
平面图形
几何性质
北方交通大学
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主要内容及重点：

静矩与形心

惯性矩和惯性积

平行移轴定理

转轴公式

几何性质：仅与截面形状、尺寸相关的几何量

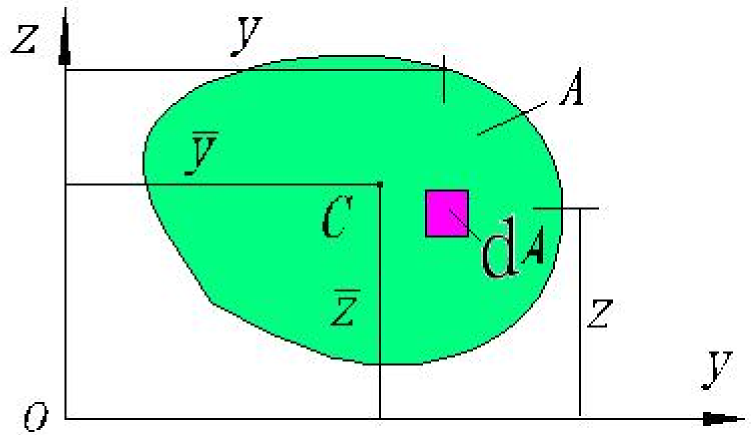
无物理意义

对杆件的强度、刚度等有重要影响

1.静矩和形心

静矩：$S_Z=\int_A ydA$

—面积A对z轴的静矩



$$S_y=\int_A zdA$$

—面积A对y轴的静矩

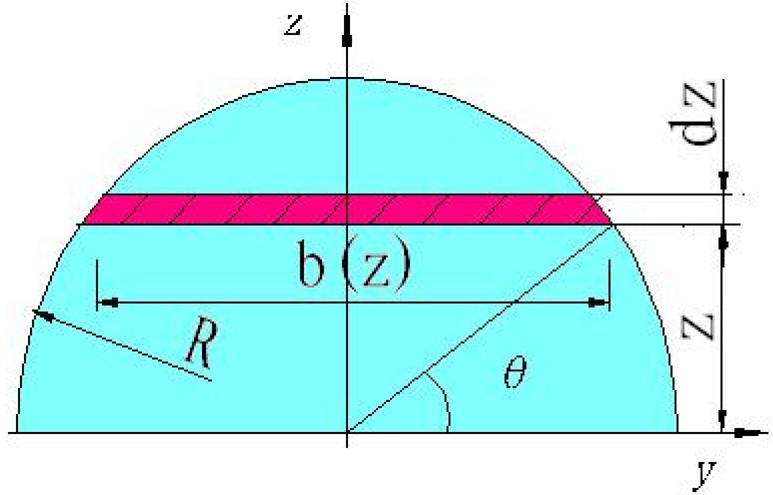
形心坐标：$\bar{y}=\frac{\int_A ydA}{A}=\frac{S_Z}{A}$ $\bar{z}=\frac{\int_A zdA}{A}=\frac{S_y}{A}$

$$S_z=\bar{y}A \qquad S_y=\bar{z}A$$

Sz=0 则 z轴过形心；
Sy=0 则 y轴过形心。

例题 求半圆形对y轴的静矩Sy及半圆形心

$$S_y = \int_A z dA = \frac{2R^3}{3}$$



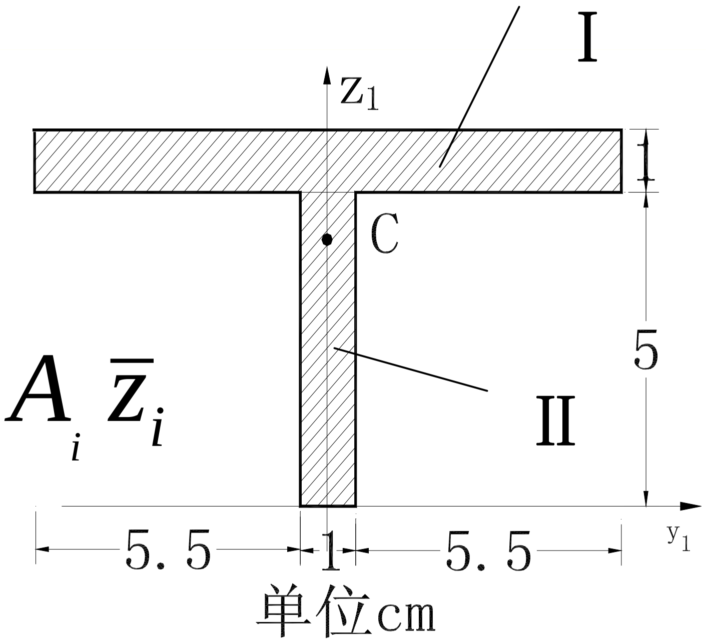
$$\bar{y} = 0$$

$$\bar{z} = \frac{\int_A z dA}{A} = \frac{S_y}{A} = \frac{4R}{3\pi}$$

$$S_y = \int_A z dA$$

$$\bar{y} = \frac{\int_A y dA}{A} = \frac{S_z}{A} \qquad \bar{z} = \frac{\int_A z dA}{A} = \frac{S_y}{A}$$

组合截面静矩



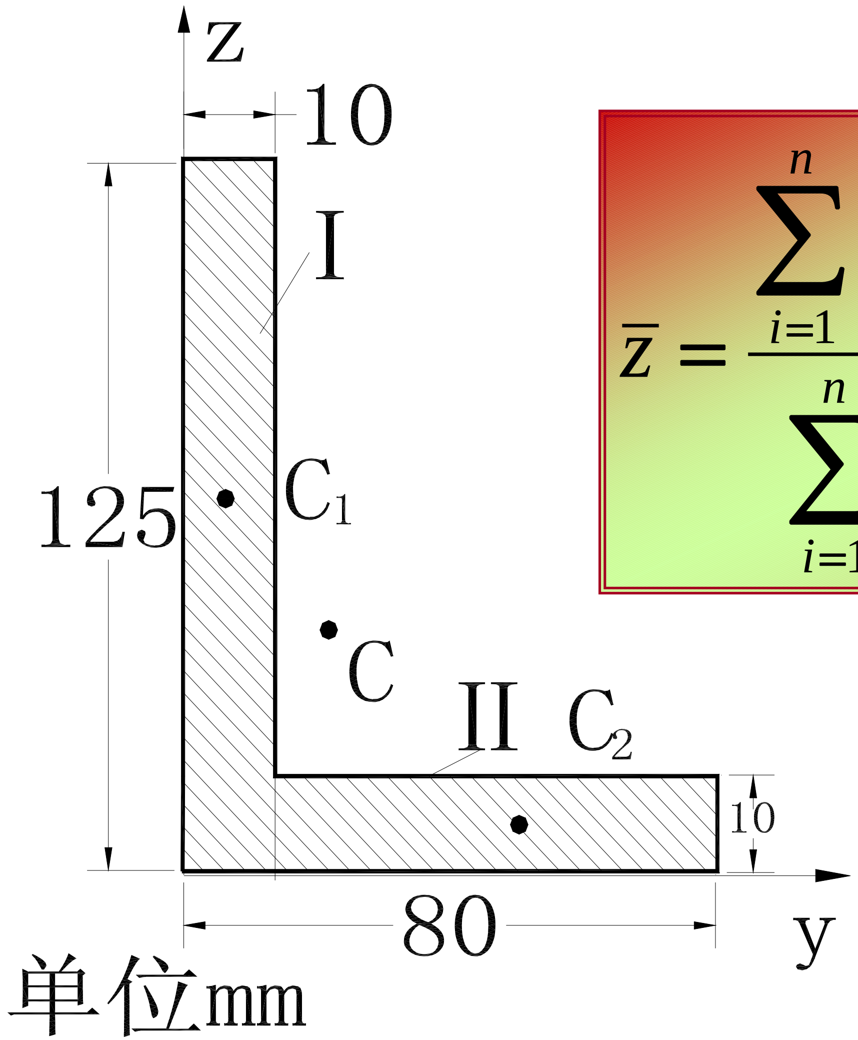
$$S_Z = \sum_{i=1}^{n} A_i \bar{y}_i \qquad S_y = \sum_{i=1}^{n} A_i \bar{z}_i$$

$$\bar{z} = \frac{\sum_{i=1}^{n} A_i \bar{z}_i}{\sum_{i=1}^{n} A_i} \qquad \bar{y} = \frac{\sum_{i=1}^{n} A_i \bar{y}_i}{\sum_{i=1}^{n} A_i}$$

$$S_z = \bar{y}A \qquad S_y = \bar{z}A$$

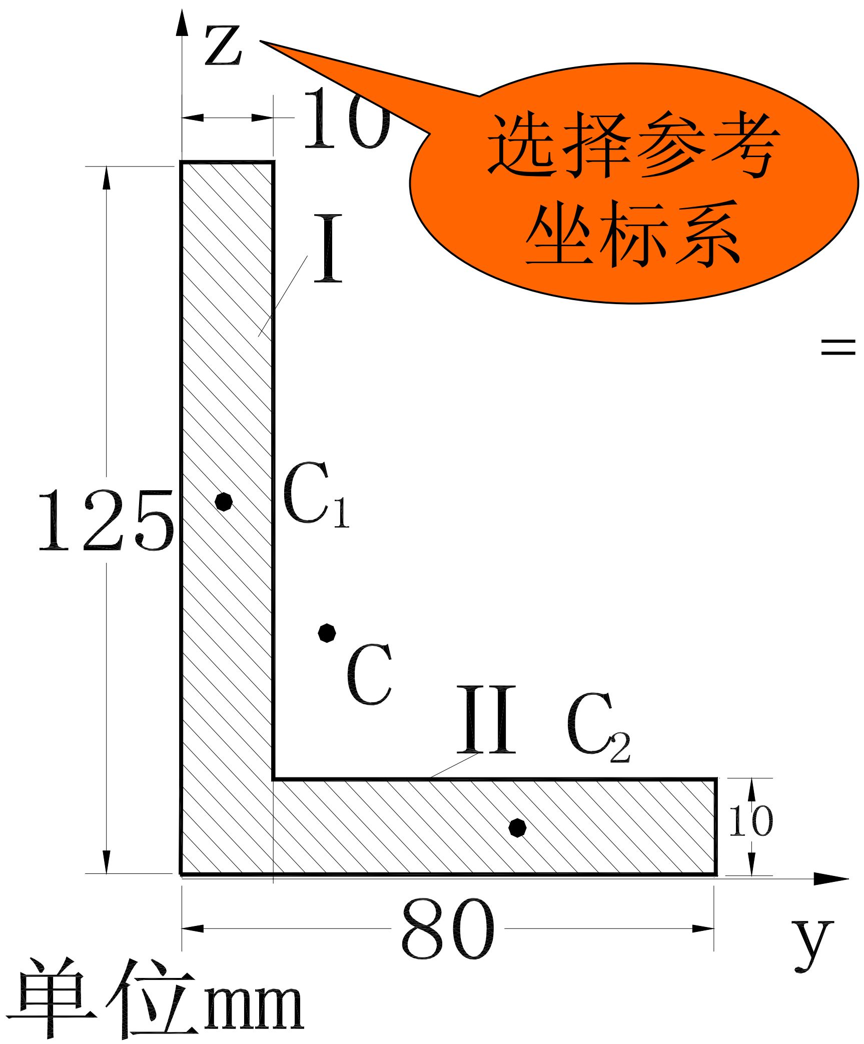
$$\bar{y} = \frac{\int_A y dA}{A} = \frac{S_Z}{A} \qquad \bar{z} = \frac{\int_A z dA}{A} = \frac{S_y}{A}$$

例题 L形截面尺寸如图所示,求其形心位置。



$$\bar{z} = \frac{\sum_{i=1}^{n} A_i \bar{z}_i}{\sum_{i=1}^{n} A_i} \qquad \bar{y} = \frac{\sum_{i=1}^{n} A_i \bar{y}_i}{\sum_{i=1}^{n} A_i}$$

例题 L形截面尺寸如图所示,求其形心位置。



$$\bar{y}=\frac{\sum_{i=1}^{n}A_i\bar{y}_i}{\sum_{i=1}^{n}A_i}=\frac{1250\times5+700\times45}{1250+700}$$

$$=19.36mm$$

$$\bar{z}=\frac{\sum_{i=1}^{n}A_i\bar{z}_i}{\sum_{i=1}^{n}A_i}$$

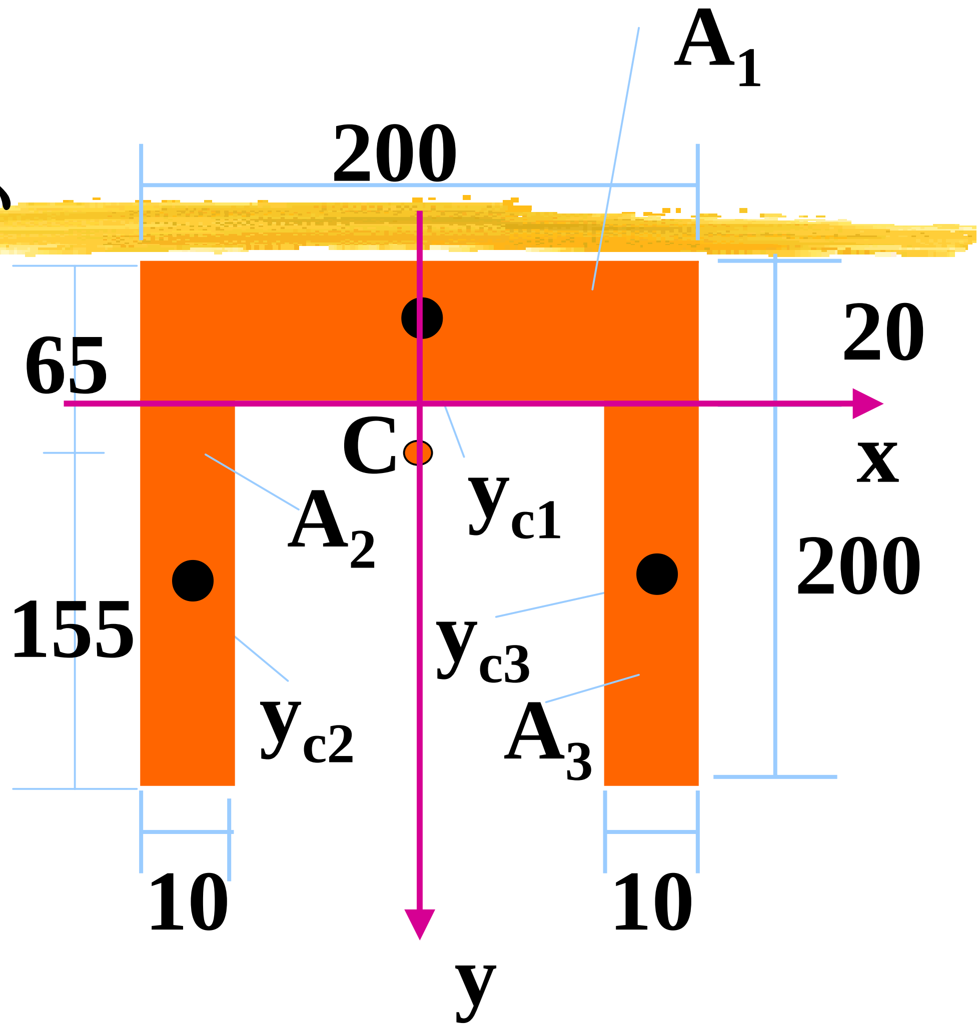
$$=\frac{1250\times62.5+700\times5}{1250+700}$$

$$=41.9\text{cm}$$

例题

求图示图形的形心

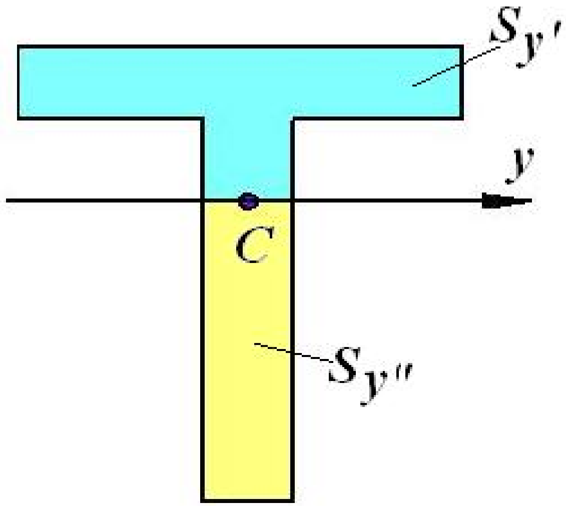
$$x_c = 0$$



$$y_c = \frac{A_1 y_{c1} + A_2 y_{c2} + A_3 y_{c3}}{A_1 + A_2 + A_3}$$

$$y_c = \frac{200\times 20\times(-10) + 200\times 10\times 100\times 2}{200\times 20 + 200\times 10\times 2} = 45$$

思考： 图示T形截面，形心轴y上下面积对形心轴y静矩s_y′与s_y″间的关系?



$$s'_y = s''_y$$

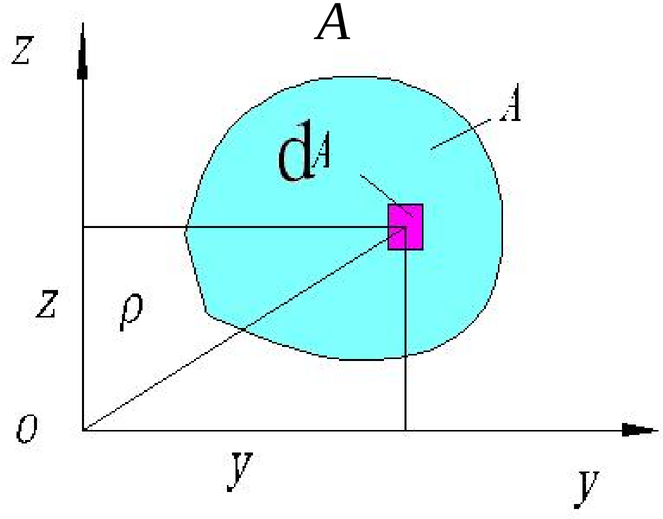
$$S_Z = \sum_{i=1}^{n} A_i \bar{y}_i \qquad S_y = \sum_{i=1}^{n} A_i \bar{z}_i$$

2.惯性矩 惯性半径 惯性积

$$I_z = \int_A y^2 \mathrm{d}A$$ —面积A对z轴的惯性矩

$$I_y = \int_A z^2 \mathrm{d}A$$ —面积A对y轴的惯性矩

$$I_P = \int_A \rho^2 \mathrm{d}A$$ —面积A对该坐标系的极惯性矩

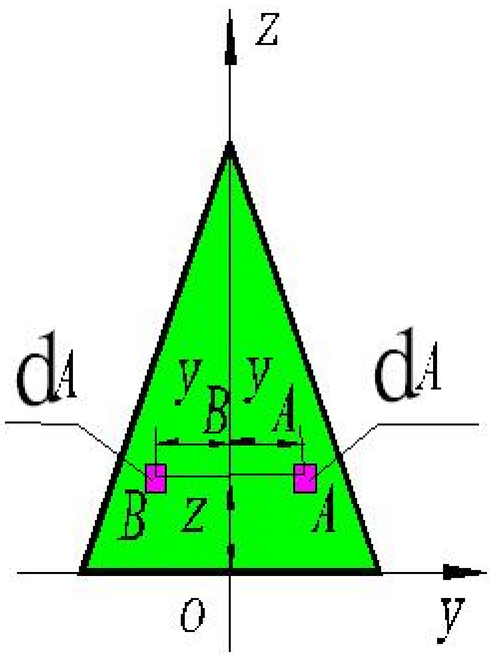


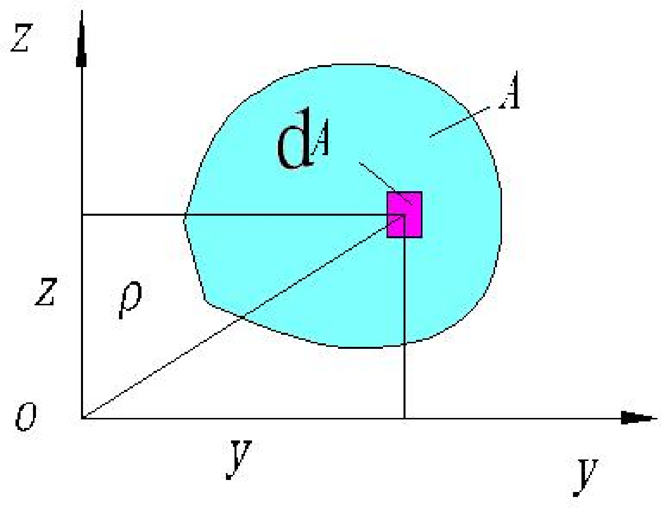
$$I_P = I_z + I_y$$

$$I_y = i_y^2 A \qquad i_y = \sqrt{\frac{I_y}{A}}$$ —面积A对y轴的惯性半径

$$I_z = i_z^2 A \qquad i_z = \sqrt{\frac{I_z}{A}}$$ —面积A对Z轴的惯性半径

$$I_{yz} = \int_A yz\mathrm{d}A$$ —面积A对该坐标系的惯性积



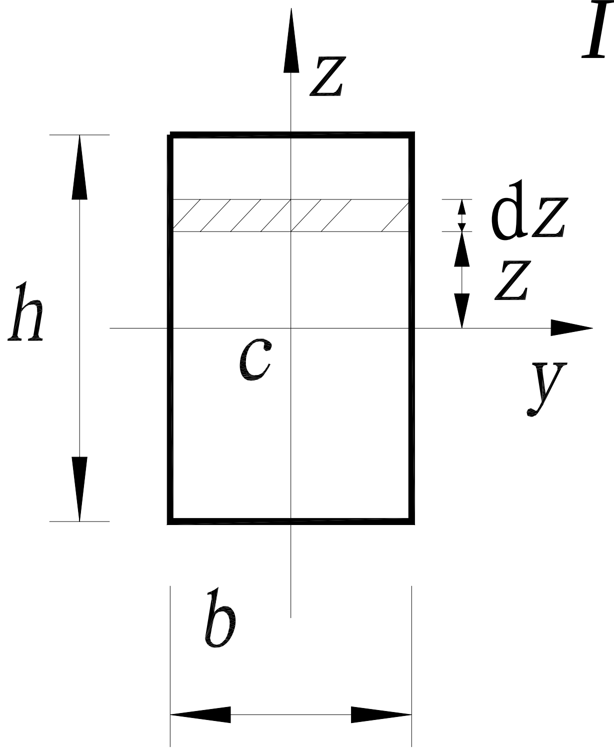


例题 求矩形截面对其形心轴y、z的惯性矩。

Iy: $dA = b\,dz$

$$I_y = \int_A z^2 dA = \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 b\,dz = \frac{bh^3}{12}$$

$$I_z = \int_A y^2 dA = \int_{-\frac{b}{2}}^{\frac{b}{2}} y^2 h\,dy = \frac{b^3 h}{12}$$

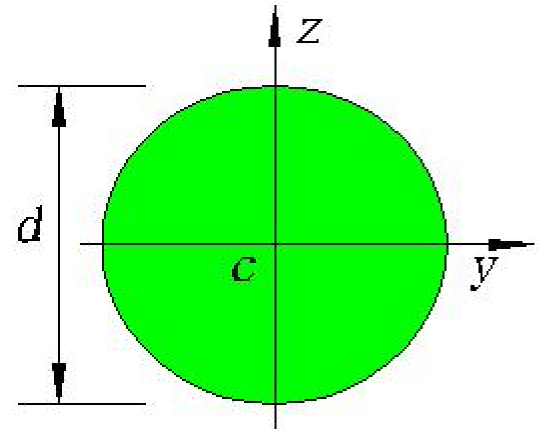


$$I_y = \int_A z^2 dA$$

$$I_z = \int_A y^2 dA$$

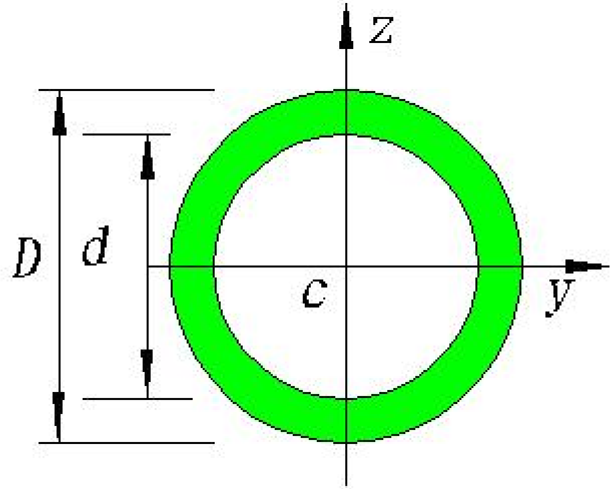
例题 求圆形截面对过形心*C*的***y***、***z***轴的惯性矩。

$$I_P=\frac{\pi d^4}{32}$$



$$I_P=I_y+I_z \qquad I_y=I_z$$

$$I_y=I_z=\frac{I_P}{2}=\frac{\pi d^4}{64}$$



$$I_y=I_z=\frac{\pi D^4}{64}(1-\alpha^4)$$

$$\alpha=\frac{d}{D}$$

3.平行移轴公式

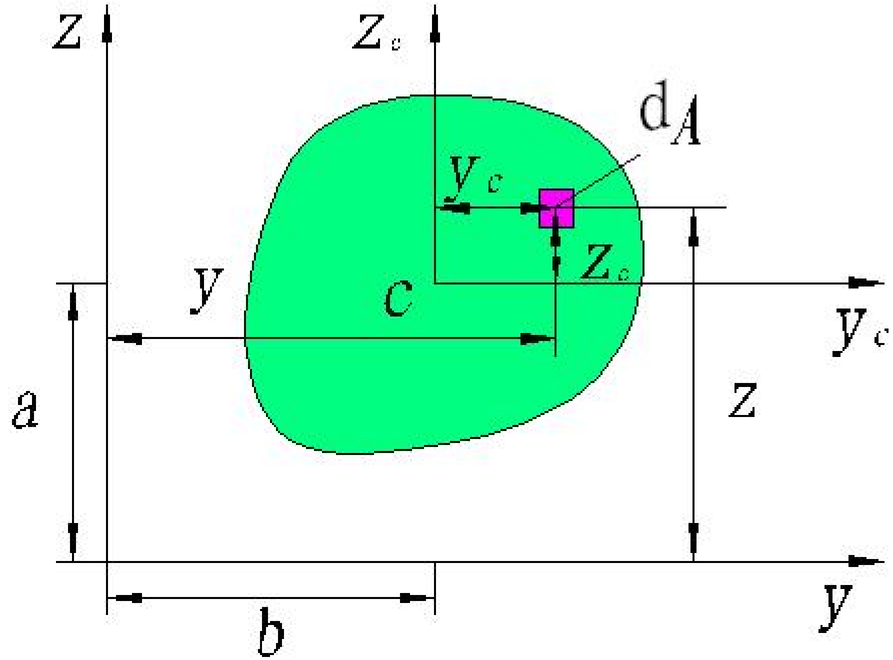
$$y = y_C + b$$

$$z = z_C + a$$

$$I_y = \int_A (z_C + a)^2 \mathrm{d}A$$

$$= \int_A z_C{}^2 \mathrm{d}A + \int_A 2az_C \mathrm{d}A$$

$$+ \int_A a^2 \mathrm{d}A$$



$$I_y = I_{yc} + a^2 A$$

$$I_z = I_{zc} + b^2 A$$

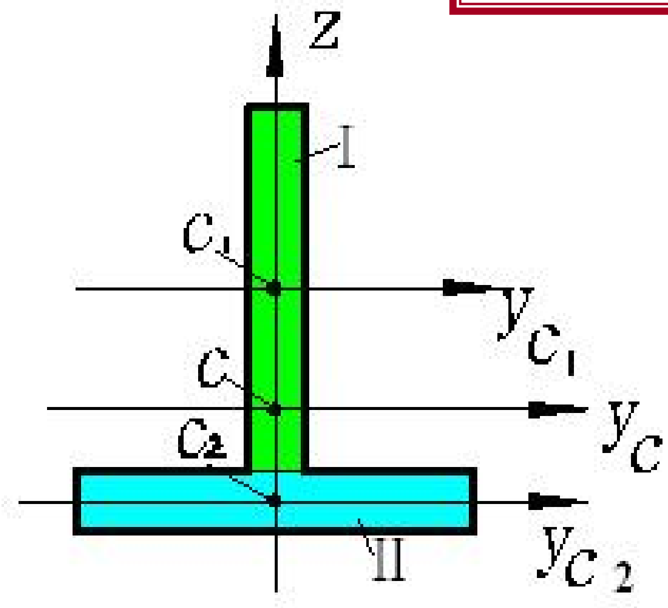
$$I_{yz} = I_{yczc} + abA$$

—平行移轴公式

对于组合截面图形：

$$I_y = I_{yc} + a^2 A$$

$$I_z = I_{zc} + b^2 A$$

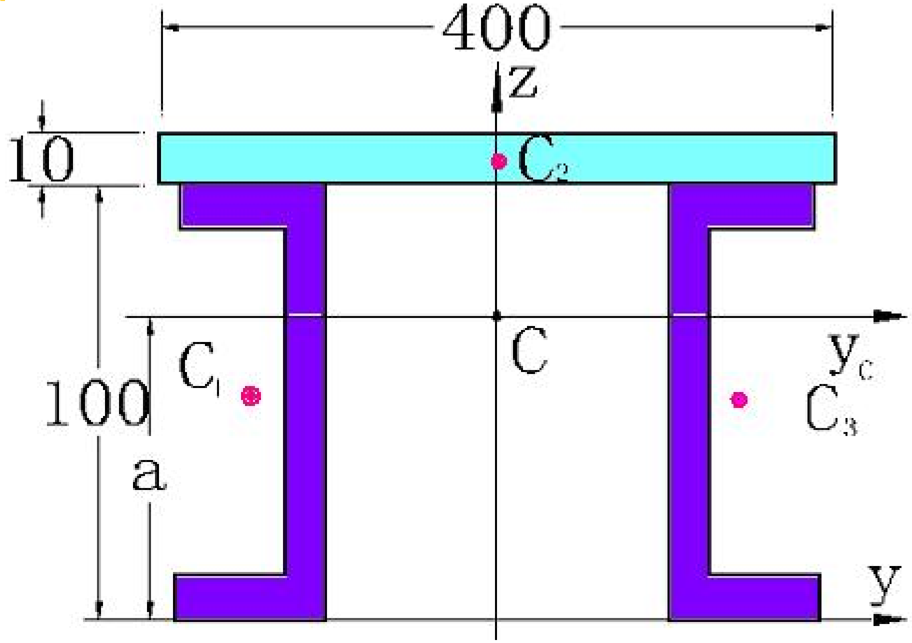


$$I_y = \sum I_{yC_i} + \sum a_i^2 A_i$$

$$I_z = \sum I_{zC_i} + \sum b_i^2 A_i$$

例题　图示截面由两个**10**号槽钢和一个**400mm×10mm**板钢组成，计算此截面对截面形心轴$\mathbf{y_c}$的惯性矩。

$$\bar{z}=\frac{\sum_{i=1}^{n} A_i \bar{z}_i}{\sum_{i=1}^{n} A_i} \qquad \bar{y}=\frac{\sum_{i=1}^{n} A_i \bar{y}_i}{\sum_{i=1}^{n} A_i}$$



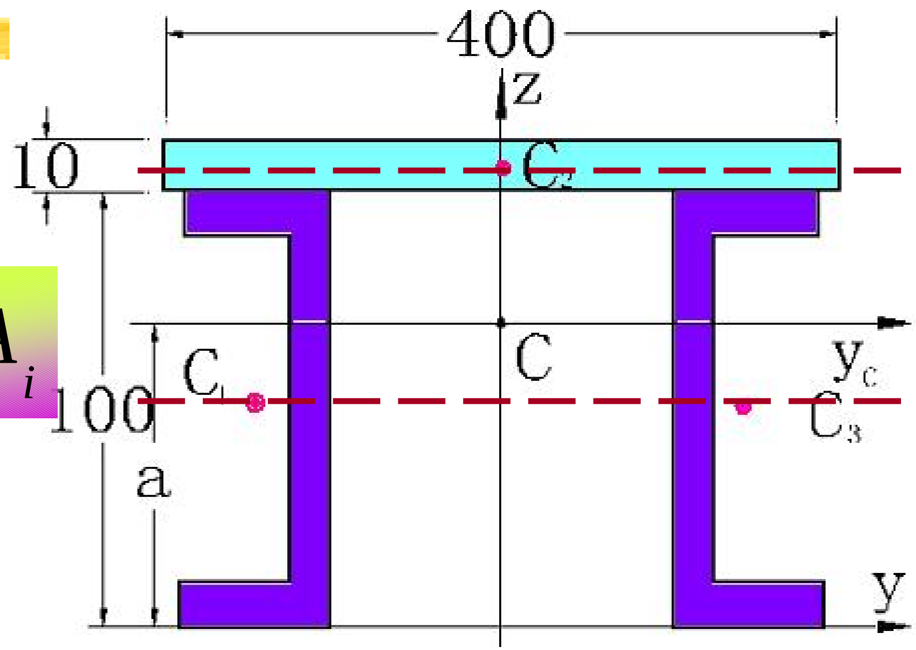
1. 选取**y**轴作为参考坐标轴求组合图形的形心：

$$a=\frac{\sum S_y}{\sum A}=83.5mm$$

例题　图示截面由两个10号槽钢和一个400mm×10mm板钢组成，计算此截面对截面形心轴y_c的惯性矩。

2. 求I_{yC}

$$I_y=\sum I_{yC_i}+\sum a_i^2 A_i$$

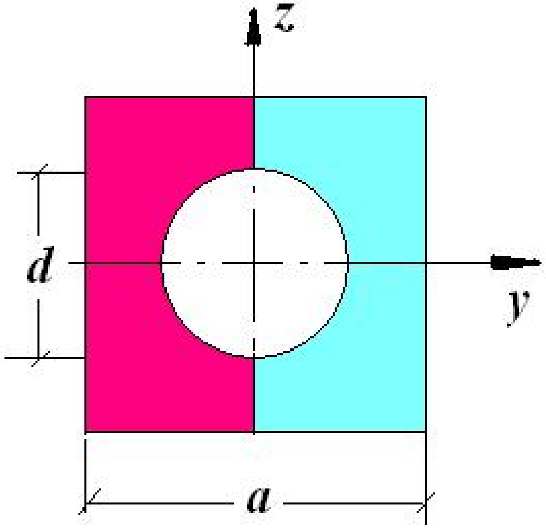


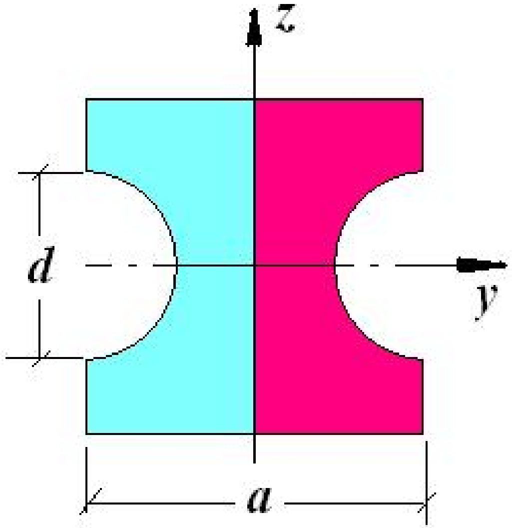
$$I_{yc}=\sum I_{yci}=\left[198\times10^4+(83.5-50)^2\times12.748\times10^2\right]\times2$$

$$+\frac{1}{12}400\times10^3+(105-83.5)^2\times400\times10=870\times10^4 mm^4$$

思考：两图形对各自形心轴y、z的惯性矩I_{ya}与I_{yb}、I_{za}与I_{zb}间的关系？

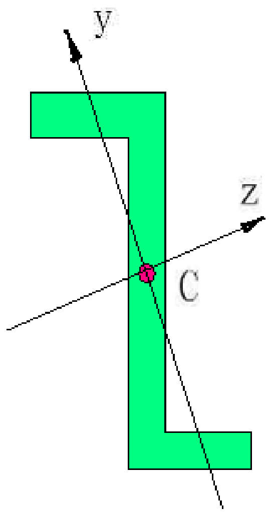
$$I_{ya} = I_{yb} \qquad I_{za} > I_{zb}$$

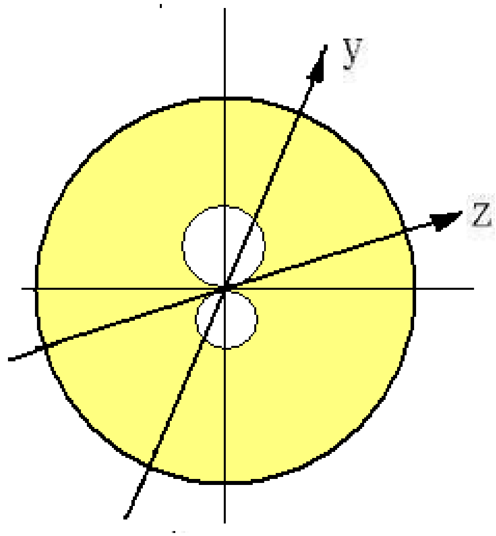




$$I_y = \int_A z^2 \mathrm{d}A \quad I_z = \int_A y^2 \mathrm{d}A$$

直观判断以下图形惯性矩I_y、I_z大小的关系





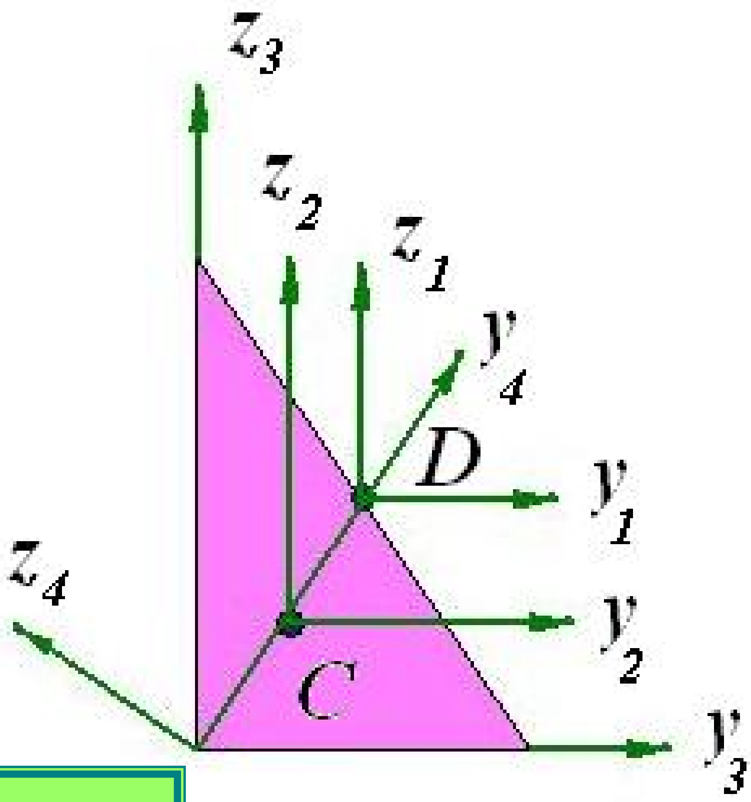
$$I_y < I_z$$

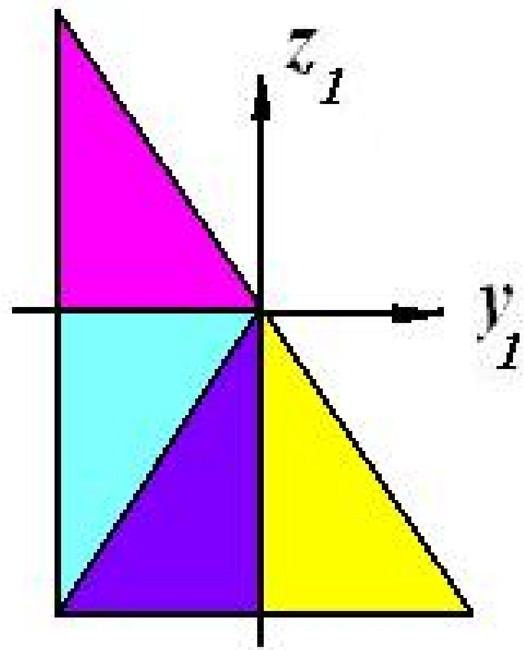
$$I_y > I_z$$

$$I_y = \int_A z^2 \mathrm{d}A \quad I_z = \int_A y^2 \mathrm{d}A$$

思考：若C为图形形心、D为斜边中点，试判断图形对哪一对坐标轴的惯性积为零？

$$y_1 - z_1$$



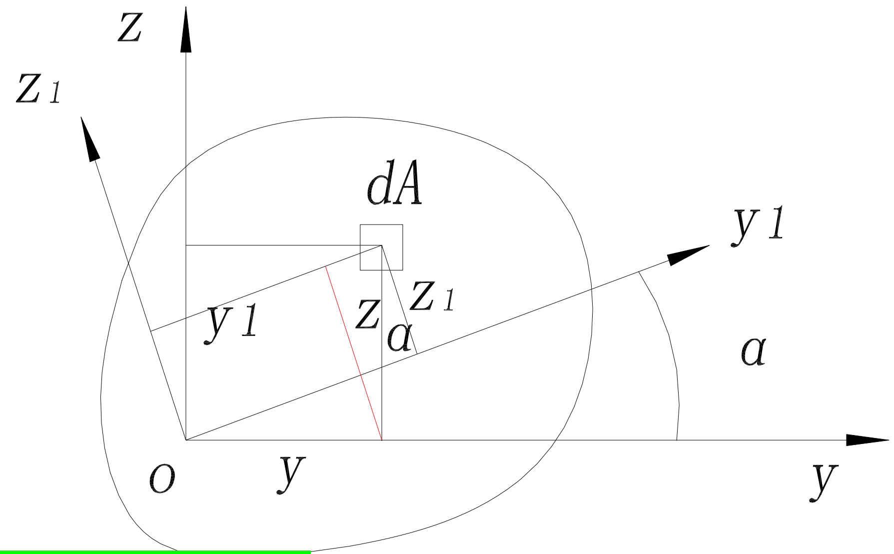


$$I_{yz} = \int_A yz\mathrm{d}A$$

4.转轴公式　主惯性轴

$y_1 = y\cos\alpha + z\sin\alpha$

$z_1 = z\cos\alpha - y\sin\alpha$



$$I_{y1} = \int_A z_1^2 dA$$

$$= \int_A (z\cos\alpha - y\sin\alpha)^2 dA$$

$$= \frac{I_y + I_z}{2} + \frac{I_y - I_z}{2}\cos 2\alpha - I_{yz}\sin 2\alpha$$

$$I_{z1} = \frac{I_y + I_z}{2} - \frac{I_y - I_z}{2}\cos 2\alpha + I_{yz}\sin 2\alpha$$

—转轴公式

$$I_{y1z1} = \frac{I_y - I_z}{2}\sin 2\alpha + I_{yz}\cos 2\alpha$$

设当α＝α$_0$时，I_{y1}有极值

$$\tan 2\alpha_0 = -\frac{2I_{yz}}{I_y - I_z}$$

惯性积为零 → 主惯性轴

主惯性矩

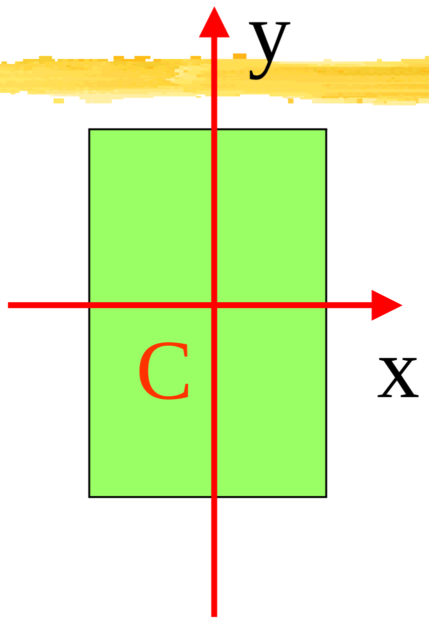
$$I_{max} = \frac{I_y + I_z}{2} + \frac{1}{2}\sqrt{\left(I_y - I_z\right)^2 + 4I_{yz}}$$

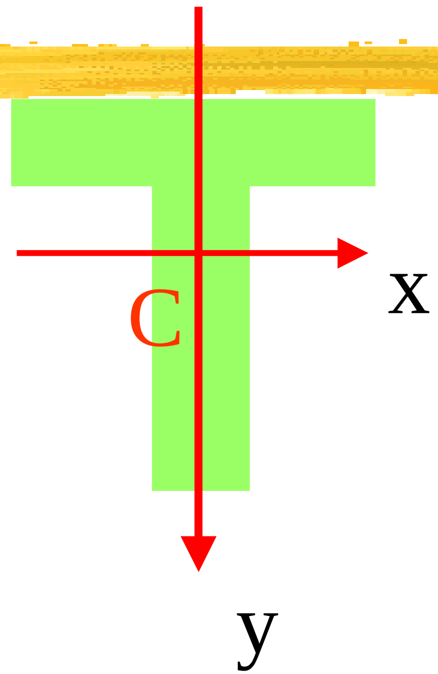
$$I_{min} = \frac{I_y + I_z}{2} - \frac{1}{2}\sqrt{\left(I_y - I_z\right)^2 + 4I_{yz}}$$

形心主惯性轴：过形心的主惯性轴

形心主惯性矩：

形心主惯性平面：由形心主惯性轴与杆件轴线组成的纵向平面





对称轴一定是主惯性轴

主惯性轴不一定是对称轴