

# Lecture 13 Special class for exercise in Chapter 1

**Example 1** If for any  $x \in D$ , domain of  $f(x)$   $\text{sgn}(f(x)) = 1$  (or  $\text{sgn}(f(x)) = -1$ ), then the constant  $T$  is period of  $f(x)$  if and only if it is a period of  $f^2(x)$ .

**Hint** The proof follows from the following equality:

$$f^2(x+T) - f^2(x) = (f(x+T) - f(x))(f(x+T) + f(x)).$$

**Example 2** Show that  $\sin x^2$  is not a periodic function.

**Proof** We prove this by contradiction.



Suppose not. Then there exists a constant  $T \neq 0$  such that

$$\sin(x+T)^2 = \sin x^2 .$$

This shows that  $(x+T)^2 = x^2 + n\pi$ , where  $n \in \mathbb{Z}$ .

It follows that  $T = 0$ . This is the desired contradiction.

