The Response of Worker Effort to Piece Rates
Evidence from the Midwest Logging Industry

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ABSTRACT

Using firm-level payroll data from the Midwest logging industry, I compute a worker's productivity response to a change in piece-rate pay, an elasticity of effort, using an empirical specification developed in Paarsch and Shearer (1999). Maximum-likelihood estimation of an agency-based structural econometric model of worker choice yields elasticities ranging from 0.413 to 1.507. These estimates are smaller than, but qualitatively similar to, those reported in Paarsch and Shearer, suggesting that their model has perhaps more general applicability than their British Columbia tree-planting example.

I. Introduction

Recently, researchers contributing to the human-resource literature have written a number of papers in which the incentive effects thought to be inherent in pay-for-productivity mechanisms are identified and estimated. One such paper, Paarsch and Shearer (1999), investigates the incentive effects of piece-rate pay using a rich firm-level data set from the British Columbia tree-planting industry. Paarsch and Shearer frame their inquiry by first considering reduced-form regression techniques that yielded negative elasticity estimates, a counter-intuitive result suggesting

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the importance of controlling for variables that affect both piece rates and productivity such as job-site conditions. Since job-site conditions were unobservable, Paarsch and Shearer, following Grossman and Hart (1983), constructed a decision theoretic structural econometric model that accounted for the relationship between planting conditions and piece rates, thus permitting consistent estimation of an elasticity of effort.

A common objection to the use of firm-level data is that the results are not general. I examine the sensitivity of the results of Paarsch and Shearer by applying their model to data gathered from the logging industry of the Midwest United States. My results are qualitatively similar to those of Paarsch and Shearer, suggesting their model is somewhat more general than a case study. Quantitatively my results differ: Loggers respond more modestly to changes in the piece rate than do tree planters.

In the next section I discuss briefly the Midwest logging industry and the unique firm-level data set I have constructed. In Section III, I describe the reduced-form regression model initially employed by Paarsch and Shearer, and then highlight its failure to estimate an elasticity of effort. In Section IV, I outline an alternative: The nonlinear structural econometric model developed in Paarsch and Shearer, which I estimate and discuss in Sections V and VI.

II. Planting Seedlings and Harvesting Trees

In the Paarsch and Shearer paper, workers are paid a per-tree-planted piece rate, a payment scheme aimed at increasing productivity by directly linking worker effort and earnings. In the United States, most notably in the South and the Midwest, logging firms employ a labor-intensive logging technology, commonly called a ''strip cutter,'' that fits well into this same pay-for-productivity framework. Strip cutters are of particular relevance for a comparison study not only because of how they are paid, which is by per-log piece rate, but because their output is easily measured, they work independently, they are not unionized, and they each use the same production process.

The strip cutter, like a tree planter, performs simple yet physically demanding tasks: he fells, limbs, tops, measures, crosscuts, stacks, and counts. Felling refers to cutting the tree down and ''limbing'' is the process of severing the limbs flush with the trunk. When the logger has limbed to where the tree is approximately 3.5 inches thick, the unmarketable crown is cut off, or, in the parlance of a lumberman, the tree is ''topped.'' Next the logger measures the limbless trunk into 100-inch lengths and crosscuts at the corresponding points. Finally, the logger stacks the logs into small piles. The only tools needed to successfully complete these tasks are a chain saw, a crude measuring device (usually a slender sapling that has been relieved of its bark), and physical effort.

A. Data

The data set (which includes piece rates and weekly production as well as logger and tract identification numbers) was constructed from the payroll records of a medium-sized strip-cutting firm operating in the Midwest United States. Piece rates

Table 1
Summary Statistics: Weekly Earnings above Minimum-Wage Weekly Earnings

Variable	Mean	Standard Deviation	Minimum	Maximum
Number of logs	1,102.50	405.44	550.00	2,311.00
Piece rate	0.28	0.03	0.25	0.35
Weekly earnings	311.93	115.05	165.00	663.00

Sample size = 231.

are measured in cents per 100-inch log and the number of such logs is the measure of productivity. The panel spans 18 months, beginning the first week of 1997 and ending the last week of May 1998, with a short discontinuity from late January to early March 1997 due to heavy snowfall. Observing eleven workers over ten separate contracts yielded 310 observations. Paarsch and Shearer truncated their data at the daily minimum wage; similarly, I eliminated all observations with weekly earnings less than $165.[1] Additionally, workers observed fewer than seven times were discarded because estimating worker-specific effects would be unreliable in such circumstances. After these screens, 231 observations for seven workers over eight tracts remain; in Table 1, I summarize these data.

III. Reduced-Form Regression Estimates

A natural empirical specification for estimating the elasticity of mean output with respect to changes in the piece rate is the following log-log regression model:

$$(1)\quad \log Y_{it} = \beta_{i0} + \beta_1 \log r_{it} + U_{it}$$

where Y_{it} is weekly production of 100-inch logs for worker i on tract t, r_{it} is worker i's piece rate on tract t, β_{i0} is a (possibly worker-specific) constant term, and U_{it} is a mean-zero innovation term assumed to be uncorrelated with the piece rate. Within this framework β_1 represents the elasticity of mean output with respect to the piece

1. For the first half of the sample the minimum wage was $4.75; during the second half of the sample (starting 1 September 1997) the minimum wage was raised by the *Fair Labor Standards Act* to $5.15. However, the 1997 change brought with it a ''subminimum'' wage for workers less than 20 years of age equal to $4.25, applicable for the first 90 days of employment. I chose a 35-hour week as per discussions with firm representatives. The calculation is as follows:

$$35 \times \left(\frac{\frac{\$5.15 + \$4.25}{2} + \$4.75}{2} \right) \approx \$165$$

Table 2
Regression Results
(a) Without Individual-Specific Effects
(b) With Individual-Specific Effects

Independent Variable	(a)	(b)
Constant	6.473	6.219
	(0.255)	(0.230)
Logarithm of piece rate	−0.373	−0.380
	(0.201)	(0.178)
Maximum individual-specific effect		0.445
		(0.057)
Minimum individual-specific effect		0.004
		(0.118)
R-squared	0.014	0.312

Standard errors are in parentheses below the point estimate.
Sample size = 231.

rate. In Table 2, I summarize the regression results. Elasticity estimates obtained with and without individual-specific indicator variables (−0.380 and −0.373) are similar in sign, magnitude, and significance to their counterparts reported in Paarsch and Shearer (−0.893 and −0.858). However, all these estimates are counter-intuitive: Productivity should not fall when piece rates rise. What can explain this?

Paarsch and Shearer argue that the piece rate is a function of job-site conditions; discussions with officials from both the tree-planting (Paarsch and Shearer) and strip-cutting (myself) industries verified this dependence. In fact, a site is viewed before setting the piece rate to ensure that working conditions are reflected in the pay schedule. For example, logging in dense undergrowth or in timber with high limb retention requires more effort per log than harvesting in a well-maintained plantation; the piece rate must adapt to these realities. Failing this, workers may be reluctant to accept logging contracts. Specification 1 fails to control for this relationship, hence violating the weak exogeneity of the covariates, a standard regression assumption.[2] Paarsch and Shearer address this problem by formalizing the interaction between job-site conditions and piece rates within a decision theoretic model of worker effort.

IV. Theoretical Model

Given that a logger has decided to work on a particular harvesting site, how much effort should he expend? Assume the logger has preferences over

2. In particular, omitted tract characteristics (which show up in U_{it}) are correlated with the piece rate, making $E(r_{it}U_{it}) \neq 0$.

two quantities: Earnings W and effort E. Let him maximize some utility function $U(W, E)$ of the form $V(W) - C(E)$. For this application V and C are as follows:

$$V(W) = W$$

$$C(E) = \frac{\kappa E^{\eta}}{\eta} \qquad \kappa > 0 \qquad \eta > 1$$

Here, worker aptitude is captured by κ.

Assume a logger produces logs Y according to a Leontief production technology consisting of his labor, a chain saw, and an effort level. Mimicking job-site conditions is a productivity shock S which is distributed lognormal with mean μ and variance σ^2. Generally,

$$Y = ES \min(labor, saw)$$

Choosing units appropriately,

$$Y = ES$$

The timing of the model is as follows:

1) The firm views a tract, evaluates conditions such as terrain, species, density, and quality, then bids;
2) The firm offers a contract consisting of average cutting conditions μ_t, variability in cutting conditions σ_t^2, and a piece rate r_t;
3) The worker accepts or rejects the contract;
4) If the worker accepts, he is assigned to a subsection of the tract;
5) The worker inspects his work area and chooses an effort level; and
6) Finally, the firm observes the worker's output and pays him according to the conditions of the contract.

The definitions above imply the logger is behaving as if he is solving

$$\max_{<E>} U(W, E) = W - \frac{\kappa E^{\eta}}{\eta}$$

$$\textit{subject to } W = rES$$

The solution, in the notation of Paarsch and Shearer, is an optimally chosen value e of E as a function of the realized value s of the productivity shock S:

$$(2) \quad e = \left(\frac{rs}{\kappa}\right)^{\gamma} \quad \gamma = \frac{1}{\eta - 1}$$

Then use Equation 2 to write

$$(3) \quad y = \left(\frac{r}{\kappa}\right)^{\gamma} s^{\gamma+1}$$

V. Identification and Estimation Strategies

To estimate the empirical specification, I follow Paarsch and Shearer by first considering a log-log version of Equation 3:

$$\log y = \gamma \log r - \gamma \log \kappa + (\gamma + 1)\log s$$

or, in terms of random variables, worker-specific index i, and tract-specific index t,

$$(4) \quad \log Y_{it} = \gamma \log r_t - \gamma \log \kappa_i + (\gamma + 1)\log S_{it}$$

Rewriting Equation 4 as

$$(5) \quad \log Y_{it} = \gamma \log r_t - \gamma \log \kappa_i + \mu_t(\gamma + 1) + (\gamma + 1)(\log S_{it} - \mu_t)$$

means that if the model is correct, the error term U_{it} in Equation 1 comprises the tract-specific effect $\mu_t(\gamma + 1)$ (which is correlated with r_t) and the random component $(\gamma + 1)(\log S_{it} - \mu_t)$. Because there is perfect collinearity between the vector of piece rates and the set of tract-specific indicators (deriving from the fact that piece rates do not vary on a specific tract), estimation based on Equation 5 cannot separately identify the effects of different piece rates and different tracts. Since the $\mu_t s$ are a necessary part of the model, and since they are unobservable, an identifying restriction is needed. Paarsch and Shearer note that there is additional information to be exploited in the fact that the least-able worker accepts contracts. Combining the logarithm of the expected-utility constraint of this least-able worker,

$$\bar{u} = \frac{E(S^{\gamma+1})r_t^{\gamma+1}}{\kappa_h^{\gamma}(\gamma + 1)}$$

where

$$\kappa_h = \max\{\kappa_i : i = 1, 2, \ldots, 7\},$$

with the fact that

$$E(S^{\gamma+1}) = \exp\left[\mu_t(\gamma + 1) + \frac{(\gamma + 1)^2\sigma_t^2}{2}\right]$$

allows $\mu_t(\gamma + 1)$ to be written in terms of the other parameters of the problem:

$$(6) \quad \mu_t(\gamma + 1) = \log \bar{u} - \frac{(\gamma + 1)^2\sigma_t^2}{2} - (\gamma + 1)\log r_t + \gamma \log \kappa_h + \log(\gamma + 1)$$

Substituting Equation 6 into Equation 5 yields an equation that identifies the marginal effect of the different piece rates:

$$(7) \quad Y_{it}^* = \log(\gamma + 1) + \gamma(\log \kappa_h - \log \kappa_i) - \frac{(\gamma + 1)^2\sigma_t^2}{2} + (\gamma + 1)(\log S_{it} - \mu_t)$$

where

$$Y_{it}^* \equiv \log Y_{it} + \log r_t - \log \bar{u}$$

Note that in Equation 7 tract-specific levels are captured by a function of individual-specific effects (the $\kappa_i s$) and tract-specific variability (σ_t^2). The latter also determines the *variability* of Y_{it}^*; it is by exploiting this nonlinear dependence that an elasticity of effort is identified. Since

$$Y_{it}^* \sim N\left[\log(\gamma + 1) - \frac{(\gamma + 1)^2\sigma_t^2}{2} + \gamma(\log \kappa_h - \log \kappa_i), (\gamma + 1)^2\sigma_t^2\right]$$

deriving the likelihood function is straightforward.

Note that Y_{it}^* requires a measure of reservation utility. While Paarsch and Shearer use zero-effort welfare payments, I am constrained to find an alternative because Midwest welfare laws prohibit zero-effort subsidies. Instead, I assume that zero-effort subsidies ''exist'' in the same proportion to the minimum wage in both locales. This ratio equals (\$27.05/\$48.00) per day in Canada, suggesting that a zero-effort subsidy, if one existed in the Midwest, would be approximately \$92.98 per week.[3]

A. Calculations and Results

Following Paarsch and Shearer I first estimated a version of Equation 7 with all the $\kappa_i s$ set equal to a common value κ; the restricted parameter set is then $\{\gamma, \sigma_1, \ldots, \sigma_8\}$.[4] The tract-specific moments and the elasticity γ are estimated using the method of maximum likelihood; the results are collected in Table 3, Column a. Of the eight tracts, only the minimum, maximum, and average tract-specific variances are included. The estimate of γ is 2.364, suggesting an elastic worker response to changes in the piece rate.

Estimation of the full parameter set, $\{\gamma, \kappa_{i \neq h}, \sigma_j: i = 1, 2, \ldots, 7; j = 1, 2, \ldots, 8\}$, which controls for worker-specific effects, produces a lower elasticity estimate equal to 1.507, suggesting that a 1 percent increase in the piece rate above its mean results in a 1.507 percent increase in productivity.[5] In terms of logs, raising the piece rate 1 cent above its mean of 28 cents results in 36 more logs being harvested. Paarsch and Shearer report a similar effect: controlling for individual-specific abilities reduces their estimate from 5.876 to 2.135. Additionally, both studies show a significant increase in the logarithm of the likelihood function, suggesting that the individual effects are jointly significant; in Table 3, Column b, I summarize these results.

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3. Note that the ratio of prices obviates the need to distinguish between currencies. The calculation is as follows:

$$\frac{\$27.05}{\$48.00} \approx \frac{\$92.98}{\$165.00}$$

4. This assumption reduces Equation 7 to Equation 5 in Paarsch and Shearer. Note also that, given these parameter estimates, Equation 6 could be used to estimate $[\mu_t(\gamma + 1) - \gamma \log \kappa]$ for $t = 1, 2, \ldots, 8$. Then, if desired, one of the $\mu_t s$ could be normalized (say, to zero); this permits recovery of an estimate of κ that can then be used in Equation 6 to estimate the remaining $\mu_t s$.

5. Estimation of the heterogeneous-worker case is carried out by adding worker-specific indicator variables for each worker except $i = h$; from Equation 7 the worker effect is *relative* to the least-able worker, and drops out when $i = h$.

Table 3
Parameter Estimates and Sensitivity Analysis
(a) Without Individual-Specific Effects
(b) With Individual-Specific Effects
(c) Sensitivity Analysis Estimates

Parameter	(a)	(b)	(c)
γ	2.364	1.507	0.413
	(0.089)	(0.362)	(0.204)
Maximum σ	0.131	0.206	0.366
Minimum σ	0.074	0.072	0.127
Average σ	0.101	0.128	0.227
Maximum individual-specific effect		0.421	1.535
Minimum individual-specific effect		0.010	0.037
Average individual-specific effect		0.159	0.580
Log-likelihood function	−81.070	−27.160	−27.160

Standard errors are in parentheses below the point estimate.
Sample size = 231.

B. Prediction

In order to evaluate the model's performance I predict the average weekly output of logs for each contract using the parameter estimates from the structural model. In Table 4, I compare the actual mean productivity per contract to the corresponding 95 percent and 99 percent prediction intervals. At the 95 percent level, five of the eight intervals contain the observed mean; at the 99 percent level six of the eight intervals contain the observed mean. These prediction results contrast with those reported by Paarsch and Shearer where only seven and nine of the 31 contracts contained the observed mean for the 95 percent and 99 percent prediction intervals. This difference may be due in part to the larger sample size found in Paarsch and Shearer.

C. Rent

Paarsch and Shearer estimate a lower bound on the profit losses accruing from the firm's failure to implement the optimal static contract. This can be accomplished by backing out the rent captured by each worker i relative to the lowest-ability worker. I estimate that these other workers earn an average of $60 in rent per week. The average price per log is approximately $0.84 and the average piece rate is known to be 28 cents, so average profits per worker per week are equal to $2rY$, or $617. Reallocation of this rent to the firm through worker-specific base fees, as the static optimal contract prescribes, would increase profits by about 9.7 percent; a similar reallocation (calculated in Paarsch and Shearer) at the tree-planting firm would garner around a 17.25 percent increase in profits.

Table 4
Prediction Results: Logarithm of Average Productivity of 100-inch Logs
(a) 95 Percent Confidence Intervals
(b) 99 Percent Confidence Intervals
(c) Observed Average

Tract	(a)	(b)	(c)
tract 1	(6.71,7.30)	(6.56,7.45)	6.95
tract 2	(6.66,7.20)	(6.53,7.33)	6.94
tract 3	(6.79,6.96)	(6.76,7.00)	7.08
tract 4	(6.36,6.97)	(6.21,7.19)	6.81
tract 5	(6.65,6.79)	(6.62,6.83)	6.83
tract 6	(6.41,7.48)	(6.14,7.75)	6.79
tract 7	(7.00,7.13)	(6.96,7.16)	7.32
tract 8	(6.60,7.07)	(6.49,7.19)	6.93

Sample size = 231.

D. Sensitivity Analysis

By appealing to the inverse relationship between the reservation level of utility and the elasticity estimate, a strategy for approximating a lower bound on the elasticity emerges. This can be accomplished by using the minimum wage as the measure of alternative utility. Using this strategy, Paarsch and Shearer estimate the lower bound on the elasticity for tree planters to be 0.767. Assuming weekly minimum earnings to be $165 per week, I estimate a lower bound equal to 0.413, suggesting that a 1 percent increase in the piece rate above its mean results in *at least* a 0.413 percent increase in production of logs; further details are collected in Table 3, Column c.

VI. Discussion and Conclusion

I have examined the sensitivity of the elasticity estimates reported in Paarsch and Shearer (1999) by applying their methods to data from a similarly structured industry. My results are qualitatively similar for both the reduced-form and structural models; this is evidence for the broader applicability of their model. However, our respective elasticity estimates differ moderately in magnitude: mine suggest a logger's effort response to changes in the piece rate to be at least 0.413 and perhaps as high as 1.507. The results reported by Paarsch and Shearer suggests tree planters to be somewhat more responsive to changes in the piece rate: estimates range from 0.767 to 2.135. This difference could suggest that piece-rate incentive effects are somewhat smaller than previously thought, or it could be an artifact of the physical realities unique to each industry. I speculate that a worker's response to an output incentive will be tempered by the risks associated with working faster.

Since harvesting trees is considerably more dangerous (both in frequency and severity of injury) than planting trees, a logger will be more apprehensive about increasing his pace than will a tree planter for a given increase in the piece rate.

References

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