Lecture 36 Special class for exercises in Chapters 19 and 20

Example 1 (Page 298: 10) Find the volume of the solid in R^3 bounded by

$$z = \frac{h}{R}\sqrt{x^2 + y^2}, \quad z = 0 \text{ and } x^2 + y^2 = R^2.$$

Solution We know that

$$V = \frac{h}{R}\iint_D \sqrt{x^2 + y^2}\,dxdy,$$

where $D=\{(x,y):x^2+y^2\le R^2\}$.

Let $\begin{cases}x=r\cos\theta\\ y=r\sin\theta\end{cases}$. Then $|J|=r$ and

$$V=\frac{h}{R}\int_0^{2\pi}d\theta\int_0^1 r^2dr=\frac{2}{3}\pi R^2h.$$

Example 2 (Page 309: 2(1)) Suppose

$$I=\int_0^1dx\int_0^{1-x}dy\int_0^{x+y}f(x,y,z)dz.$$

Find the integral region and change the order of integral variables.

Solution The integral region is the solid in R^3 bounded by $z=x+y$, $y=1-x$, $x=1$, $z=0$, $y=0$ and $x=0$, and

$$I=\int_0^1 dx\left[\int_0^x dz\int_0^{1-x} f(x,y,z)\,dy+\int_x^1 dz\int_{z-x}^{1-x} f(x,y,z)\,dy\right]$$

$$=\int_0^1 dz\left[\int_0^z dy\int_{z-y}^{1-y} f(x,y,z)\,dx+\int_z^1 dy\int_0^{1-y} f(x,y,z)\,dx\right].$$