutrient-dense foods are affordable and that the optimized minimum energy intake (ie, 2.32–2.62 MJ) is met. On the other hand, the analysis suggested that nutritional inadequacy will persist during the nonharvest season, irrespective of the combination of local foods, because of the low availability of riboflavin- and zinc-rich animal or vegetable foods and the high-phytate content of other foods. This is a problem that is even more serious for children who consume less than the average energy intake reported here. For this reason, these results suggest that changes in the range of foods available might be needed to provide a nutritionally adequate diet in the nonharvest season.

The applicability of linear programming depends on the validity of the nutritional constraints introduced into individual models. These constraints were introduced to ensure that optimized diets meet the nutrient needs of most people in the population. However, the constraints were based on assumptions that can be challenged, including assumptions that the recommendations used were appropriate for the population, the population is healthy, bioavailability factors are correct, nutrient requirements are independent, the food-composition database is accurate, and nutrients and dietary factors not included in the constraints are unimportant for dietary adequacy (ie, will have Lagrange values of 0).

The effect of choosing a particular set of recommendations can be evaluated by using different sets of nutritional constraints based on recommendations from different committees to compare results (Table 5). As shown in the present study, riboflavin was consistently identified as the first limiting nutrient—irrespective of recommended intakes—and animal products and vegetables as the limiting food groups. This consistency suggests that the analysis was relatively robust, ie, relatively insensitive to the nutritional constraints selected. It does, however, assume that riboflavin intakes of ≥0.6 mg/d (ie, the lowest recommended take) are required for optimal health and nutritional status, an affirmation that has been challenged (25).

In the present analysis it was assumed that the absorption of nutrients was independent of their food origin. There is increasing evidence, however, that this is not the case for many nutrients, such as calcium (26), iron (27), zinc (28), and carotene-derived retinol (29). Also, in the present study, the P:Z and formula used to confirm adequate intakes of bioavailable iron and zinc were only crude proxies of bioavailability. In future studies, more elaborate nonlinear programming models, including complex mathematical formulas of nutrient availability such as those recently proposed for iron (30), will reduce these limitations. Another noteworthy limitation is that dietary factors not included in the nutritional constraints might prove problematic, ie, the dietary factors will have nonzero Lagrange values if included. For example, a maximum dietary fiber intake was not included in the nutritional constraints; however, the dietary fiber content of the optimized diets was above the upper intake usually recommended for children (ie, the child's age plus 10 g/d) (31) and often above the mean intake observed in Malawian children (ie, 24.9 g/d) (15). Also, inclusion of constraints on essential fatty acids and vitamin E may have increased the difficulty of designing nutritionally adequate diets, notably because of the observed low availability and consumption of vegetable oil in this group of Malawian children.

An assumption was also made that the requirement for each nutrient is independent of the intakes of energy and other nutrients in the optimized diet. However, there are many examples of nutrient interactions, such as a possible effect of riboflavin (24) and vitamin A (32) on iron metabolism and the need for zinc for vitamin A metabolism (33). Nevertheless, the clinical effect of these interactions will be low in the optimized diet because adequate intakes of all nutrients are ensured. However, all nutrients were assigned an equivalent weighting, which is a simplistic interpretation of dietary adequacy. A recommended intake is usually derived from the estimated average requirement plus 2 SDs to ensure that the requirements of 95% of the population are met (34). If the CV in the requirement distribution is not identical across all nutrients, it is incorrect to assign the same weight to the same deviation (expressed in tenths) from recommended intakes for different nutrients. In addition, in physiologic terms, an insufficient intake of some nutrients may have more severe functional consequences than will an inadequate intake of other nutrients. In the present study, for example, riboflavin was consistently identified as the first limiting nutrient. However, until evidence of negative health consequences of suboptimal riboflavin status is shown in this population, these results must be interpreted carefully. In contrast, evidence of zinc deficiency (ie, the second limiting nutrient) does exist for this population (15). Zinc deficiency in children was shown previously to increase the risk of persistent diarrhea and pneumonia, 2 major causes of morbidity and mortality in poor African communities (35). As our understanding of nutrient needs for different populations improves (ie, requirements, interaction, and health consequences), some of these limitations can be accounted for in future models, notably when standardizing nutrients units in the analysis.

The applicability and validity of linear programming in nutrition studies is also dependent on the availability of data for defining food consumption constraints. One of the limitations of the above analysis is the need for detailed food consumption data to ensure the palatability of the optimized diets (6). Nevertheless, these data may not be essential. In Colombia, linear programming was used to identify a "food basket" providing a low-cost diet that supplied recommended energy and protein intakes for the average family (36). Simple information on family food purchases was used to design this diet. Similar pragmatic approaches could also be used, such as interviewing key informants or measuring the food consumption of a limited number of children to quickly identify foods and the maximum quantities effectively eaten by children during different seasons. Alternatively, published daily portions could be used, such as those available in the WHO document on complementary feeding for different age groups in Peru and Mexico (37). In the future, an international database of food constraints (ie, daily portions and food group patterns) could be compiled to help circumvent the need for detailed data from food consumption surveys if linear programming were widely used.

The results of the present study strengthen field observations showing that cereal-based diets low in animal products, vegetables, and fruit cannot meet the nutritional recommendations for children (1). These diets are consistent with poor growth in Malawian children (15) and in children fed macrobiotic diets devoid of animal products (38). The results emphasize the importance of vegetable consumption in meeting the nutritional needs of children to ensure optimal health status, as recently suggested in Sudanese children (39).

As shown in the present study, linear programming can be used to identify dietary patterns and limiting nutrients and to assess whether a nutritionally adequate diet is achievable with locally available foods in different seasons. It can also be used to identify combinations of foods and portion sizes needed to achieve a nutrient-dense diet and desirable modifications to observed food patterns. Thus, we strongly recommend the use of linear programming in designing alternative nutrition intervention strategies such as nutrition education, food fortification, and agricultural projects, keeping in mind the limitations outlined above. In addition, the inclusion of cost constraints in the models may further increase the strength of the analysis, especially when a decision is required among alternative intervention strategies.

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