Racial Labor Market Gaps The Role of Abilities and Schooling Choices

Sergio Urzúa

ABSTRACT

This paper studies the relationship between abilities, schooling choices, and black-white differentials in labor market outcomes. The analysis is based on a model of endogenous schooling choices. Agents' schooling decisions are based on expected future earnings, family background, and unobserved abilities. Earnings are also determined by unobserved abilities. The analysis distinguishes unobserved abilities from observed test scores. The model is implemented using data from the NLSY79. The results indicate that, even after controlling for abilities, there exist significant racial labor market gaps. They also suggest that the standard practice of equating observed test scores may overcompensate for differentials in ability.

I. Introduction

The existence of black-white gaps in a variety of labor market and educational outcomes has been extensively documented. It is well established that, on average, blacks are less educated, have lower income, and accumulate less work experience than whites.¹ This paper studies whether the differences in labor market outcomes and schooling attainment can be interpreted as the manifestation of blackwhite ability differentials. Although this idea is not new, the analysis presented is a

[Submitted May 2006; accepted June 2007]

ISSN 022-166X E-ISSN 1548-8004 © 2008 by the Board of Regents of the University of Wisconsin System

THE JOURNAL OF HUMAN RESOURCES • XLIII • 4

^{1.} See, for example, Altonji and Blank (1999), Neal (2008), and Carneiro, Heckman, and Masterov (2006).

Sergio Urzuía is a professor of economics at Northwestern University, Department of Economics, 2001 Sheridan Road, Office 302, Evanston IL 60208; telephone: 847-491-8213; email:

s-urzua@northwestern.edu. The author is indebted to James Heckman for his encouragement and guidance throughout this project. He acknowledges helpful discussions with Derek Neal, Glenn Loury, Robert Townsend, Lars Hansen, Chris Taber, Greg Duncan, Jora Stixrud, Paul LaFontaine, three anonymous referees, and the participants in workshops at the University of Chicago, Brown University, Northwestern University, the 2007 ASSA/AEA Meeting, and at the Third Chicago Workshop on Black-White Inequality. The usual disclaimer applies. Supplementary material: http://

faculty.wcas.northwestern.edu/~ssu455/research/gapWebAppendix.pdf. The data used in this article can be obtained beginning May 2009 through April 2012 from Sergio Urzúa, Northwestern University, Department of Economics, 2001 Sheridan Road, Office 302, Evanston IL 60208; telephone: 847-491-8213; email: s-urzua@northwestern.edu

comprehensive one that takes into account several aspects that have been only partially recognized in the literature.

The empirical strategy utilized in this paper treats both schooling decisions and labor market outcomes as endogenous variables. This represents an important difference relative to previous studies, as schooling decisions are usually either excluded from the analysis on the grounds that they might be influenced by discrimination (Neal and Johnson 1996), or included under the presumption that they can be treated as exogenous variables (Lang and Manove 2006).² The omission of schooling obviously prevents the study of black-white differences in educational decisions and the extent to which those differences explain the observed gaps in labor market outcomes. Their inclusion as exogenous variables, on the other hand, limits the scope of the empirical analysis because of potential endogeneity bias.

Also addressed is the extent to which black-white gaps can be explained by noncognitive, as well as cognitive, ability differentials. This is particularly relevant since recent studies have demonstrated that noncognitive abilities are as important as, if not more important than, cognitive abilities in determining labor market, educational, and behavioral outcomes (for example, Bowles, Gintis, and Osborne 2001a,b; Farkas 2003; and Heckman, Stixrud, and Urzua 2006). However, to date very little is known about the role these abilities play in explaining racial differentials.

Importantly, the analysis distinguishes observed cognitive and noncognitive *measures* from unobserved cognitive and noncognitive *abilities*. This distinction is based on the idea that observed (or measured) abilities are the outcome of a process involving familial inputs, schooling experience, and pure (unobserved) ability. The relevance of this distinction comes from the claim that racial gaps in observed achievement tests (interpreted as observed cognitive abilities or premarket factors) can explain most of the racial differences in labor market outcomes (see, for example, Neal and Johnson 1996). If racial differences in achievement test scores do not emerge exclusively as the result of differences in abilities but also as the result of differences in family background and schooling environment, then by comparing the labor market outcomes of blacks and whites with similar observed abilities (test scores), we are not necessarily understanding or identifying the real factors behind the racial gaps. The analysis of this paper sheds light on this point.³

Finally, although the analysis mainly focuses on black-white differences in labor market outcomes and schooling decisions, it also addresses whether ability

^{2.} An important exception is the analysis of Keane and Wolpin (2000). Keane and Wolpin (2000) analyze racial labor market gaps using a dynamic model of schooling, work, and occupational choice decisions with unobserved heterogeneity (endowments). However, unlike the analysis in this paper, they do not link the unobserved endowments to specific abilities.

^{3.} The literature studying racial gaps recognizes some of the limitations associated with the direct use of test scores (measured abilities) as proxies for ability, particularly in the case of cognitive test scores. Neal and Johnson (1996) and Cameron and Heckman (2001), for example, restrict their analysis to the sample of respondents 18 or younger at the time of the tests from the National Longitudinal Survey of Youth 1979 (NLSY79). This tries to control for the fact that individuals in the NLSY79 sample take the same tests at different ages, and consequently, at different schooling levels. Carneiro, Heckman, and Masterov (2005), also using NLSY, utilize age and family background adjusted test scores by constructing the residuals from OLS regression of test scores on those variables. The approach in this paper differs from these alternatives.

differentials can explain racial differences in incarceration. The racial differences in this dimension of social behavior have received increasing attention in the literature.⁴

The empirical results of the paper establish the existence of racial differences in the distributions of cognitive and noncognitive abilities. They also demonstrate that, regardless of the racial group analyzed, these abilities are important determinants of labor market outcomes and schooling attainment, and document the existence of significant differences across racial groups in the way these abilities determine each of these dimensions. This is particularly clear in the case of schooling attainment, where noncognitive abilities have stronger positive effects among blacks than among whites.

The results also confirm that racial gaps in labor market outcomes reduce after controlling for cognitive abilities. However, the percentage explained by these abilities is significantly smaller than what has been previously claimed in the literature. This is a direct consequence of the distinction between observed and unobserved abilities. Moreover, although there are significant black-white differences in noncognitive abilities, they play a minor role in explaining gaps in labor market outcomes. However, noncognitive abilities help to explain a significant fraction of the racial gaps in incarceration rates.

It is important to notice that it is not an objective of this paper to provide a comprehensive explanation of the factors explaining the racial differences in unobserved abilities. Specifically, in the context of the empirical model described in this paper, and given the data limitations,⁵ the estimated racial differences in unobserved abilities could be the result of a variety of unobserved factors (unmeasured racial differences in early family environment including prenatal family environment, unmeasured racial differences in early schooling environment, cultural differences between groups, biological/genetic differences between groups, or, most likely, a combination of all of these). There is nothing in this paper that contradicts this argument, and consequently, the existence (and explanatory power) of the ability differentials must be interpreted in this context.

The paper is organized as follows. Section II presents evidence on the black-white wage gap using the standard empirical approach. The results from Section II motivate the main ideas of the paper. Section III introduces the model of endogenous labor market outcomes, schooling decisions, and unobserved abilities, while Section IV analyzes the relationship between test scores and abilities in the context of the model. This section also introduces the model for incarceration. Section V discusses the empirical implementation of the model. Section VI presents the main results and examines whether black-white gaps in labor market outcomes, schooling choices, and incarceration rates can be explained by racial differences in abilities. Section VII concludes.

II. Background and Motivation

This section motivates the main ideas of the paper by utilizing a conventional reduced-form approach to analyze black-white differentials in labor market outcomes. Specifically, consider the following linear model for a labor market outcome *Y* (usually hourly wages or earnings):

^{4.} Freeman (1991), Bound and Freeman (1992), Grogger (1998), Western and Pettit (2000).

^{5.} The information utilized for the identification of the (unobserved) abilities comes from a sample of individuals 14 years and older. See Appendix 2 for details.

(1)
$$\ln Y = \varphi Black + \gamma Test + \sum_{s=1}^{S} \phi_s D_s + U$$

where *Black* represents the race dummy, D_s represents a dummy variable that takes a value of one if the individual's schooling level is *s* (with *s*=1,..,*S*), *Test* represents an ability measure (observed ability),⁶ and *U* is the error term in the regression. Different versions of Equation 1 can be found in the literature studying black-white inequality in the labor market.⁷ Here, the coefficient associated with the race dummy can be written as

(2)
$$\varphi = E[\ln Y | Black = 1, Test, D_s] - E[\ln Y | Black = 0, Test, D_s]$$

for any schooling level *s*, so φ can be interpreted as the mean racial difference in (log) labor market outcome *Y* after controlling for measured ability and schooling decisions. In other words, φ represents the difference between two individuals that share the same levels of education and measured ability, and differ only in their races.

Although the logic behind Equation 1 is simple and intuitive, its empirical implementation requires some non-trivial considerations. The first concern is the existence of unobserved variables simultaneously affecting schooling decisions and labor market outcomes. The consequences of this potential endogeneity on the estimates of the returns to schooling (each ϕ_s in Equation 1) have received the attention of many for more than 50 years (Mincer 1958 and Becker 1964). The instrumental variable approach has emerged as the most popular method to deal with this endogeneity problem (Card 2001). However, less attention has been paid to the consequences of endogeneity bias in the estimates of ϕ .

Neal and Johnson (1996) propose a different empirical strategy that, in principle, avoids concerns about endogeneity biases. They addressed a specific counterfactual. Namely, if two young people with the same basic reading and math skills reach the age at which schooling is no longer mandatory, how different will their labor market outcomes be when they are prime working age adults? Neal and Johnson (1996) were not interested in how choices made concerning education, labor supply, or occupation might shape the wage and earnings profiles of blacks and whites differently. Rather, they focused only on the average differences in outcomes among persons who began making adult choices concerning education and labor supply given the same endowments of basic skills (proxied by achievement test scores).⁸

^{6.} In general, Test could represent a vector containing both observed cognitive and noncognitive abilities.

^{7.} See Neal (2008), Farkas et al. (1997), Altonji and Blank (1999), and Farkas (2003).

^{8.} Neal and Johnson (1996) exclude the schooling dummies from Equation 1 on the grounds that they can be influenced by discrimination. In this way, the endogenous variables are absorbed into the error term of the regression. However, the exclusion of schooling opens the door to a new source of potential problems in the estimation of φ , due to the omission of relevant variables. But, under the logic of Neal and Johnson (1996), this does not represent a problem. Because the biased OLS estimator of $\varphi(\varphi^{OLS})$ would contain, in this case, the indirect effect of race on the schooling dummies (controlling for the observed ability), φ^{OLS} could still be interpreted as an estimate of the overall mean racial difference in the outcome (log) *Y* even if the schooling dummies are excluded. Specifically, in the Neal and Johnson's specification, the OLS estimate of the coefficient associated with the race dummy would identify the following object: $\varphi + \sum \varphi_s [\Pr(D_s = 1|Black = 1, Test) - \Pr(D_s = 1|Black = 0, Test)]$, where the second term represents the bias. This approach does not identify (estimate) the mean difference in the outcome (log) *Y* for two individuals sharing the same ability and schooling level but differing in race.

By contrast, φ in Equation 2 defines a different racial gap in labor market outcomes that has been the focus of a large literature. If two young persons of different races begin their adult lives with similar ability levels and then make comparable investment in their human capital, how much will their wages differ as adults? As the analysis of this paper will show, a fixed racial gap in wages will not provide a satisfactory answer to this question. Among black and white persons who begin their adult lives with comparable abilities, racial differences in adult wages and earnings will vary among groups that choose different levels of educational investment. The empirical approach utilized in this paper will allow me to estimate these different racial gaps and also make progress toward understanding why changes in investment behavior among blacks have not equalized black and white returns to different levels of schooling.

A second concern when implementing Equation 1 is whether observed ability (*Test*) measures ability accurately. Several studies have demonstrated that observed ability measures cannot be interpreted as pure abilities and that they are influenced by home and school environments (see Neal and Johnson 1996; Todd and Wolpin 2003; and Cunha et al. 2006 for evidence). This simple consideration has important consequences for the interpretation of the OLS estimates of φ . In fact, if pure ability were the determinant of the outcome *Y*, and if, in the estimation of Equation 1, a proxy for ability (*Test*) were used instead, the OLS estimator of the racial gap φ would be biased in an unpredictable way.

An additional concern regarding the estimation of Equation 1, which has a direct implication for the way the gap is defined (Equation 2), comes from the assumptions on the parameters of the model. A specification like Equation 1 assumes that black and white subjects face the same returns to observed ability and schooling, that is, the same γ , ϕ_1, \dots, ϕ_s . The convenience of this assumption is clear: if the returns are the same, the black-white gap can be measured directly by a single φ . However, the assumption of equal returns can—and should—be tested. The assumption of a single φ represents a simplification imposed a priori in Equation 1.

A natural extension of Equation 1 would be a specification in which φ is allowed to vary across schooling levels. However, the implementation of such a model also would require taking into account the fact that individuals may decide the schooling levels based on the potential differences in these returns. Therefore, this approach would naturally generate concerns about the endogeneity of schooling decisions.⁹

The main objective of this paper is the estimation of the black-white gaps in labor market outcomes using an approach that takes into account each of these issues. That is, the empirical model used in this paper deals with the endogeneity of schooling choices, the measurement error problem in abilities (cognitive and noncognitive abilities), and the unnecessary restrictions that are usually imposed a priori in the empirical literature. But, before introducing the model and its results, it is informative to follow the standard approach and to present the estimated black-white gaps in labor market outcomes (wages and earnings) as computed using OLS on some of the traditional specifications of Equation 1 found in the literature. These results will serve as a comparison later in the paper.

^{9.} Neal and Johnson (1998) present evidence on differences in the coefficients associated with measured ability when (log) earning equations are estimated separetely by schooling levels (see Table 14.8 in Neal and Johnson 1998). However, the regressions do not take into account the potential endogenous selection of schooling in the sample.

Table 1 presents the black-white gaps in log hourly wages and log annual earnings obtained from four different specifications of Equation 1. The gaps are estimated using a representative sample of males from the National Longitudinal Survey of Youth 1979 (NLSY79).¹⁰ The specifications differ exclusively in the set of controls included in the equations.

The first specification (Model A in Table 1) presents the baseline model. It includes only variables associated with an individual's place of residence as controls.¹¹ The estimated black-white gaps in log hourly wages and log earnings are 0.294 and 0.567, respectively. Using the traditional interpretation of these results, it is possible to conclude that, on average, blacks make approximately 25 percent less per hour and 43 percent less per year than whites.¹² Both numbers are substantial in magnitude and similar to what has been found in the literature (Neal and Johnson 1996; Neal 2008). They are also statistically significant (as are all of the numbers presented in the table). When schooling dummies are included as controls (Model B), the gaps reduce to 0.230 and 0.482 for hourly wages and annual earnings, respectively.¹³ These numbers imply a significant reduction in the gaps when compared with the estimates from the baseline model (Model A).

However, the estimated gaps should not be compared across models, as each model represents a different specification. Specifically, in order to correctly quantify the reduction in the gap due to schooling, we must compare the estimated gap from Model B with the mean black-white difference in outcome after expecting out the effects of the schooling dummies from Model B. More precisely, in the context of the two models

(3)
$$\ln Y = \varphi^{A} Black + U^{A} \qquad (Model A)$$
$$\ln Y = \varphi^{B} Black + \sum_{s=1}^{S} \varphi_{s} D_{s} + U^{B} \qquad (Model B),$$

we must compare φ^B versus $\varphi^B + \sum_{s=1}^{S} \phi_s(E[D_s | Black = 1] - E[D_s | Black = 0])$, instead of φ^B versus φ^A . Under Model B in Table 1, Row 1 presents φ^B whereas Row 2 presents the gap after expecting out the schooling dummies, that is, $\varphi^B + \sum_{s=1}^{S} \phi_s (E[D_s | Black = 1] - E[D_s | Black = 0])$. By comparing these numbers,

^{10.} The NLSY79 is widely used for the analysis of black-white gaps in wages, earnings, and employment. It contains panel data on wages, schooling, and employment for a cohort of young persons, aged 14 to 22 at their first interview in 1979. This cohort has been followed ever since. See Data Appendix 2 for details of the sample used in this paper.

^{11.} Specifically, Model A includes as controls the dummy variables: northcentral region, northeast region, south region, west region and urban area.

^{12.} The 25 percent is calculated as 1-exp(-0.29). Likewise, 43 percent is calculated as 1-exp(-0.567). Notice that these calculations omit the fact that $E[\ln Y]$ is not the same as $\ln E[Y]$.

^{13.} The schooling levels considered are: high school dropouts (including recipients of the General Educational Development (GED) certification), high school graduates, some college (including two-year college degrees) and four-year college graduates. For each individual in the sample, the schooling level is defined as the maximum schooling level ever reported. The results are robust to the particular classification of the schooling levels considered in the analysis.

			Schooling Dummies	Cogr	- Cognitive Test Score	Scho Dum Cogn	Schooling Dummies + Test Score
	(A)	D	(B)		(C)	D	(D)
Wages	Earnings	Wages	Earnings	wages	Earnings	wages	Earnings
(1) Black-White Gap -0.294	-0.567	-0.230	-0.482	-0.099	-0.287	-0.125	-0.326
 Black-White Gap Conditional on: (2) Baseline Variables (3) Baseline Variables Schooling Dummies (4) Baseline Variables and Cognitive Test Score (5) Baseline Variables, Schooling Dummies, and (6) Convintion Test Score 	-0.567	-0.292 - 0.230	-0.572 - 0.482	-0.302 -0.099	-0.578 -0.287	-0.299 -0.265 -0.159 - 0.125	-0.579 -0.532 -0.373 - 0.326

Urzúa 925

we can conclude that schooling seems to reduce the coefficient associated with the race dummy (the gap) by 21 percent or 16 percent, depending on the labor market outcome considered.

Model C in Table 1 presents the results from the specification proposed by Neal and Johnson (1996). Thus, in addition to the baseline variables, the model includes a proxy for an individual's cognitive ability or intelligence. This proxy is a standardized average computed using six achievement tests available in the NLSY79 sample.¹⁴ The estimated gaps are, in this case, 0.099 and 0.287 for wages and earnings, respectively. These numbers represent reductions in the estimated gaps for (log) wages and (log) earnings of 67 percent and 50 percent (Row 1 versus Row 4 under Model C), respectively, which are in the range of what has been found in the literature (see Carneiro, Heckman, and Masterov 2005; Neal and Johnson 1996).

The evidence from Models B and C suggests that both schooling and cognitive ability help to reduce the black-white gaps in wages and earnings. Model D studies the effects on the gap when they are simultaneously included in the regressions. The results in this case indicate that, when schooling and cognitive ability are controlled for, the estimated black-white gaps are 0.125 (wages) and 0.326 (earnings) (see Row 1 under Model D), with associated gap reductions of 58 percent and 44 percent (Row 1 versus Row 5 under Model D), respectively. Notice that these reductions are smaller in magnitude than the ones obtained when schooling variables are omitted from the analysis (Model C), and so the results seem to indicate that schooling has unequal effects on labor market outcomes.¹⁵ However, this would (again) be the wrong comparison. A closer look at the evidence from Model D suggests that when the contribution of schooling is measured correctly (Row 3) versus Row 2), it implies a reduction of 11 percent in the wage gap (0.265 versus (0.299) and 8 percent in the earnings gap (0.532 versus (0.579). Thus, schooling variables seem to explain sizeable proportions of the gaps. Likewise, when only the contribution of cognitive ability is analyzed (Row 4 versus Row 2), I obtain reductions of 47 percent in the wage gap (0.159 versus 0.299) and 36 percent in the earnings gap (0.373 versus 0.579). Overall, the evidence from Model D suggests that cognitive ability reduces the gap the most, although the contribution of cognitive ability is less than the one obtained from Model C.

In summary, the results in Table 1 suggest that the proxy for cognitive ability (average achievement test score) is the most important explanatory variable of the black-white gaps in wages and earnings. This is consistent with previous findings in the literature. Its explanatory power is maximized when it is the only variable included in the model (other than the baseline variables), and it decreases when

^{14.} The achievement test scores used in this paper are: Arithmetic Reasoning, Word Knowledge, Paragraph Composition, Math Knowledge, Numerical Operations, and Coding Speed. These tests belong to the Armed Services Vocational Aptitude Battery (ASVAB) and are used to construct the Armed Forces Qualification Test (AFQT), which is a widely used measure of cognitive skill or intelligence. See Neal and Johnson (1996), Herrnstein and Murray (1994) and Carneiro, Heckman, and Masterov (2005), among others.

^{15.} Carneiro, Heckman, and Masterov (2005) and Neal and Johnson (1996) compare results from models similar to C and D and conclude that schooling seems to increase black-white wage inequality.

schooling is included as a control. Schooling on the other hand, explains an important fraction of the gaps in wages and earnings.¹⁶

However, as previously explained, these results are subject to important qualifications. Firstly, it is not completely clear what the proxy for cognitive ability is really measuring. Achievement test scores are known to not only be the results of pure ability, but also of home and school environments (see Neal and Johnson 1996; Todd and Wolpin 2003; and Cunha et al. 2006). Additionally, the results do not consider the potential role of noncognitive abilities (such as self-motivation, self-esteem, and self-control, among others) as explanatory factors of the black-white inequality. Furthermore, by estimating an overall gap and treating schooling as an exogenous variable, these results do not provide a deep and precise understanding of the extent to which blacks and whites differ in terms of labor market outcomes. An integrated approach in which schooling choices are modeled jointly with wages is needed. This approach is discussed below.

III. The Model of Labor Market Outcomes and Schooling Choices

This section presents a model that integrates labor market outcomes (hourly wages, annual hours worked, and annual earnings) with schooling choices for the analysis of racial labor market gaps. The model assumes that individuals make their schooling choices based on their expectation about future labor market outcomes and schooling costs.

For sake of notational simplicity, I omit the supra-index for race in the exposition of the model, but the reader should be aware that every parameter in the model is defined separately for blacks and whites; that is, that the model applies separately to each race.

A. The Schooling Decision

The model considers T+1 time periods (t=0,1,...,T) and *S* possible schooling levels (s=1,...,S). Each individual chooses his final schooling level at t=0 and receives labor income at the end of each period (except period 0). The stream of labor income depends on the schooling level selected.

^{16.} Table A0 in the web appendix extends the analysis of Table 1 (and of the previous literature) by presenting the estimated black-white gaps in wages and earnings when *Test* in (1) is a multidimensional object rather than a single ability measure. Specifically, *Test* in this case (Model E in Table A0) includes proxies of an individual's cognitive and noncognitive abilities as controls. The particular measure of noncognitive ability used is the standardized average of two attitudes scales: Rosenberg Self-Esteem and Rotter Locus of Control scales (see data appendix for details). These scales have been shown to be good predictors of labor market outcomes and social behaviors (see Heckman, Stixrud, and Urzua 2006). The results suggest that when the two proxies for abilities and schooling levels are kept constant, the estimated black-white gaps are 0.131 (wages) and 0.335 (earnings). The contributions of cognitive abilities and schooling to the reduction in the overall gap are substantial. For wages, cognitive abilities explain 42 percent of the overall gap, whereas schooling explains 11 percent. For annual earnings, the numbers are 32 percent and 8 percent. Again, the proxy for cognitive ability is the most important factor explaining the gaps. However its contribution is even smaller than that obtained in Table 1. Finally, the results suggest that, for wages and earnings, only a modest 2 percent of the overall gaps can be explained by the proxy for noncognitive abilities.

Individuals make their schooling decisions based on a comparison of the expected benefits and costs associated with each alternative. Specifically, if V_s denotes the expected benefit associated with schooling level *s*, then

$$\mathcal{V}_s = E\left[\sum_{t=1}^T \rho^{t+1} u(\mathcal{E}_s(t)) | \mathcal{I}_0\right],$$

where $u(\cdot)$ represents the per period utility function, $\mathcal{E}_s(t)$ represents the total earnings received in period t given schooling level s, ρ is the discount factor, and \mathcal{I}_0 represents the information set available to the agent at t=0. The information contained in \mathcal{I}_0 is discussed below.

Total earnings received at the end of period t are simply the product of hourly wages $(Y_s(t))$ and total number of hours worked during the period $(H_s(t))$, that is, $\mathcal{E}_s(t) = Y_s(t) \times H_s(t)$. Notice that both wages and hours depend on the schooling level and time period.

Additionally, each schooling level has attached a schooling cost Γ_s . This cost can include not only monetary expenses associated with the specific schooling level (tuition, for example), but also associated psychic costs.¹⁷ Γ_s must be *paid* by the individual at the time the decision is made. Thus, the net expected value (\tilde{V}_s) associated with schooling level *s* is

(4)
$$\widetilde{\mathcal{V}}_s = E\left[\sum_{t=1}^T \rho^{t+1} u(Y_s(t) \times H_s(t)) - \Gamma_s | \mathcal{I}_0\right] \text{ for } s = 1, \dots, S.$$

The individual selects his schooling level s^* at period t=0 by comparing the expected net utility levels \tilde{V}_s across the different S alternatives.

B. Labor Market Outcomes and Schooling Costs

The models for (log) hourly wages $(Y_s(t))$ and (log) hours worked $(H_s(t))$ are

(5)
$$Y_s(t) = \varphi_{Y_s,t} + \beta_{Y_s,t} X_t + U_{Y_s,t}$$

(6)
$$H_s(t) = \varphi_{H_{s,t}} + \beta_{H_{s,t}} Q_t + U_{H_{s,t}}$$
 for $s = 1, \dots, S$ and $t = 1, \dots, T$,

where X_t and Q_t represent the exogenous vectors of variables determining the labor outcomes, and $U_{Y_{s,t}}$ and $U_{H_{s,t}}$ represent the associated unobserved components in the equations. Equations 5 and 6 show how the individual's labor market outcomes depend on the specific schooling level and time period considered.

Schooling costs associated with schooling level s are modeled as

(7)
$$\Gamma_s = \varphi_{\Gamma_s} + \gamma_s \boldsymbol{P}_s + \varepsilon_s \text{ for } s = 1, \dots, S,$$

where P_s represent the vector of observables in Γ_s , and ε_s is the unobserved cost component.

^{17.} In other words, this cost can be interpreted as the utility or disutility of education.

Notice that no assumptions have been made on the unobservables in Equations 5, 6, or 7. In principle, they can be correlated over time, across schooling levels, and across outcomes. In fact, the distinction between observable and unobservable components is made only based on the information available in the data (the econometrician's point of view). Individuals may have information about variables contained in the unobservable components of the model $(U_{Y_s,t}, U_{H_s,t}, \varepsilon_s \text{ with } s=1,...,S$ and t=1,...,T) and they can use such information to decide which schooling level to select. This is the idea developed next.

C. Incorporating Unobserved Components

The model assumes that every agent is born with a vector of ability endowments **f**. These abilities include both individual cognitive (for example, intelligence) and noncognitive (for example, extraversion) traits. Thus, $\mathbf{f} = \{f_C, f_N\}$ where f_C and f_N represent the unobserved cognitive and noncognitive abilities, respectively. The levels of these traits are assumed to be known to the agent and to be constant over time. Direct information on these abilities is assumed not to be available, so they are interpreted as unobserved abilities.¹⁸

The model also considers the presence of a third unobserved component: uncertainty (θ). Uncertainty is intended to capture information that is revealed or learned by the individuals after they decide their schooling level. Therefore, unlike the vector of endowments **f**, uncertainty θ does not belong to the information set of the agent at t=0, that is, $\theta \notin \mathcal{I}_0$, but it is revealed during t=1.¹⁹ This implies that the agent's schooling problem does not depend on θ in any way since he is not aware of its existence. Direct measures of θ are assumed to be unavailable.

Initially, cognitive and noncognitive unobserved abilities (**f**) and uncertainty (θ) are assumed to be independent random variables with zero means. The zero mean assumption for **f** is relaxed below.

Unobserved abilities and uncertainty are incorporated in the model in the following manner:

$$U_{Y_{s,t}} = \alpha_{Y_{s,t}} \mathbf{f} + \lambda_{Y_{s,t}} \theta + e_{Y_{s,t}}$$
$$U_{H_{s,t}} = \alpha_{H_{s,t}} \mathbf{f} + \lambda_{H_{s,t}} \theta + e_{H_{s,t}} \text{ for } s = 1, \dots, S \text{ and } t = 1, \dots, T,$$

^{18.} From an empirical perspective, the longitudinal stability of cognitive ability has been well established in the literature (Jensen 1998; Conley 1984; Carroll 1993). However, there is no clear agreement regarding the stability of noncognitive abilities. For noncognitive traits such as neuroticism and extraversion, the evidence supports the idea of strong longitudinal stability. On the other hand, the evidence is not as strong when it comes to the stability of variables such as self-opinion (see Conley 1984; and Trzesniewski et al. 2004). However, since the analysis of this paper distinguishes observed measures (which would be allowed to change over time had they been repeatedly observed) from unobserved abilities, the idea of a fixed vector of unobserved endowments is not inconsistent with the literature.

^{19.} Cunha, Heckman, and Navarro (2005), using a sample of white males from the NLSY79, estimate a model in which the agents choose between two schooling levels (high school and college) based on limited information about the future. As in this paper, uncertainty is revealed only after the schooling decisions are made. Thus, as an aside contribution, this paper analyzes whether blacks and whites face different distributions of uncertainty as defined by Cunha, Heckman, and Navarro (2005).

where $e_{Y_{s,t}}$ and $e_{H_{s,t}}$ are *iid* idiosyncratic shocks for any schooling level *s* and time period *t*.²⁰

The vector of abilities also determines the costs of schooling. In particular, **f** is assumed to enter Γ_s through its error term:

 $\varepsilon_s = \alpha_{\varepsilon_s} \mathbf{f} + e_{\varepsilon_s}$ for s = 1, ..., S,

where e_{ε_s} is an *iid* idiosyncratic shocks for s=1,..,S. Uncertainty does not affect schooling costs since by assumption $\theta \notin \mathcal{I}_0$.

It is important to emphasize that what is considered unobserved by the econometrician may in fact be known to the agents. This has important consequences. Since agents base their schooling decisions on the comparison of expected benefits from different alternatives and because those benefits depend on \mathbf{f} , which is known to the agent but not by the econometrician, schooling decisions must be treated as endogenous variables. If information on \mathbf{f} were available, the econometrician could deal with the endogeneity of the schooling decisions by simply including \mathbf{f} in the analysis as an additional explanatory variable.

Finally, all error terms (*e* variables with subscripts) are mutually independent, independent of (f_C, f_N, θ) and independent of all the observable characteristics (*X*, *Q*, *P*). The unobserved components are also independent of all the observable characteristics.

D. The Information Set \mathcal{I}_0

As previously mentioned, the information set of the agent at the time the schooling decision is made, \mathcal{I}_0 , contains the vector of endowments **f**. Additionally, since the agents are assumed to know the schooling costs $\{\Gamma_s\}_{s=1}^{s}$, it must be the case that $\{\mathbf{P}_s, e_{\varepsilon_s}\}_{s=1}^{s} \in \mathcal{I}_0$. Furthermore, the model assumes that the agent knows the values of X and Q, the variables determining the labor market outcomes, at t=0, that is, $(\mathbf{X}_0, \mathbf{Q}_0) \in \mathcal{I}_0$.

The rest of the observables and unobservables in the model are not in the agent's information set at t=0.

IV. Additional Ingredients: The Models of Test Scores and Incarceration

Notice that since **f** is unobserved, I cannot empirically distinguish $\alpha_{Y_{s,t}}$, $\alpha_{H_{s,t}}$, and α_{ε_s} from $\alpha_{Y_{s,t}}$, **f**, $\alpha_{H_{s,t}}$, **f**, and α_{ε_s} **f** for any *s* and *t*. The same logic applies to θ and its associated parameters. This illustrates the fact that, without further structure, the model introduced in Section III is not fully identified. This section presents two additional ingredients of the model that secure its identification. Appendix 1

^{20.} An implicit assumption is the existence of mechanisms through which **f** and θ can be communicated or learned by potential employers. In this way, they can be priced in the labor market and, consequently, enter the models for labor market outcomes.

presents the formal arguments of identification. As before, I omit the index for race in what follows.

A. Test Scores versus Unobserved Abilities

As explained in the introduction, in general, test scores (measured abilities) cannot be directly interpreted as abilities. Thus, and following previous notation, let f_C and f_N denote cognitive and noncognitive abilities, and C_i and N_j denote the *i*th and *j*th cognitive and noncognitive tests, respectively. The model assumes the availability of n_C cognitive measures (that is, $i=1,...,n_C$) and n_N noncognitive measures (that is, $j = 1,...,n_N$). Finally, let s_T represent the schooling level at the time of the test ($s_T = 1,...,S_T$).

The model for the *i*th cognitive test score taken at schooling level $s_T(C_i(s_T))$ is

(8)
$$C_i(s_T) = \varphi_{C_i}(s_T) + \beta_{C_i}(s_T) X_C + \alpha_{C_i}(s_T) f_C + e_{C_i}(s_T),$$

where $e_{C_i}(s_T) \perp (f_C, X_C)$ and $e_{C_j}(s_T) \perp e_{C_j}(s_T')$ for any $i, j \in \{1, ..., n_C\}$ and s_T, s_T' such that either $i \neq j$ for any (s_T, s_T') or $s_T \neq s_T'$ for any (i, j).²¹ The vector X_C in Equation 8 represents the set of observable characteristics affecting test scores (for example, family background variables).

Likewise, the model for the noncognitive measure N_j taken at schooling level s_T $(j=1,...,n_N$ and $s_T=1,...,S_T$) is

(9)
$$N_j(s_T) = \varphi_{N_j}(s_T) + \beta_{N_j}(s_T)X_N + \alpha_{N_j}(s_T)f_N + e_{N_j}(s_T),$$

where $e_{N_i}(s_T) \perp (f_N, X_N)$ and $e_{N_i}(s_T) \perp e_{N_j}(s'_T)$ for any $i,j \in \{1,...,n_N\}$ and s_T, s'_T such that either $i \neq j$ for any (s_T, s'_T) or $s_T \neq s'_T$ for any (i,j). All error terms (*e* variables with subscripts) are mutually independent, independent of (f_C, f_N) and independent of all the observable X's.

Notice that in Equation 8 noncognitive ability f_N is not included as a determinant of $C_i(s_T)$. Similarly, in Equation 9 cognitive ability f_C is not considered as a determinant of $N_j(s_T)$. In principle, these cross-constraints can be relaxed, allowing (f_C, f_N) to appear in both $C_i(s_T)$ and $N_j(s_T)$. However, in this case the interpretation, or labeling, of the components of **f** would not be straightforward. What may be interpreted as cognitive ability could actually be noncognitive ability, and vice versa. Therefore, the exclusions in Equations 8 and 9 are justified because they allow for a clean interpretation of the unobserved abilities in the model.²²

Equations 8 and 9 clearly illustrate that measured test scores (C_i and N_j) and unobserved ability ($\mathbf{f} = (f_C, f_N)$) can be understood as different concepts. Besides their dependency on \mathbf{f} , test scores are also determined by the individual's characteristics (X_C and X_N) as well as by the individual's schooling at the time of the test (s_T). This latter dependency is particularly important, since the model can control for the possibility of

^{21.} I use " $A \perp B$ " to denote that "A and B are statistically independent."

^{22.} Notice that there are no intrinsic units for the latent or unobserved abilities. Therefore, I need to assume $\alpha_{C_i} = \alpha_{N_i} = 1$ for some *i* (*i*=1,...,*n_C*) and *j* (*j* = 1,...,*n_N*). A similar normalization needs to be used in the case of uncertainty θ . These assumptions set the scale of (f_C, f_N, θ). The actual equations used when implementing these normalizations are discussed in Section V. The formal identification argument, including the role of these normalizations, is described in Appendix 1.

reverse causality of schooling on test scores. Finally, uncertainty does not appear in Equations 8 or 9 because test scores are assumed to be taken during t=0.

A.1. Using Test Scores to Identify Racial Differences in the Means of Unobserved Abilities

Up to this point, unobserved abilities have been assumed to have zero means for both races. However, it is possible to identify racial differences in the means of cognitive and noncognitive abilities under one additional assumption. I illustrate this idea by analyzing the case of cognitive ability.

Consider the cognitive test score C_i for whites and blacks

$$\begin{split} C_i^W &= \varphi_{C_i}^W + \alpha_{C_i}^W f_C^W + e_{C_i}^W \\ C_i^B &= \varphi_{C_i}^B + \alpha_{C_i}^B f_C^B + e_{C_i}^B, \end{split}$$

where $E(e_{C_i}^W) = E(e_{C_i}^B) = 0$ and, for a simpler exposition, the dependency of the scores on schooling (s_T) and observables (X_C) is omitted.

Suppose that blacks and whites have different means of unobserved cognitive abilities. Let μ_C^B and μ_C^W denote the means of the distributions of cognitive ability for blacks and whites, respectively, and Δ_C denote their difference, that is, $\Delta_C = \mu_C^B - \mu_C^W$. Then, under the assumption that $\varphi_{C_i}^W = \varphi_{C_i}^B$, it is easy to show that $E(C_i^W) - E(C_i^B) = (\alpha_{C_i}^W - \alpha_{C_i}^B) \mu_C^W - \alpha_{C_i}^B \Delta_C$.

Notice that the left-hand side of this equation can be directly computed from data on test scores. Finally, by normalizing μ_C^W to be equal to zero (or any other number), and since $\alpha_{C_i}^B$ is identified, I can directly obtain Δ_C . A similar logic can be applied in the case of noncognitive abilities (Δ_N).

The question then becomes which cognitive and noncognitive test scores to use for the computation of Δ_C and Δ_N , respectively. This is discussed in the empirical section of the paper (Section VI.B).

B. Incarceration as a Manifestation of Cognitive and Noncognitive Abilities

The racial differences in incarceration rates have received significant attention in the literature. Blacks are considerably more likely to be incarcerated than whites regardless of the age considered. I can use the structure of the model to study whether cognitive and noncognitive abilities can explain these differences. I do so by including in the analysis a binary model for whether or not an individual has been incarcerated during a particular time period *t*. Specifically, let J(t) denote a binary variable that takes a value of one if the individual has been incarcerated during period *t*, and zero otherwise. Thus, the model for J(t) is

 $J(t) = 1[I_J(t) > 0]$ for t = 0, ..., T,

where 1[A] denotes an indicator function that takes a value of one if the argument A is true, and zero otherwise, and $I_J(t)$ represents the associated latent variable, which is assumed to depend on the set of observable variables (K_t), abilities (f), and uncertainty (θ) according to the following equation $I_J(t) = \varphi_{J,t} + \beta_{J,t} \mathbf{K}_t + \alpha_{J,t} \mathbf{f} + \lambda_{J,t} \theta + e_{J,t},$

where $e_{J,t}$ is an idiosyncratic shock for t=0,...,T. Finally, in order to be consistent throughout the paper with the definition of \mathcal{I}_0 , θ is excluded from $I_J(0)$.

V. Implementing the Model

The source of information used in this paper is the National Longitudinal Survey of Youth 1979 (NLSY79). The sample is designed to represent the entire population of youth aged 14 to 21 as of December 31, 1978. Data was collected annually until 1994, then biannually until 2002 (the last year used in this paper). The NLSY79 collects extensive information on each respondent's labor market outcomes and educational experiences. The survey also includes data on the youth's family and community backgrounds. I use the nationally representative cross-section of black and white males, and the supplemental sample designed to oversample civilian black males. Additional details on the sample and variables used in this paper are presented in Appendix 2.

The model is estimated separately by race using Monte Carlo Markov Chain (MCMC) techniques. A two-component mixture of normals is used to model the distribution of each unobserved component. More precisely,

$$\begin{split} f_C &\sim p_1^C N(\mu_1^C, (\sigma_1^C)^2) + (1 - p_1^C) N(\mu_2^C, (\sigma_2^C)^2) \\ f_N &\sim p_1^N N(\mu_1^N, (\sigma_1^N)^2) + (1 - p_1^N) N(\mu_2^N, (\sigma_2^N)^2) \\ \theta &\sim p_1^\theta N(\mu_1^\theta, (\sigma_1^\theta)^2) + (1 - p_1^\theta) N(\mu_2^\theta, (\sigma_2^\theta)^2). \end{split}$$

These distributions provide enough flexibility in the estimation and do not impose normality a priori.

In the empirical implementation of the model, the theoretical time periods (*t*) are replaced by ranges of ages. Period 0 represents ages between 14 and 22, Period 1 between 23 and 27, Period 2 between 28 and 32 years of age, and Period 3 between 33 and 37.²³ This has implications for the definition of the labor market outcomes. Since hours worked and hourly wage depend on the periods (*t*), I use the average (over the given age range) hourly wage (from principal occupation) and the average annual hours worked as measures of $Y_s(t)$ and $H_s(t)$, respectively.^{24,25}

^{23.} The last period of age is selected based on the fact that every individual in the NLSY79 sample should report information at least up to age 37 by 2002.

^{24.} The sample is restricted to those individuals reporting between US\$2 and US\$150 (2000 dollars) per hour as hourly wage. The total number of hours worked was restricted to be in the range of 160 and 3,500 hours per year.

^{25.} This also has implications regarding the role of employment in the analysis. Given the five-year periods as proxies for t, the fraction of individuals without information on hours worked and hourly wages because of unemployment or inactivity is negligible, even for high school dropouts at early ages. Nevertheless, versions of the model with probit equations for employment (by age range and schooling levels) do not change the main results of this paper, but they do reflect the poor identification of the employment's equations given the lack of individuals reporting to be unemployed or inactive during five-year periods.

A multinomial probit model is used to approximate the schooling decision problem presented in Section IIIA.²⁶ The schooling levels considered in the analysis are: high school dropouts, high school graduates, some college (more than 13 years of schooling completed but without four-year college degree), and four-year college graduates.²⁷ Since there is no sequential schooling decision process in the model, the maximum schooling level reported in the sample (after age 27) is used to define the individuals' schooling levels.

The model for incarceration is estimated using a probit model.²⁸ The information on incarceration comes from the individual's description of the place of residence at the time of the interview in which "Jail" is a possible answer.

The cognitive test scores used in the measurement system are: Arithmetic Reasoning, Word Knowledge, Paragraph Composition, Math Knowledge, Numerical Operations and Coding Speed. As previously mentioned, these tests are components of the Armed Services Vocational Aptitude Battery (ASVAB) available for the NLSY79 sample and they are used to construct the Armed Forces Qualification Test (AFQT), which has been a widely used measure of cognitive skill or intelligence.

A detailed characterization of an individual's noncognitive abilities would require information on dimensions such as self-esteem, self-discipline, locus of control, motivation, and impulsiveness, among others. Instead, due to data limitations, this paper examines noncognitive abilities linked to an individual's locus of control and selfesteem. Specifically, two attitude scales are used as measures of noncognitive abilities: the Rosenberg Self-Esteem and Rotter Locus of Control scales. Both scales have been shown to be good predictors of labor market outcomes and social behaviors (see Heckman, Stixrud, and Urzua 2006).

One of the main concerns associated with the direct use of these scores is that in the NLSY79 sample, the same cognitive and noncognitive tests are answered by individuals with different ages and schooling levels. This implies, for example, that the information on cognitive test scores available for the NLSY79 sample does not control for the fact that an 18-year-old high school dropout is answering the same questions as a 20-year-old individual enrolled in college. If test scores affect schooling (a reverse causality problem), comparison of the scores would not be informative of the ability differentials between the two individuals. This is particularly relevant if we take into account that blacks report fewer years of education than whites at the time of the testing. This comparison problem is aggravated by the fact that different tests are collected at different periods. That is, while the cognitive tests are collected during the summer of 1980, the Rosenberg and Rotter scales are collected in 1980 and 1979 (both at the time of the interview), respectively.

Thus, while on average blacks report 11.17 years of education at the time cognitive test scores are collected, whites report 11.66 years of education. Likewise, at the time the Rosenberg scale is collected, blacks report 10.02 years of education versus

^{26.} More precisely, the schooling choice model can be interpreted as a multinomial probit model only after conditioning on the unobserved abilities. But since the individual's abilities are unknown by the econometrician, they must be integrated out during the estimation process.

^{27.} The some college category includes individuals obtaining two-year college degrees.

^{28.} The same argument explaining the particular characteristics of the multinomial probit model for schooling choices applies here as well.

10.42 years for whites, and at the time the Rotter scale is collected, blacks report 10.63 years of education versus 11.09 for whites. These differences do not allow for the direct interpretation of achievement test scores and attitude scales as good measures of cognitive and noncognitive abilities.

However, as discussed in Section IVA, the analysis in this paper solves this problem by controlling for the potential reverse causality of schooling at the time of the tests on test scores and attitude scales. The model does this by estimating separate equations, depending on the specific schooling level completed at the test date.²⁹ In the empirical implementation of the model, I consider the following schooling levels at the time of the test (s_T): less than tenth grade completed, between tenth and eleventh grade completed, twelfth grade completed.

Finally, I set the scale of unobserved cognitive ability by normalizing to one the coefficient associated with f_c in the equation for Coding Speed for individuals with less than 10 years of schooling at the time of the test. Similarly, I set the scale of unobserved noncognitive ability by using the same normalization on the coefficient associated with f_N in the equation for the Rosenberg Self-Esteem Scale for individuals with less than 10 years of schooling at the time of the test. Finally, for uncertainty, I normalize the loading in hours worked for high school dropouts at ages 23–27 to be equal to one.

Tables 2a and 2b present the variables included in the empirical implementation of the model, as well as the imposed exclusion and identification restrictions.

VI. Estimation Results

Tables 3 and 4 present evidence on the goodness-of-fit for (log) hourly wages and (log) annual hours worked, respectively. The model does well in predicting the means and standard deviations of hourly wages among whites for any given schooling level and age range (see Panels A and B in Table 3). Formal goodness-of-fit tests cannot reject the null hypothesis that the simulated distributions of hourly wages for whites are statistically equivalent to the actual distributions (Panel C in Table 3). The performance of the model predicting (log) hourly wages for blacks is also good, although some of the goodness-of-fit tests suggest differences between the simulated and actual distributions for high school dropouts and high school graduates.

The results for hours worked are not as positive as the ones for hourly wages. Table 4 shows that, even though the model does well in predicting the means and standard deviations of the distributions of hours worked for any given race, schooling level, and age range (Panels A and B), formal goodness-of-fit tests suggest the rejection of the hypotheses of equal distributions. However, this result does not have major consequences for the purpose of this paper, since hours worked (in combination with hourly wages) are used to construct annual earnings and, as shown below, the empirical model does a good job capturing the observed racial inequality in annual earnings.

Tables 5 and 6 analyze the performance of the model in predicting schooling choices, and incarceration rates, respectively. The results from the goodness-of-fit tests show that the model does well in these dimensions for both whites and blacks.

^{29.} See Hansen, Heckman, and Mullen (2004) for a formal exposition of this idea.

		Educational Choice Model ^b					
Variables	Hourly Wage ^a and Hours Worked	HS Dropouts	HS Graduates	Some College	4-yr. Degree		
Region of residence	Yes	Yes	Yes	Yes	_		
Urban residence	Yes	Yes	Yes	Yes	_		
Family income in 1979	_	Yes	Yes	Yes	_		
Broken home at age 14 (Dummy)	_	Yes	Yes	Yes	_		
Number of siblings at age 17 (Dummy)	_	Yes	Yes	Yes	—		
Mother highest grade completed at age 17	_	Yes	Yes	Yes	_		
Father highest grade completed at age 17	-	Yes	Yes	Yes	-		
LUR ^c of high school dropouts at age 17	_	Yes	-	_	_		
LUR of high school graduates at age 17	-	-	Yes	_	-		
LUR of attendees of some college at age 17	_	-	_	Yes	_		
LUR for college graduates at age 17	_	-	_	-	Yes		
Tuition at two year college at age 17	_	_	-	Yes	_		
Tuition at four year college at age 17	_	-	-	-	Yes		
Factors							
Cognitive	Yes	Yes	Yes	Yes	-		
Noncognitive	Yes	Yes	Yes	Yes			
Uncertainty	Yes	No	No	No	_		
Estimated by	Schooling Level and Age Range	-	-	_	-		

Table 2a

Variables in the Empirical Implementation of the Outcome Equations

Notes: a. The hourly wage and annual hours worked models are estimated for four different categories (high school dropouts, high school graduates, some college, and four-year college graduates) and for three different ranges of age (23-27, 28-32 and 33-37).

b. The educational choice model is estimated considering four different categories: high school dropouts, high school graduates, some college, and four-year college graduates.

c. LUR = Local Unemployment Rate.

Table 2b

	Measuremen	nt System	
Variables	Incarceration ^a	Test Scores (Cognitive Variables) ^b	Attitude Scales (Noncognitive Variables) ^c
Living in a urban area at age 14 (dummy)	_	Yes	Yes
Living in the south at age 14 (dummy)	_	Yes	Yes
Mother highest grade completed at age 17	Yes	Yes	Yes
Father highest grade completed at age 17	Yes	Yes	Yes
Number of siblings at age 17 (dummy)	Yes	Yes	Yes
Family income in 1979	Yes	Yes	Yes
Broken home (dummy)	Yes	Yes	Yes
Age dummies Factors	-	Yes	Yes
Cognitive	Yes	Yes	No
Noncognitive	Yes	No	Yes
Uncertainty	Yes	No	No
Estimated by	Age Range	Grade Completed at the Time of the Test	Grade Completed at the Time of the Test

Variables in the Empirical Implementation of the model

Notes: a. There are four models of incarceration, one for each age range (14-22, 23-27, 28-32, and 33-37). Uncertainty is excluded from the model estimated for the first period.

b. Test scores are standardized to have within-sample mean 0, variance 1 in the overall population. The included cognitive variables are Arithmetic Reasoning, Word Knowledge, Paragraph Comprehension, Math Knowledge, Coding Speed, and Numerical Operations;

c. The included noncognitive variables are Rotter Locus of Control Scale and Rosenberg Self-Esteem Scale. The locus of control scale is based on the four-item abbreviated version of the Rotter Internal-External Locus of Control Scale. This scale is designed to measure the extent to which individuals believe they have control over their lives through self-motivation or self-determination (internal control) as opposed to the extent that the environment controls their lives (external control). The Self-Esteem Scale is based on the ten-item Rosenberg Self-Esteem Scale. This scale describes a degree of approval or disapproval toward oneself. In both cases, I standardize the test scores to have within-sample mean zero and variance one in the overall population.

It is not possible to reject the null hypothesis that the model produces the same distributions of schooling decisions and incarceration rates as the ones observed in the actual data. These tables also show the substantial racial differences in schooling attainment and incarceration rates observed in the data.

Table 3

Goodness of Fit - (Log) Hourly Wages by Age Schooling Level, and Race

		Whites			Blacks	
Schooling Level	23-27	28-32	33-37	23-27	28-32	33-37
A. Means						
High school dropouts						
Actual	2.309	2.387	2.412	2.147	2.185	2.232
Model	2.329	2.402	2.417	2.156	2.201	2.233
High school graduates						
Actual	2.436	2.567	2.645	2.198	2.282	2.321
Model	2.435	2.563	2.640	2.204	2.300	2.324
Some college						
Actual	2.488	2.691	2.809	2.355	2.460	2.544
Model	2.481	2.686	2.797	2.350	2.438	2.561
Four-year college graduates						
Actual	2.524	2.868	3.106	2.482	2.723	2.896
Model	2.539	2.860	3.097	2.448	2.693	2.856
B. Standard deviations						
High school dropouts						
Actual	0.373	0.388	0.455	0.351	0.391	0.424
Model	0.385	0.420	0.488	0.360	0.407	0.436
High school graduates	01000	01.120	01100	0.000	01107	01100
Actual	0.368	0.402	0.453	0.356	0.388	0.429
Model	0.376	0.419	0.466	0.363	0.392	0.435
Some college						
Actual	0.376	0.429	0.517	0.375	0.408	0.407
Model	0.388	0.458	0.564	0.392	0.428	0.450
Four-year college graduates						
Actual	0.380	0.437	0.522	0.379	0.454	0.472
Model	0.385	0.445	0.540	0.439	0.532	0.550
C. Goodness of fit test (p-va	lue) ^a					
High school dropouts	0.588	0.766	0.294	0.004	0.009	0.000
High school graduates	0.431	0.448	0.816	0.177	0.014	0.029
Some college	0.143	0.934	0.262	0.055	0.502	0.440
Four-year college graduates	0.115	0.135	0.048	0.437	0.951	0.809

Notes: The simulated data (Model) contains 20,000 observations generated from the Model's estimates. The actual data (Actual) comes from the NLSY79 sample of Males. For each individual, the schooling level refers to the maximum schooling level reported in the sample. a. Goodness of fit is tested using a χ^2 test where the Null Hypothesis is *Model=Data*.

Table 4

Goodness of Fit - (Log) Annual Hours Worked by Age Range, Schooling Level, and Race

		Whites			Blacks	
Schooling Level	23-27	28-32	33-37	23-27	28-32	33-37
A. Means						
High school dropouts						
Actual	7.365	7.461	7.501	7.041	7.078	7.253
Model	7.378	7.466	7.508	7.136	7.121	7.226
High school graduates						
Actual	7.548	7.648	7.669	7.264	7.367	7.414
Model	7.549	7.635	7.664	7.291	7.381	7.388
Some college						
Actual	7.488	7.608	7.644	7.229	7.450	7.495
Model	7.470	7.590	7.638	7.189	7.400	7.675
Four-year college graduates						
Actual	7.261	7.652	7.720	7.188	7.610	7.631
Model	7.268	7.653	7.716	7.181	7.596	7.559
B. Standard deviations						
High school dropouts						
Actual	0.533	0.528	0.474	0.869	0.831	0.685
Model	0.577	0.611	0.508	0.859	0.851	0.719
High school graduates						
Actual	0.404	0.310	0.308	0.679	0.605	0.613
Model	0.412	0.320	0.313	0.677	0.604	0.652
Some college						
Actual	0.481	0.354	0.365	0.628	0.544	0.556
Model	0.518	0.391	0.400	0.668	0.586	0.686
Four-year college graduates						
Actual	0.549	0.340	0.274	0.599	0.357	0.387
Model	0.555	0.330	0.284	0.623	0.391	0.597
C. Goodness of fit test (<i>p</i> -value) ^a						
High school dropouts	0.000	0.000	0.000	0.000	0.000	0.000
High school graduates	0.000	0.000	0.000	0.000	0.000	0.000
Some college	0.000	0.000	0.000	0.000	0.000	0.000
Four-year college graduates	0.000	0.000	0.000	0.000	0.000	0.000

Notes: The simulated data (Model) contains 20,000 observations generated from the Model's estimates. The actual data (Actual) comes from the NLSY79 sample of Males. For each individual, the schooling level refers to the maximum schooling level reported in the sample. a. Goodness of fit is tested using a χ^2 test where the Null Hypothesis is *Model=Data*.

An alternative perspective of the performance of the model is presented in Figures 1 and 2. Figure 1 presents, for each age range, the fraction of blacks reporting hourly wages within different quantiles of the white distribution of wages. The model satisfactorily mimics the large inequality of wages in the sample. It captures the fact that while approximately 50 percent of blacks report hourly wages that are below the 25th percentile of the white distribution, only less than 10 percent of blacks report wages above the 75th percentile of the white distribution. This is consistent across different age groups.

Figure 2 repeats the analysis, but for annual earnings. The performance of the model is less satisfactory than in the case of wages, but the model still mimics well the large inequality observed in the data, especially for the last two age ranges.³⁰

Therefore, based on the previous results, it is possible to conclude that the model predicts well the actual racial inequality in labor market outcomes (wages and earnings), schooling choices, and incarceration rates.

A. Schooling at Test Date, Observed Test Scores, and Racial Gaps

As explained in Section IVA, this paper treats observed cognitive and noncognitive test scores as the outcomes of a process that has as inputs schooling (at the time tests are taken), family background (mother's and father's education, number of siblings, among others), and unobserved abilities. Additionally, the analysis does not constrain the parameters associated with this process to be same across races, so blacks and whites are allowed to have different production technologies of cognitive and non-cognitive test scores. This interpretation of the observed ability measures is formally established in Equations 8 and 9. These equations can be used to analyze the existence of black-white gaps in observed cognitive and noncognitive scores after controlling for unobserved abilities and schooling at test date. Additionally, they can be used to study the effect of schooling (at test date) on observed scores (controlling for unobserved abilities), and whether or not this effect differs across races.

Equation 8 is used to construct each of the panels in Figure 3. Each panel shows the significant and positive effect of schooling (at test date) on each of the observed cognitive measures utilized in this paper. The patterns are similar for blacks and whites. It is worth noting that in order to control for the levels of unobserved cognitive abilities, the average scores utilized in this figure are constructed assuming the same level on unobserved cognitive ability across races and schooling levels (at test date) ($f_C^W = f_C^B = 0$), whereas the observable characteristics are set to their respective sample means (black or white sample means). This allows us to distinguish between comparisons of test scores and abilities, controlling for schooling at test date.

In addition to the significant effect of schooling on test scores, Figure 3 also illustrates the sizeable black-white gaps in cognitive test scores. Regardless of the cognitive measure and schooling level considered, on average, whites have significantly higher test scores than blacks even after controlling for unobserved cognitive ability. The range of values for the computed black-white gaps in test scores is

^{30.} Annual earnings are constructed using the information on hours worked and hourly wages. The fact that for earnings I find some discrepancies between the model and data is due to the problems fitting hours worked reported in Table 4. Nevertheless, the evidence in Figure 2 is still satisfactory.

Table 5

	Wh	nites	Bla	icks
A. Schooling Level (100%)	Model	Actual	Model	Actual
High school dropouts	17.20	17.18	29.27	29.48
High school graduates	34.70	34.81	37.12	37.23
Some college	19.95	19.82	21.14	20.63
Four-year college graduates	28.15	28.19	12.48	12.66
B. Goodnes of fit (<i>p</i> -value) ^a	0.7	723	0.8	341

Goodness of Fit - Schooling Choices by Race

Notes: The simulated data (Model) contains 20,000 observations generated from the Model's estimates. The actual data (Actual) comes from the NLSY79 sample of Males.

a. Goodness of fit is tested using a χ^2 test where the Null Hypothesis is *Model=Data*.

between -0.5 and -1.5 (recall that the each cognitive test score is normalized to have a mean of zero and a variance of one in the overall population).

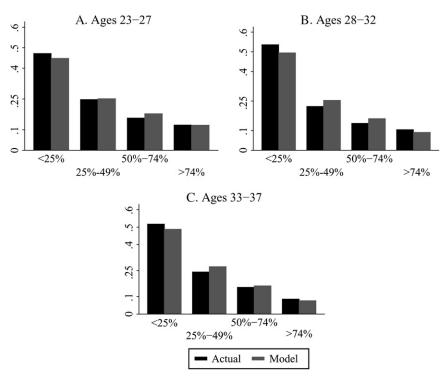
The results in Figure 3 also provide new insights about the implications associated with the standard practice of comparing white and black subjects with the same observed cognitive test scores. From the analysis of each panel in Figure 3, I can conclude that, even controlling for the level of unobserved cognitive ability, when we equate blacks and whites on the basis of test scores, we are in fact comparing blacks that have attained substantially more schooling at the test date to whites that have attained substantially less. This is particularly important if we consider the significant

Table 6

Goodness of Fit - Incarceration by Age Range and Race

		Wh	ites			Bl	acks	
Schooling Level	14-22	23-27	28-32	33-37	14-22	23-27	28-32	33-37
A. Fraction of individu in jail	als							
Actual	1.26	1.87	1.40	1.64	5.31	8.73	12.33	10.53
Model	1.60	1.59	1.09	1.50	7.94	8.31	12.02	10.42
B. Goodness of fit test (<i>p</i> -value) ^a	0.886	0.959	0.878	0.940	0.359	0.940	0.957	0.896

Notes: The simulated data (Model) contains 20,000 observations generated from the Model's estimates. The actual data (Actual) comes from the NLSY79 sample of Males. The binary variable Jail takes a value of one if the individual reports at least one episode of incarceration during the respective age range. a. Goodness of fit is tested using a χ^2 test where the Null Hypothesis is *Model=Data*.



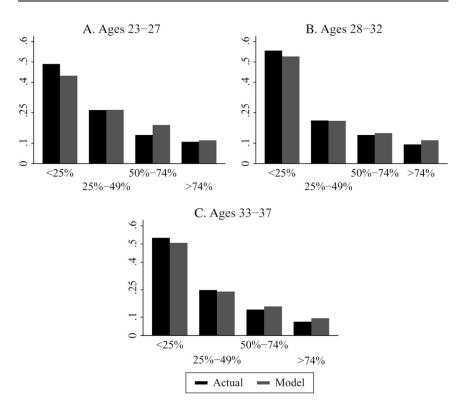
Location of Blacks in White Distribution–Hourly Wages Model Versus Data, by Age Range

Note: The panels in this figure compare the proportion of blacks with hourly wages in the respective percentile range of the white distribution obtained using simulated ("Model") and actual ("Actual") data. The simulated data represents a sample of 20,000 individuals generated from the estimates of the model. The simulated (log) hourly wages are obtained as follows. Let $Y_s^R(a)$ denote the simulated individual's log hourly wage at age *a* and schooling level *s* given individual's race *R* (*R*={White, Black}). Let D_s^R denote a dummy variable that takes a value of one if schooling level *s* is selected, and zero otherwise. Thus, at age *a*, the individual's log hourly wage is:

$$Y^{R}(a) = \sum_{s=1}^{3} D_{s}^{R} \times Y_{s}^{R}(a) \text{ where } R = \{\text{White, Black}\}.$$

racial differences in schooling attainment at the time the information on cognitive test scores is collected (see the discussion in Section V).

Figure 4 presents the same analysis but applied to noncognitive measures. The noncognitive scores are also affected by schooling at the time of the test, even after controlling for the level of unobserved noncognitive ability (which is set to 0 across races) and schooling levels. For locus of control (Rotter scale—Panel A in Figure 4), we observe that the gradient of the average score with respect to schooling is larger for whites than for blacks. On the contrary, in the case of self-esteem (Rosenberg



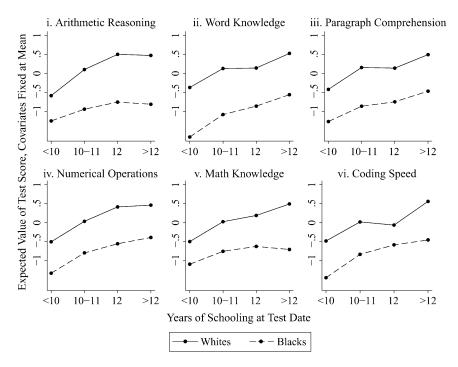
Location of Blacks in White Distribution—Annual Earnings Model versus Data, by Age Range

Note: The panels in this figure present the proportion of blacks with annual earnings in the respective percentile range of the white distribution computed using the simulated ("Model") and actual ("Actual") data. The simulated data represents a sample of 20,000 individuals generated from the estimates of the model. The simulated earnings are obtained as follows. Let $Y_s^R(a)$ and $H_s^R(a)$ denote the simulated individual's log hourly wage and log annual hours worked at age *a* and schooling level *s* given individual's race *R* (*R*={White, Black}), respectively. Let D_s^R denote a dummy variable that takes a value of one if the schooling level *s* is selected and zero otherwise. Thus, at age *a*, the individual's log earning is:

$$\mathcal{E}^{R}(a) = \sum_{s=1}^{S} D_{s}^{R} \times (Y_{s}^{R}(a) + H_{s}^{R}(a)) \text{ where } R = \{\text{Black, White}\}.$$

scale—Panel B in Figure 4), the average score for blacks presents the strongest response to the increase in schooling.

Unlike the case of the cognitive measures, Figure 4 does not show substantial black-white differences after controlling for schooling and unobserved noncognitive abilities. Nevertheless, the comparison of black and white subjects with the same



The Effect of Schooling on Observed Measures given $f_C^W = f_C^B = 0$: Black and White Males

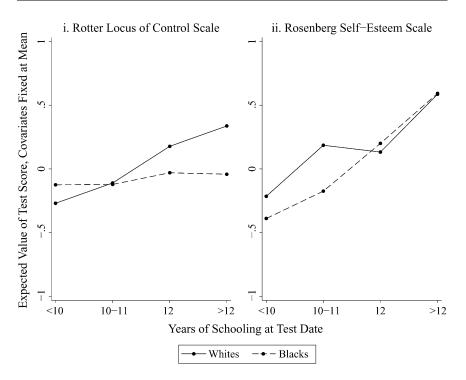
Notes: Each of the observed cognitive measures (test scores) is standardized to have mean zero and variance one in the overall population. Each panel depicts the average cognitive scores computed under the assumption of $f_C^W = f_C^B = 0$. The observable characteristics determining the observed scores are set to their respective sample means (black or white sample means). Formally, given the schooling level at the time of the test (s_T) and race (R), the panel associated with the observed cognitive measure C_i presents $\overline{C_i^R(s_T)}$ (the mean score) for $s_T = \{$ nine or less years of schooling, between ten and 11 years of schooling, 12 years of schooling, and some post-secondary education $\}$ where

$$\bar{C}_i^R(s_T) = \hat{\varphi}_{C_i}^R(s_T) + \hat{\beta}_{C_i}^R(s_T)\bar{X}_C^R + \alpha_{C_i}(s_T) \times 0,$$

and $R = \{\text{Black, White}\}$. $\hat{\varphi}_{C_i}^R(s_T)$ and $\hat{\beta}_{C_i}^R(s_T)$ represent estimated coefficients. The model is estimated using the NLSY79 samples of whites and blacks (see Appendix 2 for details).

observed noncognitive measures is still problematic. This is because of the significant racial differences in schooling attainment at the time the noncognitive test scores are collected and because the comparison of the raw scores does not consider the heterogeneity in unobserved noncognitive ability.

In summary, Figures 3 and 4 illustrate the limitations of using observed test scores as proxies for true abilities.



The Effect of Schooling on Noncognitive Scales given $f_N^W = f_N^B = 0$: Black and White Males

Notes: Each of the observed noncognitive measures (scales) is standardized to have mean zero and variance one in the overall population. Each panel depicts the average noncognitive scores computed under the assumption of $f_N^W = f_N^B = 0$. The observable characteristics determining the observed scores are set to their respective sample means (black or white sample means). Formally, given the schooling level at the time of the test (s_T) and race (R), the panel associated with the observed non-cognitive measure N_i presents $\bar{N}_i^R(s_T)$ (the mean score) for $s_T = \{$ nine or less years of schooling, between ten and 11 years of schooling, 12 years schooling, and some post-secondary education $\}$ where

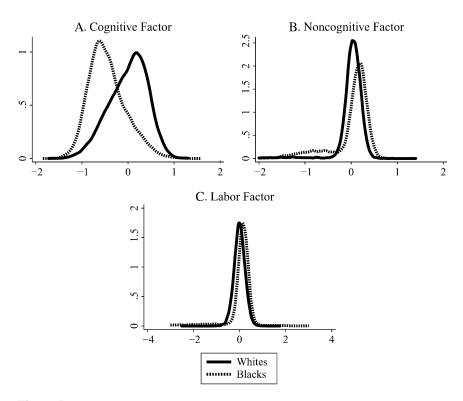
$$\bar{N}_i^R(s_T) = \hat{\varphi}_{N_i}^R(s_T) + \hat{\beta}_{N_i}^R(s_T) \bar{X}_N^R + \alpha_{N_i}(s_T) \times 0,$$

and $R = \{\text{Black, White}\}$. $\hat{\varphi}_{N_i}^R(s_T)$ and $\hat{\beta}_{N_i}^R(s_T)$ represent estimated coefficients. The model is estimated using the NLSY79 samples of whites and blacks (see Appendix 2 for details).

B. The Distributions of Abilities and Uncertainty

Figure 5 compares the estimated distributions of unobserved cognitive and noncognitive abilities (Panels A and B, respectively) and uncertainty (Panel C) across races.

Panel A shows that the distribution of cognitive abilities for blacks is dominated by the whites' distribution. The estimated difference between the means of the white and



Distribution of Unobserved Abilities and Uncertainty by Race

Note: Panel A compares the black and white distributions of unobserved cognitive ability. Panel B compares the distributions of noncognitive ability. Panel C compares the distributions of uncertainty across races. The distributions are computed using 20,000 simulated observations for each race. The simulated data is generated using the estimates of the model.

black cognitive distributions is 0.47, which represents a difference of approximately 1.1 standard deviations.³¹ This difference is consistent with evidence on racial differences in IQ tests reported elsewhere (Jensen 1998; Carroll 1993).³²

For noncognitive abilities, the results indicate that, although there are no significant differences in means, blacks and whites have different distributions of non-

^{31.} The standard deviations of cognitive ability for whites and blacks are 0.417 and 0.413, respectively.

^{32.} The difference in means is estimated using the logic described for a single cognitive test score in Section IV.A.1 but applied to all cognitive measures. Specifically, if Δ_{C_i} denotes the mean difference obtained applying the logic of Section IV.A.1 to test score C_i , then 0.47 represents the average across $\Delta_{C_1}, \dots, \Delta_{C_{n_c}}$ where n_c is the number of cognitive test scores. This difference in means is statistically significant at the five percent level.

cognitive abilities.³³ Panel B in Figure 5 shows a left fat-tailed distribution for blacks and a more symmetric distribution for whites. The estimated standard deviations are 0.347 for whites and 0.440 for blacks.

Panel C presents the distributions of uncertainty. The means of the two distributions are the same, even though by comparing the two figures it might be concluded that blacks face, on average, more uncertainty than whites. This is simply due to the visual effect produced by the asymmetries of the distributions. Overall, it is possible to conclude that uncertainty among blacks is more volatile than among whites (the standard deviations are 0.76 and 0.24, respectively).³⁴

I also use the distributions of abilities to shed light on racial differences, specifically in the way abilities affect schooling decisions. Figure 6 presents black and white distributions of abilities by schooling levels. Panels A and B show a clear sorting by cognitive ability: individuals loaded with high levels of cognitive abilities are more likely to be more educated, regardless of race. Panels C and D show that this result does not hold for noncognitive abilities. While there is a strong sorting among blacks, with individuals highly loaded with noncognitive abilities reaching higher levels of education (Panel D), the sorting is considerably weaker for whites. Interestingly, by comparing Figures 5 and 6, I conclude that most of the bimodality of the noncognitive distribution for blacks is linked to high school dropouts.

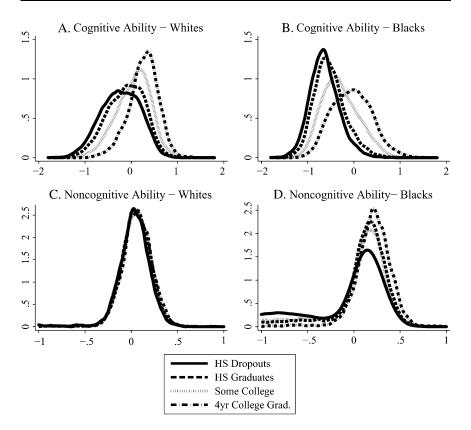
C. The Effect of Abilities and Uncertainty on Outcomes

Table 7 presents standardized estimates of the race-specific coefficients associated with abilities and uncertainty for labor market outcomes, schooling choices and incarceration model. For hours worked and hourly wages (Panel A in Table 7), the results show that cognitive and noncognitive abilities, in general, have positive and significant effects for both races. The numbers also indicate that noncognitive abilities always have stronger effects among blacks than whites. For cognitive ability, whether the effects are stronger for whites or blacks depends on the particular outcome, schooling level, and age range considered. Uncertainty on the other hand, has usually positive effects on hours and wages, and whether the effect of uncertainty is stronger among whites or blacks also depends on the labor market outcome, schooling level, and age considered.

The results from the schooling model on the other hand (Panel B in Table 7), suggest that both abilities have positive effects on individuals' schooling decisions (recall that the schooling level "Four-year-college graduate" is used as baseline level in the discrete choice model). While the effect of cognitive ability is stronger among

^{33.} The analysis of difference in the means of noncognitive abilities follows the logic used in the case of cognitive abilities. But for noncognitive abilities, the difference in means is not statistically significant (the implied p-value is 0.46).

^{34.} Formal tests reject the null hypothesis that blacks and whites share the same distributions of cognitive abilities, noncognitive abilities, and uncertainty. Additionally, formal tests of normality are strongly rejected for each unobserved component. These results are presented in Table A1 in the web appendix of the paper.



Racial Differences in schooling Sort: Distribution of Unobserved Abilities by Race and Schooling Level

Note: Each panel in this figure presents the distributions of unobserved abilities by schooling levels. The distributions are computed using 20,000 simulated observations for each race. The simulated data is generated using the estimates of the model. The schooling choices are the optimal decisions simulated from the model.

whites, the effect of noncognitive ability is much stronger among blacks. This in part explains the patterns observed in Figure 6.

It is important to note that there are significant differences across races for more than half of the estimates presented in Table 7. This is strong evidence supporting models that do not restrict parameters to be the same across races.³⁵

^{35.} Table A2 in the web appendix of the paper presents the *p*-values for the tests of equal coefficients across races for each of the parameters presented in Table 7.

Table 7 Standardized Loadings: Labor Market Outcomes, Schooling and Incarceration	rket Outcomes, S	chooling and Inc	arceration			
	Cognitiv	Cognitive Ability	Noncognit	Noncognitive Ability	Uncertainty ^a	ainty ^a
Equation	Blacks	Whites	Blacks	Whites	Blacks	Whites
A. Labor market outcomes						
A1. High school dropouts						
Wages - ages 23–27	0.03	0.08	0.06	0.03	0.04	0.23
Wages - ages 28–32	0.07	0.09	0.10	0.04	0.08	0.30
Wages - ages 33–37	0.09	0.07	0.12	0.06	0.09	0.36
Hours worked - ages 23-27	0.05	0.04	0.19	0.17	0.77	0.24
Hours worked - ages 28-32	0.17	0.05	0.41	0.32	0.35	0.16
Hours worked - ages 33-37	0.18	0.03	0.36	0.22	0.06	0.12
A2. High school graduates						
Wages - ages 23–27	0.14	0.07	0.06	0.02	0.12	0.28
Wages - ages 28–32	0.15	0.07	0.08	0.06	0.08	0.35
Wages - ages 33–37	0.18	0.07	0.19	0.07	0.07	0.35
Hours worked - ages 23-27	0.06	0.09	0.20	0.17	0.60	0.08
Hours worked - ages 28-32	0.10	0.04	0.40	0.22	0.23	0.05
Hours worked - ages 33-37	0.07	0.03	0.76	0.12	0.04	0.06
A3. Some college						
Wages - ages 23–27	0.01	0.02	0.11	0.02	-0.03	0.27
Wages - ages 28–32	0.04	0.02	0.21	0.09	0.01	0.36
Wages - ages 33–37	0.03	0.05	0.22	0.16	-0.16	0.36
Hours worked - ages 23-27	-0.09	-0.03	0.46	0.31	-0.10	0.12
Hours worked - ages 28-32	0.05	-0.01	0.56	0.20	0.00	0.10
Hours worked - ages 33-37	0.02	0.01	0.23	0.16	-0.63	0.04

Urzúa

949

A4. College graduates	000	0000	02.0			
wages - ages 23-21	0.08	60.0	0.70	/0.0	-0.09	-0.20
Wages - ages 28–32	0.10	0.13	1.02	0.05	-0.05	-0.38
Wages - ages 33–37	0.11	0.19	1.03	0.07	-0.03	-0.38
Hours worked - ages 23–27	0.13	-0.04	0.57	0.14	0.18	-0.11
Hours worked - ages 28-32	0.03	-0.04	0.42	0.29	0.20	-0.08
Hours worked - ages 33-37	0.06	0.01	0.08	0.10	0.56	-0.05
B. Schooling choices						
High school dropouts	-1.53	-1.58	-1.41	-0.45	I	I
High school graduates	-1.14	-1.26	-0.93	-0.20	I	I
Some college	-0.56	-0.84	-1.01	-0.28	I	I
C. Incarceration						
Ages 14-22	-0.37	-0.38	-0.60	-0.29	I	I
Ages 23-27	-0.37	-0.33	-0.77	-0.25	-0.12	-0.21
Ages 28-32	-0.32	-0.26	-0.93	-0.28	-0.01	-0.13
Ages 33-37	-0.30	-0.22	-0.80	-0.26	0.03	-0.09

numbers represent significant estimates (at five percent level). a. The loadings on uncertainty associated with the row "Hours Worked – Ages 23-27" for H.S. dropouts are normalized to one so the numbers are simply the standard deviations of uncertainty for blacks and whites, respectively. This normalization is necessary for the identification of the model.

The results for cognitive test scores and attitude scales (not shown in Table 7 but available in the web appendix) indicate that, in general, cognitive and noncognitive abilities have positive effects on the respective measures.³⁶

D. Understanding Racial Gaps in Labor Market Outcomes and Schooling Choices

From the previous analysis it is possible to conclude that there exist racial differences in the distributions of unobserved abilities (and uncertainty), that these abilities are important determinants of a variety of outcomes, and that the specific way they determine these outcomes depends on race. Given these facts, the question is then whether the observed racial differences in labor market outcomes and schooling choices can be interpreted as the result of these estimated differences in abilities.

We can analyze this question using the structure of the model. Specifically, by simulating data from the model, it is possible to study how the black-white gaps in outcomes change after blacks are compensated for racial differences in the distributions of unobserved abilities, uncertainty, and observed characteristics. To illustrate this, consider the case of hourly wages. Let $Y_s^R(a, X_a^R, \mathbf{f}^R, \theta^R)$ denote the potential (log) hourly wage at age *a* and schooling level *s*, given observed characteristics X_a^R , unobserved abilities \mathbf{f}^R , and uncertainty θ^R . The supra-index *R* denotes race (*R*={Black, Whit}). The individual's schooling decision on the other hand, depends on observed characteristics (X_0^R, \mathbf{Q}_0^R and \mathbf{P}^R) and unobserved abilities (\mathbf{f}^R). For notational simplicity, I define \mathbf{Z}^R as the vector of observed characteristics determining the schooling decision, that is, $\mathbf{Z}^R = (X_0^R, \mathbf{Q}_0^R, \mathbf{P}^R)$. Thus, I can denote by $D_s^R(\mathbf{Z}^R, \mathbf{f}^R)$ a dummy variable such that it is one if schooling level *s* is selected and, zero otherwise. The observed (log) hourly wage $Y^R(a, X_a^R, \mathbf{Z}^R, \mathbf{f}^R, \theta^R)$ is then:

(10)
$$Y^{R}(a, \boldsymbol{X}_{a}^{R}, \boldsymbol{Z}^{R}, \boldsymbol{f}^{R}, \boldsymbol{\theta}^{R}) = \sum_{s=1}^{S} D_{s}^{R}(\boldsymbol{Z}^{R}, \boldsymbol{f}^{R}) Y_{s}^{R}(a, \boldsymbol{X}_{a}^{R}, \boldsymbol{f}^{R}, \boldsymbol{\theta}^{R}),$$

where, for one (and only one) schooling level s^* , it is true that $D_{s^*}^R(\mathbf{Z}^R, \mathbf{f}^R) = 1$. Equation 10 contains the ingredients necessary to understand how the model can be used to assess if racial gaps can be interpreted as a manifestation of racial differentials in \mathbf{f} (as well as in X_a , \mathbf{Z} , or θ) in a framework that allows schooling decisions to be endogenously determined. Consider a case in which blacks are *compensated* for the differences in abilities. Here, the distribution of \mathbf{f} for whites would be used to generate the values of the abilities utilized when solving the model for blacks. As a result, I can construct the counterfactual hourly wage for blacks having been endowed with the white distribution of abilities. More precisely, for each individual in the sample, I can construct the variable

^{36.} The two exceptions are the loadings on Rotter for individuals with some postsecondary education at the time of the test, and on Rosenberg for individuals with less than tenth grade completed at the time of the test. In both cases, the estimated coefficients are not statistically significant (at five percent level). Table A3 in the web appendix presents these coefficients. Table A4 presents the *p*-values of the test of equal loading across races. For most of the cognitive test scores the null hypothesis of equal loadings is rejected. For the attitude scales, the null hypothesis of equal loadings across races is never rejected.

(11)
$$Y^B(a, \boldsymbol{X}_a^B, \boldsymbol{Z}^B, \boldsymbol{\mathrm{f}}^W, \boldsymbol{\mathrm{\theta}}^B) = \sum_{s=1}^S D^B_s(\boldsymbol{Z}^B, \boldsymbol{\mathrm{f}}^W) Y^B_s(a, \boldsymbol{X}_a^B, \boldsymbol{\mathrm{f}}^W, \boldsymbol{\mathrm{\theta}}^B),$$

and its associated distribution, which can be compared with the original variables represented by Equation 10 to analyze the consequences of the ability *compensation*. Similar exercises can be considered for the other determinants of Y^B .

Tables 8 and 9 present the results of several exercises on hourly wages and annual earnings, respectively, for the age range 28–32.³⁷

Panels A and B in Table 8 display the results as predicted by the model, that is, without compensations, for whites and blacks, respectively. Both present the mean (log) hourly wage by schooling level and in the population, and the distributions of schooling decisions. Panel B also presents the overall gap in mean (log) outcome as well as the contribution of each schooling level to it. Specifically, the row "Gap" in Panel B presents the terms from the following decomposition:

$$\begin{split} & E[Y^{W}(a, \boldsymbol{X}_{a}^{W}, \boldsymbol{Z}^{W}, \boldsymbol{\mathbf{f}}^{W}, \boldsymbol{\theta}^{W})] - E[Y^{B}(a, \boldsymbol{X}_{a}^{B}, \boldsymbol{Z}^{B}, \boldsymbol{\mathbf{f}}^{B}, \boldsymbol{\theta}^{B})] \\ & = \sum_{s=1}^{S} \left\{ \begin{split} & E[Y^{W}_{s}(a, \boldsymbol{X}_{a}^{W}, \boldsymbol{\mathbf{f}}^{W}, \boldsymbol{\theta}^{W}) | D^{W}_{s}(\boldsymbol{Z}^{W}, \boldsymbol{\mathbf{f}}^{W}) = 1] \Pr[D^{W}_{s}(\boldsymbol{Z}^{W}, \boldsymbol{\mathbf{f}}^{W}) = 1] \\ & -E[Y^{B}_{s}(a, \boldsymbol{X}_{a}^{B}, \boldsymbol{\mathbf{f}}^{B}, \boldsymbol{\theta}^{B}) | D^{B}_{s}(\boldsymbol{Z}^{B}, \boldsymbol{\mathbf{f}}^{B}) = 1] \Pr[D^{B}_{s}(\boldsymbol{Z}^{B}, \boldsymbol{\mathbf{f}}^{B}) = 1] \\ \end{split} \right\}. \end{split}$$

Using this decomposition, I can examine how the interaction of potential outcomes and schooling choices determine the overall gap in the population.

The results in Panels A and B show the well-known large differences between races in hourly wages. The largest gap is estimated among high school graduates (0.27), whereas the smallest is for college graduates (0.13). The large racial differences in schooling attainment are also presented in the table. Finally, the gap in hourly wages in the overall population is 0.28 with four-year-college graduates contributing the most to it.³⁸

Panel C presents analogous results but after compensating blacks for racial differences in the components of $Y^B(a, X^B_a, Z^B, \mathbf{f}^B, \theta^B)$. Specifically, Panel C1 presents the results when blacks are assumed to have the same distributions of observables and unobservables as whites; Panel C2 assumes that only the distribution of observables are equalized across races; Panel C3 assumes that the distributions of all unobserved components are equalized across races, and the results in Panels C4–C6 present the results when the distribution of each unobserved component is equalized across races. The results for whites (Panel A) are always used to compute the overall gaps and their associated decompositions.

The evidence in Panel C1 implies that when the distributions of observed and unobserved characteristics are equalized between races, the overall gap reduces 33 percent (from 0.28 to 0.19). This is a result of two effects: the significant improvement in

^{37.} The main results are similar for the other two age ranges, so they are not discussed in the text. They are available at the paper's web site.

^{38.} The contribution is defined according to decomposition of the overall gap described in the text. Notice that even small differences in hourly wages can be magnified because of the differences in schooling attainment. Among college graduates, the relatively small difference in mean (log) hourly wages (0.13) is amplified by the differences in college graduation rates across races (28 percent among whites 13 percent among blacks).

Table 8

Black-White Gap in Hourly Wages under Different Assumptions: Sample of 28-32 year old Males

 A. Outcomes for whites Hourly wages Percent in each schooling level B. Outcomes for blacks Hourly wages Percent in each schooling level Actual gap 	2.39 0.17 2.20 0.29	2.57 0.35 2.30 0.37	2.68 0.20 2.45 0.21	2.85 0.28 2.72	2.64
Hourly wages Percent in each schooling level B. Outcomes for blacks Hourly wages Percent in each schooling level	0.17 2.20 0.29	0.35 2.30	0.20 2.45	0.28	2.64
Percent in each schooling level B. Outcomes for blacks Hourly wages Percent in each schooling level	2.20 0.29	2.30	2.45		-
B. Outcomes for blacks Hourly wages Percent in each schooling level	0.29			2 72	
B. Outcomes for blacks Hourly wages Percent in each schooling level	0.29			2 72	
Hourly wages Percent in each schooling level	0.29			2 7 2	
Percent in each schooling level		0.37	0.21	2.12	2.36
	0.24		0.21	0.13	_
	0.24				
Actual gap	-0.24	0.04	0.02	0.46	0.28
C. Blacks with whites' ch	haracteristic	<i>s</i>			
C1. Observables and uno	bservables				
Hourly wages	2.25	2.42	2.52	2.47	2.45
Percent in each	0.11	0.23	0.33	0.34	_
schooling level					
Gap	0.17	0.35	-0.29	-0.04	0.19
C2. Observables					
Hourly wages	2.21	2.30	2.48	2.68	2.40
Percent in each	0.24	0.33	0.25	0.18	_
schooling level					
Gap	-0.11	0.14	-0.10	0.31	0.24
C3. Unobservables					
Hourly wages	2.26	2.43	2.47	2.50	2.44
Percent in each	0.15	0.29	0.30	0.26	_
schooling level					
Gap	0.07	0.18	-0.21	0.15	0.20
C4. Cognitive ability					
Hourly wages	2.24	2.43	2.45	2.73	2.49
Percent in each	0.15	0.28	0.29	0.28	_
schooling level					
Gap	0.07	0.21	-0.18	0.05	0.15
C5. Noncognitive ability					
Hourly wages	2.22	2.30	2.46	2.48	2.33
Percent in each	0.29	0.38	0.21	0.11	_
schooling level					
Gap	-0.24	0.02	0.01	0.53	0.31

(continued)

	H. S. Dropouts	H. S. Graduates	Some College	College Graduates	Overall
C6. Uncertainty					
Hourly wages	2.21	2.30	2.45	2.72	2.36
Percent in each schooling level	0.29	0.37	0.21	0.13	-
Gap	-0.24	0.04	0.02	0.46	0.28

Table 8 (continued)

Note: The numbers in this table present the mean (log) hourly wages (by schooling level and overall), the distribution of schooling decisions, and the racial gaps in (log) hourly wages. Panels A and B show these numbers for blacks and whites as predicted by the model. For example, for blacks (panel B), the row "Hourly wages" presents the means of $Y_s^B(a, X_a^B, f^B, \theta^B)$ for individuals selecting the respective schooling level *s*, whereas the row "Percent in each schooling level" presents the distribution of schooling decisions $D_s^B(\mathbf{Z}_s^B, \mathbf{f}^B)$ in the black population. For "Hourly wages" the last column (Overall) presents the average of (log) hourly wages $Y^B(a, X_a^B, \mathbf{z}^B, \mathbf{f}^B, \theta^B)$ in the population, where

$$Y^{B}(a, \boldsymbol{X}_{a}^{B}, \boldsymbol{Z}^{B}, \boldsymbol{\mathrm{f}}^{B}, \boldsymbol{\theta}^{B}) = \sum_{s=1}^{S} D_{s}^{B}(\boldsymbol{Z}^{B}, \boldsymbol{\mathrm{f}}^{B}) Y_{s}^{B}(a, \boldsymbol{X}_{a}^{B}, \boldsymbol{\mathrm{f}}^{B}, \boldsymbol{\theta}^{B}).$$

The numbers under the row "Gap" on the other hand, come from the following decomposition of the overall racial gap in (log) hourly wages

$$\begin{split} & E\left(Y^{W}\left(a, X_{a}^{W}, \mathbf{Z}^{W}, \mathbf{f}^{W}, \boldsymbol{\theta}^{W}\right) - Y^{B}\left(a, X_{a}^{B}, \mathbf{Z}^{B}, \mathbf{f}^{B}, \boldsymbol{\theta}^{B}\right)\right) \\ & = \sum_{s=1}^{S} \begin{pmatrix} E\left(Y_{s}^{W}\left(a, X_{a}^{W}, \mathbf{f}^{W}, \boldsymbol{\theta}^{W}\right) \middle| D_{s}^{W}\left(\mathbf{Z}^{W}, \mathbf{f}^{W}\right) = 1 \end{pmatrix} \Pr\left(D_{s}^{W}\left(\mathbf{Z}^{W}, \mathbf{f}^{W}\right) = 1\right) \\ & - E\left(Y_{s}^{B}\left(a, X_{a}^{B}, \mathbf{f}^{B}, \boldsymbol{\theta}^{B}\right) \middle| D_{s}^{B}\left(\mathbf{Z}^{B}, \mathbf{f}^{B}\right) = 1 \right) \Pr\left(D_{s}^{B}\left(\mathbf{Z}^{B}, \mathbf{f}^{B}\right) = 1\right) \end{split}$$

so, each column represents a term in the summation with the last column (Overall) presenting the total sum (or the gap). Panel C presents analogous results, but after modifying different components of $Y^B(a, X^B_a, Z^B, \mathbf{f}^B, \theta^B)$. In particular, Panel C1 presents the results when blacks are assumed to have the same distributions of observables and unobservables as whites, that is, it presents the results obtained using $Y^B_s(a, X^W_a, \mathbf{f}^W, \theta^W)$, $D^B_s(Z^W, \mathbf{f}^W)$, and $Y^B(a, X^W_a, Z^W, \mathbf{f}^W, \theta^W)$. Panel C2 assumes that only the distributions of observables are equalized across races. Panel C3 assumes that all of the distributions of the unobserved component is equalized across races. The results obtained when the distribution of each unobserved component is equalized across races. The results for whites (Panel A) are always used to compute the gaps.

the schooling attainment of blacks and their higher mean (log) hourly wage by schooling level. The reduction in the mean log hourly wage among college graduates is explained by the *compensation* of noncognitive ability and is analyzed in detail below. It is also interesting to see how the contribution of each schooling level to the overall gap is affected by the exercise. While in Panel B most of the contribution to the gap is coming from college graduates, in Panel C1 high school dropouts and high school graduates are the groups that contribute the most. This highlights the importance of considering the endogeneity of schooling decision in understanding the overall gap.

Panel C2 shows the role played by observed characteristics in the results presented in Panel C1. When the distributions of observables are equalized across races, blacks become slightly more educated, the average hourly wage by schooling level barely

Table 9

Black-White Gap in Annual Earnings under Different Assumptions Sample of 28-32 year old Males

	H.S. Dropouts	H.S. Graduates	Some College	College Graduates	Overall
A. Outcomes for whites					
Annual earnings	9.87	10.21	10.27	10.48	10.24
Percent in each schooling level	0.17	0.35	0.20	0.28	_
B. Outcomes for blacks					
Annual earnings	9.32	9.68	9.83	10.31	9.68
Percent in each schooling level	0.29	0.37	0.21	0.13	-
Actual gap	-1.05	-0.05	0.01	1.65	0.56
C. Blacks with whites' chard	acteristics				
C1. Observables and unobse	rvables				
Annual earnings	9.37	9.78	9.89	9.97	9.84
Percent in each	0.11	0.23	0.33	0.34	-
schooling level					
Gap	0.68	1.35	-1.20	-0.43	0.40
C2. Observables					
Annual earnings	9.21	9.62	9.82	10.26	9.69
Percent in each	0.24	0.33	0.25	0.18	-
schooling level					
Gap	-0.47	0.41	-0.46	1.08	0.55
C3. Unobservables					
Annual earnings	9.50	9.85	9.89	10.00	9.85
Percent in each schooling level	0.15	0.29	0.30	0.26	_
Gap	0.29	0.68	-0.93	0.36	0.39
C4. Cognitive ability					
Annual earnings	9.40	9.86	9.80	10.33	9.90
Percent in each	0.15	0.28	0.29	0.28	_
schooling level					
Gap	0.27	0.79	-0.81	0.09	0.34
C5. Noncognitive ability					
Annual earnings	9.43	9.65	9.86	9.97	9.67
Percent in each schooling level	0.29	0.38	0.21	0.11	—
Gap	-1.08	-0.12	-0.07	1.85	0.58

(continued)

	H.S. Dropouts	H.S. Graduates	Some College	College Graduates	Overall
C6. Uncertainty					
Annual earnings	9.32	9.67	9.83	10.31	9.68
Percent in each schooling level	0.29	0.37	0.21	0.13	-
Gap	-1.05	-0.05	0.01	1.65	0.56

Table 9 (continued)

Note: The numbers in this table present the mean (log) annual earnings (by schooling level and overall), the distribution of schooling decisions, and the racial gaps in (log) annual earnings. Let $\mathcal{E}_s^R(a, X_a^R, Q_a^R, \mathbf{f}^R, \theta^R)$ denote the log annual earnings given characteristics $(X_s^R, Q_a^R, \mathbf{f}^R, \theta^R)$, race R, schooling level s and age a. Formally, $\mathcal{E}_s^R(a, X_a^R, \mathbf{f}^R, \theta^R) = Y_s^R(a, X_a^R, \mathbf{f}^R, \theta^R) + H_s^R(a, Q_a^R, \mathbf{f}^R, \theta^R)$, where $Y_s^R(a, X_a^R, \mathbf{f}^R, \theta^R)$ and $H_s^R(a, Q_a^R, \mathbf{f}^R, \theta^R)$ represent the associated log hourly wage and log annual hours worked, respectively. Panels A and B show these numbers for blacks and whites as predicted by the model. For example, for blacks (panel B) the row "Annual earnings" presents the means of $\mathcal{E}_s^R(a, X_a^R, Q_a^R, \mathbf{f}^R, \theta^R)$ for individuals selecting the respective schooling level s, whereas the row "Percent in each schooling level" presents the distribution of schooling decisions $D_s^R(\mathbf{Z}^R, \mathbf{f}^R)$. The last column (Overall) presents the average log annual earning in the population. This average is constructed using the variable $\mathcal{E}^R(a, X_a^R, \mathbf{Z}^R, \mathbf{f}^R, \theta^R)$ which is generated as

$$\mathcal{E}^{B}(a, \mathbf{X}_{a}^{B}, \mathbf{Q}_{a}^{B}, \mathbf{Z}^{B}, \mathbf{f}^{B}, \theta^{B}) = \sum_{s=1}^{S} D_{s}^{B}(\mathbf{Z}^{B}, \mathbf{f}^{B}) \mathcal{E}_{s}^{B}(a, \mathbf{X}_{a}^{B}, \mathbf{Q}_{a}^{B}, \mathbf{f}^{B}, \theta^{B})$$

The row "Gap" on the other hand, presents the numbers associated with the following decomposition of the overall racial gap in (log) earnings

$$\begin{split} & E\big(\mathcal{E}^{W}\big(a, \boldsymbol{X}_{a}^{W}, \boldsymbol{Q}_{a}^{W}, \boldsymbol{Z}^{W}, \mathbf{f}^{W}, \boldsymbol{\theta}^{W}\big) - \mathcal{E}^{B}\big(a, \boldsymbol{X}_{a}^{B}, \boldsymbol{Q}_{a}^{B}, \boldsymbol{Z}^{B}, \mathbf{f}^{B}, \boldsymbol{\theta}^{B}\big)\big) \\ & = \sum_{s=1}^{S} \begin{pmatrix} E\big(\mathcal{E}_{s}^{W}\big(a, \boldsymbol{X}_{a}^{W}, \boldsymbol{Q}_{a}^{W}, \mathbf{f}^{W}, \boldsymbol{\theta}^{W}\big) \big| D_{s}^{W}\big(\boldsymbol{Z}^{W}, \mathbf{f}^{W}\big) = 1\big) \Pr\big(D_{s}^{W}\big(\boldsymbol{Z}^{W}, \mathbf{f}^{W}\big) = 1\big) \\ - E\big(\mathcal{E}_{s}^{B}\big(a, \boldsymbol{X}_{a}^{B}, \boldsymbol{Q}_{a}^{B}, \mathbf{f}^{B}, \boldsymbol{\theta}^{B}\big) \big| D_{s}^{B}\big(\boldsymbol{Z}^{B}, \mathbf{f}^{B}\big) = 1\big) \Pr\big(D_{s}^{W}\big(\boldsymbol{Z}^{B}, \mathbf{f}^{B}\big) = 1\big) \end{pmatrix} \end{split}$$

so, each column represents a term in the summation with the last column (Overall) presenting the total sum (or the gap). Panel C presents analogous results but after modifying different components of $\mathcal{E}^B(a, \mathbf{X}^B_a, \mathbf{Q}^B_a, \mathbf{Z}^B, \mathbf{f}^B, \theta^B)$. In particular, Panel C1 presents the results when blacks are assumed to have the same distributions of observables and unobservables as whites. Panel C2 assumes that only the observables are equalized across races. Panel C3 assumes that all of the distributions of unobserved components are equalized across races. Finally, Panels C4 to C6 present the results obtained when the distribution of each unobserved component is equalized across races. The results for whites (Panel A) are always used to compute the gap.

changes, and the overall gap reduces only 14 percent (from 0.28 to 0.24). Overall, the results from Panel C1 and C2 seem to indicate that observed characteristics help to reduce the gap but are not as important as unobservables. This is confirmed by the evidence presented in Panel C3. After compensating blacks for differences in unobserved characteristics, blacks are as educated as whites and the gaps in hourly wages by schooling level are smaller than the ones observed in the original samples except, again, for college graduates. As a result, the overall gap reduces to 0.20 representing a reduction of 28.5 percent, which is close to the 33 percent reduction presented in Panel C1.

Panel C4 displays the results when blacks are compensated in the distribution of unobserved cognitive ability. The results show again strong effects on schooling attainment. Blacks with the same distributions of cognitive ability as whites are more educated than whites. The compensation also has effects on hourly wages. Although still sizeable, the school-specific gaps are smaller than the original ones. The overall gap is reduced to 0.15 representing a reduction of 44 percent, larger than any of the previous results. High school graduates are the group contributing the most to the overall gap in this table.

The differences in the distributions of noncognitive abilities, on the other hand, have negligible effects on schooling attainment and hourly wages except for college graduates. In that case, the compensation leads to a reduction of 24 percent in average log hourly wages (2.72 versus 2.48). Two elements explain this result. First, the large effects of noncognitive abilities on hourly wages among blacks (see Table 7), and, second, the difference in the distributions of noncognitive ability between the original black college graduates and the compensated black college graduates. Specifically, while in the original sample of black college graduates the mean noncognitive ability is 0.20, for the compensated black college graduates (blacks with the white distribution of noncognitive abilities selecting endogenously to become college graduates) the mean is only 0.10. This difference is due to the high value of noncognitive abilities being more common among blacks than among whites.³⁹ These two considerations also explain the low average hourly wages for black college graduates reported in Panels C1 and C3. Finally, Panel C6 suggests that hourly wages are not sensitive to compensation in the distribution of uncertainty.

Table 9 presents the results for log annual earnings. The results follow basically the same patterns as the ones in Table 8, although because of the racial differences in hours worked, the gaps for earnings are larger than for wages. Panels A and B present the gaps in log earnings by schooling levels and in the overall population. The smallest gap is found for college graduates where, the estimated average black-white difference in log earnings is 0.17. This means that, on average, black college graduates make approximately 15.6 percent less in annual earnings than white college graduates per year between ages 28 and 32.⁴⁰ Analogously, the black-white differences in log earnings are 0.55 for high school dropouts, 0.54 for high school graduates, and 0.44 for some college. The overall difference in earnings is 0.56.

Panel C1 shows the effect of equalizing observed and unobserved characteristics. The effects on schooling attainment are identical to the ones reported in Table 8, but they are included in the table for completeness. Even though most of the gaps in earnings are reduced after the black-white differences in the distributions of observed and unobserved characteristics are eliminated, their magnitudes are still large. The overall gap is 0.40, which represents a reduction of 28 percent when compared to the original 0.56. Therefore, for (log) earnings, observables and unobservables can explain less of the gap compared to the results for wages.

The evidence from Panel C2 indicates that equalizing observed characteristics between races do not affect the overall gap. The overall gap computed in this case is 0.55 versus 0.56 from Panel B. However, a closer analysis of the results suggests that, although the overall gap is almost unchanged, all the schooling-specific gaps increase, with the largest gap found for high school dropouts (0.66). Thus, the final

^{39.} Panel B in Figure 3 illustrates this fact.

^{40.} The 15.6 percent is obtained as 1-exp(-0.17) and as previously mentioned, this calculation omits the fact that $E[\ln Y]$ is not the same as $\ln E[Y]$.

0.55 combines the positive effect of schooling (described in the context of Table 8) with the negative effects of schooling-specific earnings.

The results from the compensation in unobserved characteristics present a different story. Panel C3 shows that most of the school-specific gaps in earnings are reduced as a result of this exercise. The computed overall black-white gap is 0.39, which implies that unobserved characteristics explain approximately 30 percent of the actual gap. In addition, the evidence from Panel C4 suggests that cognitive abilities are the driving force behind this reduction. The gaps for high school dropouts, high school graduates, and college graduates are reduced once blacks have whites' distribution of cognitive ability, and the overall gap is only 0.34, which represents the smallest overall gap reported in Table 10. Consequently, I can conclude that the differences in cognitive ability explain approximately 39 percent of the actual gap in (log) annual earnings.

Panel C5 shows the results associated with the change in the distribution of noncognitive ability. In this case, the racial gaps in earnings are reduced for high school dropouts (from 0.55 to 0.44) and some college (from 0.44 to 0.41), but increase for high school (from 0.53 to 0.56) and college graduates (from 0.17 to 0.51). This large increase among college graduates is explained by the same argument used to explain the similar finding for wages, with the additional consideration that the noncognitive loadings on hours worked are also large for black college graduates.

Until this point, the analysis has considered only the age range between 28 and 32 years. As mentioned above, the results for the other two age ranges lead to similar conclusions and thus do not need additional discussion. I can, however, combine the results from all age ranges to analyze black-white differences in the present value of earnings (computed using information up to age 37). In order to achieve this, consider that, for each individual, the observed present value of earnings is

$$PV^{R}(\boldsymbol{X}^{R},\boldsymbol{Q}^{R},\boldsymbol{Z}^{R},\boldsymbol{f}^{R},\boldsymbol{\theta}^{R}) = \sum_{s=1}^{S} D_{s}^{R}(\boldsymbol{Z}^{R},\boldsymbol{f}^{R}) \sum_{a=1}^{\bar{A}} \rho^{a-1} \mathcal{E}_{s}^{R}(a,\boldsymbol{X}_{a}^{R},\boldsymbol{Q}_{a}^{R},\boldsymbol{f}^{R},\boldsymbol{\theta}^{R})$$

where, for one (and only one) schooling level s^* , it is true that $D_{s^*}^R(\mathbf{Z}^R, \mathbf{f}^R) = 1$, $\mathcal{E}_s^R(a, \mathbf{X}_a^R, \mathbf{Q}_a^R, \mathbf{f}^R, \theta^R)$ represents the earnings at age *a* given observed $(\mathbf{X}_a^R, \mathbf{Q}_a^R)$ and unobserved (\mathbf{f}^R, θ^R) characteristics, and the individual's race $R^{.41} \rho$ is the discount factor, which is assumed equal to 0.97; that is, the implicit discount rate is 0.03. Then, I can apply the same strategy previously utilized for wages and earnings for the analysis of black-white gaps in present value of earnings. Table 10 displays these results. The evidence in Panels A and B confirms the existence of sizeable blackwhite differentials regardless of the schooling level analyzed. The black-white gap is 34 percent in the overall population (378,006 versus 249,783), 32 percent for high school dropouts (290,430 versus 197,986), 35 percent for high school graduates (366,076 versus 237,463), 30 percent for some college (395,520 versus 277,939), and 17 percent for college graduates (433,795 versus 360,681). Again, given the

^{41.} In order to go from logs to levels, I take the exponential of the individual's log earning simulated from the model at each age. In this way, the simulated log earnings include the idiosyncratic disturbances. This strategy avoids taking exponential excluding the idiosyncratic disturbances that may lead to different results. Therefore, individual's earning (in levels) are used in the computation of the present values.

Table 10

Black-White Gap in Present Value of Annual Earnings under Different Assumptions: Ages 23 to 37

	H. S. Dropouts	H. S. Graduates	Some College	College Graduates	Overall	(Percent)
A. Outcomes for whites PV of earnings	290,430	366,076	395,520	433,795	378,006	
Percent in each schooling level	0.17	0.35	0.20	0.28	_	
B. Outcomes for blacks						
PV of earnings	197,986	237,463	277,939	360,681	249,783	
Percent in each schooling level	0.29	0.37	0.21	0.13	-	
Actual gap	-8,460	39,024	21,053	76,606	128,223	(0.34)
C. Blacks with whites' chara C1. Observables and unobser						
PV of earnings	191,647	241,041	279,250	271,533	258,539	
Percent in each schooling level	0.11	0.23	0.33	0.34	-	
Gap	29,198	72,980	-12,867	30,156	119,467	(0.32)
C2. Observables	.,	,)	,	.,	()
PV of earnings	181,601	228,256	276,369	354,081	252,529	
Percent in each schooling	0.24	0.33	0.25	0.18		
level						
Gap	7,111	52,712	8,236	57,419	125,477	(0.33)
C3. Unobservables						
PV of earnings	210,439	251,494	278,968	277,084	260,275	
Percent in each schooling level	0.15	0.29	0.30	0.26	-	
Gap	18,646	53,945	-5,340	50,480	117,731	(0.31)
C4. Cognitive ability						
PV of earnings	216,389	283,793	274,267	399,580	302,903	
Percent in each schooling level	0.15	0.28	0.29	0.28	-	
Gap	17,091	47,771	-1,283	11,523	75,103	(0.20)
C5. Noncognitive ability						
PV of earnings	203,710	218,922	264,053	269,541	229,685	
Percent in each schooling level	0.29	0.38	0.21	0.11	-	
Gap	-10,056	43,912	22,056	92,408	148,321	(0.39)
C6. Uncertainty						
PV of earnings	184,218	225,047	288,906	334,602	240,062	
Percent in each schooling level	0.29	0.37	0.21	0.13	-	
Gap	-4,402	43,646	18,791	79,909	137,944	(0.36)

Note: The numbers in this table present the mean present value of earnings (by schooling level and overall), the distribution of schooling decisions, and the racial gaps in present value of earnings. Let

 $\mathcal{E}_s^r(a, X_a^r, Q_a^r, \mathbf{f}^r, \theta^R)$ denote the log annual earnings given characteristics $(X_a^r, Q_a^r, \mathbf{f}^r, \theta^R)$, race *R*, schooling level *s* and age *a*. Panels A and B show these numbers for blacks and whites as predicted by the model. For example, for blacks (panel B) the row "PV of earnings" presents the means of

$$PV_s^B(a, X_a^B, \boldsymbol{Q}_a^B, \mathbf{f}^B, \boldsymbol{\theta}^B) = \sum_{a=1}^A \rho^{a-1} \mathcal{E}_s^B(a, X_a^B, \boldsymbol{Q}_a^B, \mathbf{f}^B, \boldsymbol{\theta}^B)$$

for individuals selecting the respective schooling level (s), whereas the row "percent in each schooling level" presents the distribution of schooling decisions $D_s^B(\mathbf{Z}^B, \mathbf{f}^B)$. The last column (Overall) presents the mean present value of earnings in the population. The row "Gap" on the other hand, presents the numbers associated with the following decomposition of the overall racial gap in PV of earnings

$$\begin{split} & E(PV^{W}(a, X_{a}^{W}, \mathbf{Q}_{a}^{W}, \mathbf{Z}^{W}, \mathbf{f}^{W}, \mathbf{\theta}^{B}) - PV^{B}(a, X_{a}^{R}, \mathbf{Q}_{a}^{B}, \mathbf{Z}^{B}, \mathbf{f}^{B}, \mathbf{\theta}^{B})) \\ & = \sum_{s=1}^{S} \begin{pmatrix} E(PV_{s}^{W}(a, X_{a}^{W}, \mathbf{Q}_{a}^{W}, \mathbf{f}^{W}, \mathbf{\theta}^{W}) | D_{s}^{W}(\mathbf{Z}^{W}, \mathbf{f}^{W}) = 1) \Pr(D_{s}^{W}(\mathbf{Z}^{W}, \mathbf{f}^{W}) = 1) \\ & -E(PV_{s}^{B}(a, X_{a}^{B}, \mathbf{Q}_{a}^{B}, \mathbf{f}^{B}, \mathbf{\theta}^{B}) | D_{s}^{B}(\mathbf{Z}^{B}, \mathbf{f}^{B}) = 1) \Pr(D_{s}^{W}(\mathbf{Z}^{B}, \mathbf{f}^{B}) = 1) \end{pmatrix} \end{split}$$

so, each column represents a term in the summation with the last column (Overall) presenting the total sum (or the gap). Panel C presents analogous results but after modifying different components of $PV^B(a, X^B_a, Q^B_a, Z^B, f^B, \theta^B)$. In particular, Panel C1 presents the results when blacks are assumed to have the same distributions of observables and unobservables as whites. Panel C2 assumes that all of the distributions associated with the observables are equalized across races. Panel C3 assumes that all of the distributions of unobserved components are equalized across races. Finally, Panels C4 to C6 present the results obtained when the distribution of each unobserved component is equalized across races. The results for whites (Panel A) are always used to compute the gap.

differences in schooling attainment, the difference for college graduates contributes the most to the overall gap.

The evidence in Panel C follows the same patterns previously discussed in the context of wages and earnings. That is, while the compensations in observables and (all) unobservables have minor effects on the racial gaps, when blacks are compensated only for the racial differences in unobserved cognitive ability the gaps, although still large, are reduced. Specifically, on average, blacks with the same distribution of cognitive ability as whites, still earn 20 percent less than whites (378,006 versus 302,903). The black-white gap is 25 percent for high school dropouts (290,430 versus 216,389), 22 percent for high school graduates (366,076 versus 283,793), 31 percent for some college (which represents a negligible increase in the gap) (395,520 versus 274,267), and 8 percent for college graduates (433,795 versus 399,580).

In summary, Tables 8, 9, and 10 show the existence of sizeable black-white differences in the means of labor market outcomes by schooling level and in the overall population even after controlling for the racial differences in the distributions of observed and unobserved characteristics. The results also indicate that racial differences in cognitive abilities are the most important force driving the black-white gaps in schooling attainment and labor market outcomes.

The previous analysis focuses on black-white gaps in means. This emphasis is useful for comparison to evidence in the literature, but it is unnecessary in the context of the model. Figure 7 presents a more general approach for the analysis of black-white gaps in hourly wages. It depicts the location of blacks in the white distribution of hourly wages under the different scenarios discussed above. Each panel in the figure presents the analysis for a specific age range. The bars are labeled accordingly to the different scenarios (compensations). The bars under "Model" present the location of blacks in the white distribution as predicted by the model (that is, without any compensation). The bars under "Observables and Unobservables" show the location of blacks in the

white distribution when blacks are assumed to have whites' distributions of observed and unobserved characteristics. The same logic applies for the other cases.

The comparison of the results across the different panels provides a different perspective of the strong and robust effects of the compensation for unobserved cognitive abilities. For example, while 49 percent of blacks report hourly wages in the first quartile of the white distribution between ages 28 and 32 (Panel B), the percentage falls to 37 percent after differences in cognitive ability are eliminated. Likewise, while only 10 percent of blacks have wages above the 74th percentile of the white distribution for the same age range, the percentage is 17 percent after the compensation. The results are similar for the other two age ranges.

Figure 8 repeats the analysis for earnings. In general, the results are qualitatively similar to the ones in Figure 7. The only significant difference is observed for the first range of ages. Here, when blacks have the same distributions of observables and unobservables as whites, the resulting distribution of earnings is more unequal than the original. This is due to the change in the distributions of hours worked, which is more right-skewed after the compensation. Nevertheless, even in this case, the compensation for cognitive abilities has positive effects on the black distribution of earnings.

Finally, Figure 9 presents a similar analysis for the present value of earnings. The results are similar and the importance of cognitive ability is again highlighted. For example, while 52 percent of blacks report present value of earnings in the first quartile of the white distribution, that percentage falls to 40 percent after the differences in cognitive ability are eliminated. Likewise, while only 7 percent of blacks have wages above the 74th percentile of the white distribution, the percentage is 14 percent after the compensation.

E. Incarceration and the role of noncognitive abilities

From the previous analysis, it is possible to conclude that black-white differentials in the distributions of noncognitive ability do not play a significant role in explaining racial gaps in labor market outcomes. Most of the reductions come from cognitive ability. The question is then whether this result also applies to the behavioral outcome analyzed in the model. The results from the incarceration model suggest a different story.

The evidence presented in Table 7 (Panel C) shows that cognitive and noncognitive abilities have large negative effects on the probability of incarceration for blacks and whites. The comparison across races, however, indicates that the effects of cognitive abilities are stronger among whites, whereas noncognitive ability has the stronger effects among blacks. Based on these results, and given the racial differences in the distributions of abilities, it is possible to infer that noncognitive abilities should play an important role in explaining the observed gaps in incarceration rates. Table 11 evaluates this idea by repeating the strategy used in the previous section but now applied to incarceration.

The results in Table 11 show that the large black-white gaps in incarceration rates are significantly reduced when the racial differences in observed and unobserved characteristics are eliminated. In particular, the percentage of blacks reporting at least one episode of incarceration between ages 14 and 22 reduces from 8 percent to 3.8 percent as a result of the change. This represents a significant reduction in the gap if we consider that the incarceration rate among whites is 1.6 percent for

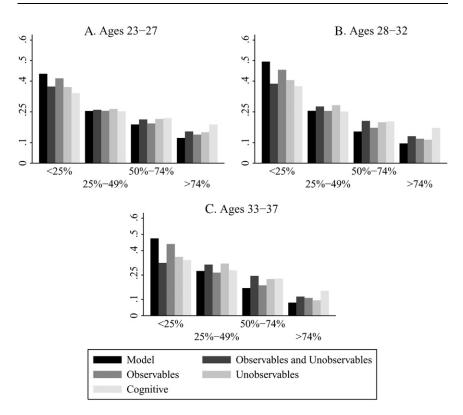


Figure 7

Location of Blacks in White Distribution under Different Scenarios: Hourly Wages

Note: The panels in this figure present the proportion of blacks with hourly wages in the respective percentile range of the white distribution under different scenarios. The bars under "Model" show the location of blacks in the white distribution as predicted by the model. The bars under "Observables and Unobservables" show the location of blacks in the white distribution when blacks are assumed to have white's distributions of observed and unobserved characteristics. Specifically, given individual's race R, let $Y_s^R(a, X_a^R, \mathbf{f}^R, \theta^R)$ denote potential (log) hourly wage at age a and schooling s, given observed (\mathbf{X}_a^R) and unobserved (\mathbf{f}^R, θ^R) characteristics, and let $D_s^R(\mathbf{Z}^R, \mathbf{f}^R)$ denote a dummy variable such that, given characteristics ($\mathbf{Z}^R, \mathbf{f}^R$), takes a value of one if the schooling level s is selected, and zero otherwise. Thus the bars under "Observables and Unobservables" compare the distribution of

$$Y^{B}(a, \boldsymbol{X}_{a}^{W}, \boldsymbol{Z}^{W}, \boldsymbol{f}^{W}, \boldsymbol{\theta}^{W}) = \sum_{s=1}^{S} D^{B}_{s}(\boldsymbol{Z}^{W}, \boldsymbol{f}^{W}) Y^{B}_{s}(a, \boldsymbol{X}_{a}^{W}, \boldsymbol{f}^{W}, \boldsymbol{\theta}^{W})$$

with the distribution of $Y^W(a, X_a^W, \mathbf{Z}^W, \mathbf{f}^W, \theta^W)$. The same logic applies for "Unobservables", "Observables" and "Cognitive" where $Y^B(a, X_a^B, \mathbf{Z}^B, \mathbf{f}^W, \theta^W), Y^B(a, \mathbf{X}_a^W, \mathbf{Z}^W, \mathbf{f}^B, \theta^B)$ and $Y^B(a, \mathbf{X}_a^B, \mathbf{Z}^B, f_C^W, f_N^B, \theta^B)$ are used instead of $Y^B(a, \mathbf{X}_a^W, \mathbf{Z}^W, \mathbf{f}^W, \theta^W)$, respectively.

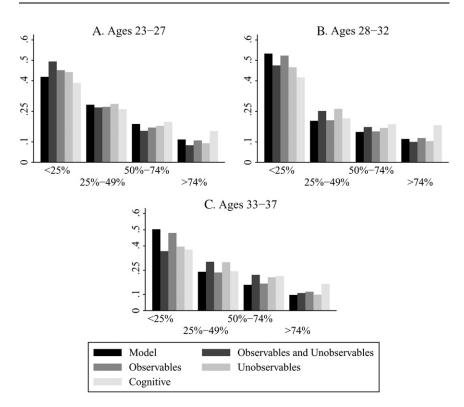


Figure 8

Location of Blacks in White Distribution under Different Scenarios: Annual Earnings

Note: The panels in this figure present the proportion of blacks with annual earnings in the respective percentile range of the white distribution under different scenarios. The bars under "Model" show the location of blacks in the white distribution as predicted from the model. The bars under "Observables and Unobservables" show the location of blacks in the white distribution when blacks are assumed to have whites' distributions of observed and uobserved characteristics. Specifically, given individual's race R, let $Y_s^R(a, X_a^R, \mathbf{f}^R, \theta^R)$ and $H_s^R(a, Q_a^R, \mathbf{f}^R, \theta^R)$ denote potential (log) hourly wage and potential (log) annual hours worked at age a and schooling s, given observed $(\mathbf{X}_a^R, \mathbf{Q}_a^R)$ and unobserved (\mathbf{f}^R, θ^R) characteristics, and let $D_s^R(\mathbf{Z}^R, \mathbf{f}^R)$ denote a dummy variable such that, given characteristics ($\mathbf{Z}^R, \mathbf{f}^R$), takes a value of one if the schooling level s is selected, and zero otherwise. Thus, the bars under "Observables" compare the distribution of the variable:

$$\mathcal{E}^{B}(a, \boldsymbol{X}_{a}^{W}, \boldsymbol{Q}_{a}^{W}, \boldsymbol{Z}^{W}, \boldsymbol{f}^{W}, \boldsymbol{\theta}^{W}) = \sum_{s=1}^{S} D_{s}^{B}(\boldsymbol{Z}^{W}, \boldsymbol{f}^{W})(Y_{s}^{B}(a, \boldsymbol{X}_{a}^{W}, \boldsymbol{f}^{W}, \boldsymbol{\theta}^{W}) + H_{s}^{B}(a, \boldsymbol{Q}_{a}^{W}, \boldsymbol{f}^{W}, \boldsymbol{\theta}^{W}))$$

with the distribution of $\mathcal{E}^{W}(a, X_{a}^{W}, Q_{a}^{W}, Z^{W}, \mathbf{f}^{W}, \theta^{W})$. The same logic applies for "Observables", "Unobservables" and "Cognitive" where the respective elements are equalized across races.

the same period. Similar and even stronger changes in gaps are found for the other ages. Furthermore, the comparison of the numbers in rows B1, B2, and B3 suggests that most of the reductions in the incarceration rates are due to compensations

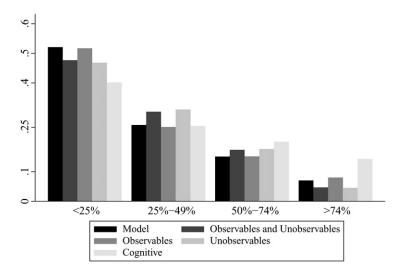


Figure 9

Location of Blacks in White Distribution under Different Scenarios: Present Value of Earnings

Note: This figure presents the proportion of blacks with present value of earnings in the respective percentile range of the white distribution of present value of earnings under different scenarios. In each case, the present value of earnings was created as follows. Let $\mathcal{E}_s^R(a, X_a^R, Q_a^R, \mathbf{f}^R, \theta^R)$ denote the potential (log) annual earnings at age *a* and schooling *s*, given observed (X_a^R, Q_a^R) and unobserved (\mathbf{f}^R, θ^R) characteristics, and race *R*. Likewise, let $D_s^R(\mathbf{Z}^R, \mathbf{f}^R)$ denote a dummy variable such that, given characteristics $(\mathbf{Z}^R, \mathbf{f}^R)$ takes a value of one if the schooling level *s* is selected, and zero otherwise. Thus if ρ denotes the discount rate, the present value of earnings is:

$$PV^{R}(\boldsymbol{X}^{R},\boldsymbol{Q}^{R},\boldsymbol{Z}^{R},\boldsymbol{f}^{R},\boldsymbol{\theta}^{R}) = \sum_{a=1}^{A} \rho^{a-1} \sum_{s=1}^{S} D_{s}^{R}(\boldsymbol{Z}^{R},\boldsymbol{f}^{R}) \mathcal{E}_{s}^{R}(a,\boldsymbol{X}_{a}^{R},\boldsymbol{Q}_{a}^{R},\boldsymbol{f}^{R},\boldsymbol{\theta}^{R}).$$

The discount factor (ρ) used in this figure is 0.97. The bars under "Model" depict the location of blacks is the white distribution as predicted by the model. The bars under "Observables and Unobervables" show the location of blacks in the white distribution when blacks are assumed to have whites' distributions of observed and unobserved characteristics. Specifically, these bars compare the distribution of:

$$PV^{B}(\boldsymbol{X}^{W},\boldsymbol{Q}^{W},\boldsymbol{Z}^{W},\boldsymbol{f}^{W},\boldsymbol{\theta}^{W}) = \sum_{a=1}^{\bar{A}} \rho^{a-1} \sum_{s=1}^{\bar{S}} D_{s}^{B}(\boldsymbol{Z}^{W},\boldsymbol{f}^{W}) \mathcal{E}_{s}^{B}(a,\boldsymbol{X}_{a}^{W},\boldsymbol{Q}_{a}^{W},\boldsymbol{f}^{W},\boldsymbol{\theta}^{W})$$

With the distribution of $PV^W(X^W, Q^W, Z^W, \mathbf{f}^W, \mathbf{\theta}^W)$. The same logic applies of "Obervables", "Unobservables" and "Cognitive" where the respective elements are equalized across races.

involving unobserved characteristics. Specifically, when the differences in all unobserved characteristics are eliminated, the incarceration rate becomes 3.75 percent for the age range 14–22. On the contrary, the rate is 8.07 percent if only the differences in observables are eliminated. The results are (again) similar for the other ages.

	Ages 14-22	Ages 23-27	Ages 28-32	Ages 33-37
A. Predicted				
Whites	1.60	1.59	1.09	1.50
Blacks	7.94	8.31	12.02	10.42
B. Blacks with whites'	characteristics			
B1. Observables and unobservables	3.81	2.73	4.76	4.28
B2. Observables	8.07	6.34	10.65	9.07
B3. Unobservables	3.75	3.52	5.55	5.24
B4. Cognitive	4.72	5.12	9.05	7.62
B5. Noncognitive	6.62	5.84	8.46	7.59

Table 11

Probability of Incarceration among Whites, Blacks and Blacks under Different Assumptions, by Age Range

Note: Let $J_a^B(a, K_a^B, \mathbf{f}^B, \theta^B)$ represents a dummy variable that takes a value of one if the black individual is incarcerated at age *a* given characteristics $(K_a^B, \mathbf{f}^B, \theta^B)$. Analogously, let $J_a^W(a, K_a^W, \mathbf{f}^W, \theta^W)$ denote the same dummy variable but for whites. Then, the numbers in this table present the proportions of individuals reporting to be incarcerated under different scenarios. Panels A presents the proportions among blacks and whites as predicted by the model. That is, the means of $J_a^W(a, K_a^W, \mathbf{f}^W, \theta^W)$ and $J_a^B(a, K_a^B, \mathbf{f}^B, \theta^B)$ in each population. Panel B presents the proportions for blacks when the distributions of K_a , f and θ are equalized across races. For example, "Observables and Unobservables" presents the proportions obtained using $J_a^B(a, K_a^W, \mathbf{f}^B, \theta^B)$. The same logic applies for rest of the rows in Panel B.

Finally, when the analysis is carried out separately by cognitive and noncognitive abilities, I obtain that each one contributes to the reductions in the gaps, but it is their interaction that reduces the gaps the most. Thus, unlike the case of labor market outcomes, differences in cognitive ability do not, by themselves, account for the large reductions in incarceration rates presented in B3. Black-white differences in noncognitive ability are as important determinants of these changes as differences in cognitive ability.

VII. Conclusions

This paper integrates schooling decisions and labor market outcomes to study whether or not the observed black-white gaps in labor market outcomes can be interpreted as a manifestation of racial differentials in unobserved abilities. Cognitive and noncognitive abilities are considered in the analysis.

The results from the empirical model provide strong evidence of differences in the distributions of unobserved abilities between blacks and whites. The effects of these abilities on schooling decisions, hourly wages and annual hours worked also differ by race. In particular, the effects of noncognitive ability are uniformly stronger for blacks than whites. This pattern is not observed for cognitive ability, and depending on the age range and outcome, the effect of cognitive ability can be stronger for either blacks or whites.

Unobserved cognitive ability is the most important variable explaining racial gaps in schooling attainment and labor market outcomes. When blacks are assumed to have the white distribution of unobserved cognitive ability, they achieve equal (or greater) education levels as whites. The overall racial gaps in wages and earnings fall by approximately 40 percent after this compensation. However, this is smaller than expected compared to previous evidence, which reports reductions in the range of 50 to 75 percent when black-white gaps in observed cognitive ability (achievement test scores) are controlled for (see Carneiro, Heckman, and Masterov 2005; and Neal and Johnson 1996). Racial differences in family background and schooling at the time of the tests are the determinants of the larger explanatory power of observed cognitive ability. The standard practice of equating cognitive test scores overcompensates for differentials in ability, resulting in underestimates of unexplained racial gaps.

On the other hand, even though the results indicate that unobserved noncognitive ability is an important determinant of schooling decisions and labor market outcomes, its role in explaining the black-white gaps in labor market outcomes is negligible. Nevertheless, unobserved noncognitive ability does play an important role in closing the black-white gaps in incarceration rates.

Finally, I consider it necessary to point out that, as always, the results analyzed in this paper are conditional on the assumptions of the empirical model being true and on the quality of the data available. In this context, future research should extend my analysis to more general models allowing, for example, for multiple and correlated cognitive and noncognitive abilities and for a direct role of incarceration over schooling attainment and/or labor market outcomes. Additionally, the study of a more comprehensive set of noncognitive measures also should be part of a future research agenda. The inclusion of these elements might provide an even better understanding of the observed and unobserved factors behind the racial gaps in labor market outcomes.

Appendix 1

Identification of the Model

This appendix presents the identification of the empirical model estimated in this paper. The argument follows the same logic utilized in Carneiro, Hansen, and Heckman (2003).

Let C_1 , C_2 and C_3 denote three cognitive measures. Following the structure of the model,

$$C_i = \alpha_{C_i} f_C + e_{C_i}$$
 for $i = 1, 2, 3$

where e_{C_i} represents an *iid* random variable, and f_C is the unobserved cognitive ability. Notice that from the covariances,

$$Cov(C_1, C_2) = \alpha_{C_1} \alpha_{C_2} Var(f_C)$$
$$Cov(C_2, C_3) = \alpha_{C_2} \alpha_{C_3} Var(f_C)$$

it is possible to identify $\alpha_{C_3}/\alpha_{C_1}$ since:

$$\frac{Cov(C_2,C_3)}{Cov(C_1,C_3)} = \frac{\alpha_{C_3}}{\alpha_{C_1}}.$$

Analogously, I can identify $\alpha_{C_2}/\alpha_{C_1}$ from the ratio of $Cov(C_1,C_3)$ and $Cov(C_2,C_3)$. Finally, by normalizing $\alpha_{C_1} = 1$, I obtain α_{C_2} and α_{C_3} . The following theorem, due to Kotlarski (1967), provides the conditions to secure the identification of the distribution of the unobserved cognitive ability.

Theorem 1.1 If

$$T_1 = f_C + e_1$$
$$T_2 = f_C + e_2$$

and $f_C \perp e_1 \perp e_2$, the means of f_C , e_1 and e_2 are finite, the conditions of Fubini's theorem are satisfied for each random variable, and the random variables possess non-vanishing characteristics functions, then the densities of f_C , e_1 and e_2 are identified.

Proof. See Kotlarski (1967). Thus, by writing

$$C_1 = f_C + e_1$$

$$\frac{C_2}{\lambda_2} = f_C + \frac{e_2}{\lambda_2},$$

the identification of distribution of f_C follows directly from Kotlarski (1967), given that assumptions are satisfied.

For the identification of the distribution of noncognitive ability I use a similar argument. In particular, consider the two noncognitive test scores and the latent variable associated with the incarceration model for period t

$$N_1 = \alpha_{N_1} f_N + e_{N_1}$$

$$N_2 = \alpha_{N_2} f_N + e_{N_2}$$

$$I_J(t) = \alpha_{J,C}(t)f_C + \alpha_{J,N}(t)f_N + \lambda_J(t)\theta + e_J(t),$$

where e_{N_1} , e_{N_2} , and $e_J(t)$ are *iid* random variables. Thus, since $f_C \perp f_N \perp \theta$, I have that

$$\frac{Cov(N_2, I_J(t))}{Cov(N_1, I_J(t))} = \frac{\alpha_{N_2}}{\alpha_{N_1}},$$

so the normalization $\alpha_{N_1} = 1$ ensures the identification of the loading α_{N_2} . With α_{N_2} in hand, I secure the identification of the distribution of f_N using Kotlarski's theorem.⁴²

The identification of the distribution of uncertainty can also be established using a different version of the same logic.

^{42.} Notice that the covariances between observed noncognitive test scores and the latent variable cannot be directly computed from the data. However, as long as the joint distribution of $(N_i, I_j(t))$ is identified the computation of the covariances is feasible (up to a scale). I can apply Theorem 1 in Carneiro, Hansen, and Heckman (2003) to prove the identification of this distribution.

Appendix 2

Data Description

This appendix contains details of sample construction as well as brief descriptions of the schooling, family background and family income variables, cognitive and noncognitive test scores, local labor market variables, and measures of local tuition, utilized in this paper.

A. Background on the NLSY Data

The National Longitudinal Survey of Youth (NLSY79) comprises three samples that are designed to represent the entire population of youth aged 14 to 21 as of December 31, 1978, and residing in the United States on January 1, 1979. I use the nationally representative cross-section and the set of supplemental samples designed to oversample civilian blacks. The military sample, the sample of civilian Hispanics, and the sample of economically disadvantaged Non-Black/non-Hispanic youths are excluded. Data was collected annually until 1994, then biannually until 2002 (last release utilized in this paper). NLSY79 collects extensive information on respondents' family background labor market behavior and educational experiences. The survey also includes data on the youth's family and community backgrounds.

Sample Characteristics

The sample used in this paper excludes females, Hispanics, the military sample, individuals reporting less than 160 or more than 3,500 hours worked per year, individuals reporting hourly wages less than \$2 and more than \$150, individuals for whom information on schooling attainment is not available, and individuals who are not interviewed after age 27. This produces a sample of 1,264 blacks and 2,159 whites. The racial classifications of blacks and whites are obtained directly from the NLSY79 guidelines.

B. Schooling Choices

The schooling levels considered in the analysis are: high school dropouts, high school graduates, some college (more than 13 years of schooling completed but without four-year college degree), and four-year college graduates. The some college category includes individuals obtaining two-year college degrees. Since there is no sequential schooling decision process in the model, the maximum schooling level reported in the sample (after age 27) is used to define the individuals' schooling levels.

C. Socioeconomic Status and Family Structure of the Sample

Family income and background variables include mothers and father's education in 1979, parental family income in 1979 dollars, whether the respondent came from a broken home at age 14 (that is, did not live with both biological parents), number of siblings in the household, and geographic information such as region of residence and urban residence at age 14.

D. Cognitive and Noncognitive Test Scores

The NLSY79 contains the Armed Services Vocational Aptitude Battery (ASVAB), which consists of ten tests that were developed by the military to predict performance in the armed forces training programs. The battery involves achievement tests designed to measure knowledge of general science, arithmetic reasoning, word knowledge, paragraph comprehension, numerical operations, coding speed, auto and shop information, mathematics knowledge, mechanical comprehension, and electronics information. This paper uses six of these ten tests: word knowledge, paragraph comprehension, mathematics knowledge, arithmetic reasoning, and coding speed.

The two attitude scales utilized as measures of noncognitive abilities are the Rosenberg Self-Esteem and Rotter Locus of Control scales. The Rotter Internal-External Locus of Control scale is designed to measure the extent to which individuals believe they have control over their lives through self-motivation (internal control) as opposed to the extent that the environment controls their lives (external control). The scale used in this paper is scored in the internal direction—the higher the score, the more internal the individual. The Rosenberg Self-Esteem scale describes a degree of approval or disapproval toward oneself. The scale contains ten statements of self-approval and disapproval with which respondents are asked to strongly agree, disagree, or strongly disagree. Higher scores are associated with higher levels of self-approval (self-esteem).

E. Local Variables

Direct and opportunity costs of attending school affect schooling decisions and must be included in the schooling choice equations. Local wage, unemployment, and tuition variables were constructed at the state and Metropolitan Statistical Area (MSA) levels and merged to each individual in NLSY79 sample using the NLSY's Geocode information that provides respondents' geographical location at each interview date. Local wage and local variables were constructed by four education levels in each MSA and attached to each individual at age 17 in the sample according to their education level at a given year. The schooling categories include high school dropouts, high school graduates, those with some college or associate degrees, and college graduates.

Local Unemployment

Local unemployment rates for each MSA and four schooling groups were generated from the monthly Current Population Survey (CPS) from 1978 to 2000. Without conditioning on education status, the constructed unemployment rates are highly consistent with the local unemployment rates available in the NLSY79 (NLSY79 includes local unemployment for each individual in the sample but these estimates do not vary between individuals within an MSA).

Tuition Data

Local tuition at age 17, for two-year and four-year public colleges (including universities) was constructed from annual records on tuition and enrollment from the Higher Education General Information Survey (HEGIS) and the Integrated Postsecondary Education Data System (IPEDS). By matching location with a person's county of residence, I was able to determine the presence of both two- and four-year colleges in an individual's county of residence. Public colleges were divided into two- and four-year programs, and a weighted average of tuition was generated for each county (college enrollment was used for weighting). This process was repeated at the state level.

References

- Altonji, Joseph G. and Rebecca M. Blank. 1999. "Race and Gender in the Labor Market." In Handbook of Labor Economics, vol. 3C, ed. O. Ashenfelter and D. Card, 3143–3259. Amsterdam: Elsevier Science.
- Becker, Gary S. 1964. *Human Capital: A Theoretical and Empirical Analysis, with Special Reference to Education.* New York: National Bureau of Economic Research, distributed by Columbia University Press.
- Bound, John, and Richard B. Freeman. 1992. "What Went Wrong? The Erosion of Relative Earnings and Employment among Young Black Men in the 1980s." *Quarterly Journal of Economics* 107(1):201–32.
- Bowles, Samuel, Herbert Gintis, and Melissa Osborne. 2001a. "The Determinants of Earnings: A Behavioral Approach." *Journal of Economic Literature* 39(4):1137–76.
 2001b. "Incentive-Enhancing Preferences: Personality, Behavior, and Earnings."
- American Economic Review 91(2):155–58.
- Cameron, Stephen V., and James J. Heckman. 2001. "The Dynamics of Educational Attainment for Black, Hispanic, and White Males." *Journal of Political Economy* 109(3):455–99.
- Card, David. 2001. "Estimating the Return to Schooling: Progress on Some Persistent Econometric Problems." *Econometrica* 69(5):1127–60.
- Carneiro, Pedro, Karsten Hansen, and James J. Heckman. 2003. "Estimating Distributions of Treatment Effects with an Application to the Returns to Schooling and Measurement of the Effects of Uncertainty on College Choice." *International Economic Review* 44(2):361–422.
- Carneiro, Pedro, James J. Heckman, and Dimitriy V. Masterov. 2005. "Labor Market Discrimination and Racial Differences in Pre-Market Factors." *Journal of Law and Economics* 48(1):1–39.

——. 2006. "Understanding the Sources of Ethnic and Racial Wage Gaps and Their Implications for Policy." In *Handbook of Employment Discrimination Research: Rights* and Realities, eds. R. Nelson and L. Nielsen, 99-136. New York: Springer.

- Carroll, John B. 1993. *Human Cognitive Abilities: A Survey of Factor-Analytic Studies*. New York: Cambridge University Press.
- Conley, James J. 1984. "The Hierarchy of Consistency: A Review and Model of Longitudinal Findings on Adult Individual Differences in Intelligence, Personality, and Self-Opinion." *Personality and Individual Differences* 5(1):11–25.
- Cunha, Flavio, James J. Heckman, Lance J. Lochner, and Dimitriy V. Masterov. 2006. "Interpreting the Evidence on Life Cycle Skill Formation." In *Handbook of the Economics of Education*, ed. Eric A. Hanushek and Frank Welch, 697–812. Amsterdam: North-Holland.
- Cunha, Flavio, James J. Heckman, and Salvador Navarro. 2005. "Separating Uncertainty from Heterogeneity in Life Cycle Earnings, The 2004 Hicks Lecture." *Oxford Economic Papers* 57(2):191–261.

- Farkas, George. 2003. "Cognitive Skills and Noncognitive Traits and Behaviors in Stratification Processes." *Annual Review of Sociology* 29:541–62.
- Farkas, George, Paula England, Keven Vicknair, and Barbara Stanek Kilbourne. 1997. "Cognitive Skill, Skill Demands of Jobs, and Earnings among Young European American, African American, and Mexican American Workers." *Social Forces* 75(3):913–38.
- Freeman, Richard B. 1991. "Crime and the Employment of Disadvantaged Youths." Working Paper 3875. National Bureau of Economic Research, Cambridge, Mass.
- Grogger, Jeff. 1998. "Market Wages and Youth Crime." *Journal of Labor Economics* 16(4):756–91.
- Hansen, Karsten T., James J. Heckman, and Kathleen J. Mullen. 2004. "The Effect of Schooling and Ability on Achievement Test Scores." *Journal of Econometrics* 121(1-2): 39–98.
- Heckman, James J., Jora Stixrud, and Sergio Urzua. 2006. "The Effects of Cognitive and Noncognitive Abilities on Labor Market Outcomes and Social Behavior." *Journal of Labor Economics* 24(3):411–82.
- Herrnstein, Richard J., and Charles A. Murray. 1994. *The Bell Curve: Intelligence and Class Structure in American Life.* New York: Free Press.
- Jensen, Arthur R. 1998. *The g Factor: The Science of Mental Ability*. Westport, Conn.: Praeger.
- Keane, Michael P., and Kenneth I. Wolpin. 2000. "Eliminating Race Differences in School Attainment and Labor Market Success." *Journal of Labor Economics* 18(4):614–52.
- Kotlarski, Ignacy I. 1967. "On Characterizing the Gamma and Normal Distribution." *Pacific Journal of Mathematics* 20:69–76.
- Lang, Kevin and Michael Manove. 2006. "Education and Labor-Market Discrimination." Working paper 12257. National Bureau of Economic Research, Cambridge, Mass.
- Mincer, Jacob. 1958. "Investment in Human Capital and Personal Income Distribution." Journal of Political Economy 66(4):281–302.
- Neal, Derek A. 2008. "Black-White Labour Market Inequality in the United States." In *The New Palgrave Dictionary of Economics*, ed. Steven Durlauf and Lawrence Blume. New York: Stockton Press. Forthcoming.
- Neal, Derek A., and William R. Johnson. 1996. "The Role of Premarket Factors in Black-White Wage Differences." *Journal of Political Economy* 104(5):869–95.

———. 1998. "Basic Skills and the Black-White Earnings Gap." In *The Black-White Test Score Gap*, ed. Christopher Jencks and Meredith Phillips, 480-97. Washington, D.C.: Brookings Institution Press.

- Todd, Petra E., and Kenneth I. Wolpin. 2003. "On the Specification and Estimation of the Production Function for Cognitive Achievement." *Economic Journal* 113(485):F3–33.
- Trzesniewski, Kali H., Richard W. Robins, Brent W. Roberts, and Avshalom Caspi. 2004. "Personality and Self-esteem Development Across the Life Span." In *Recent Advances in Psychology and Aging*, eds. Paul T. Costa Jr. and Ilene C. Siegler, 163-86. Boston: Elsevier.

Western, Bruce, and Becky Pettit. 2000. "Incarceration and Racial Inequality in Men's Employment." *Industrial and Labor Relations Review* 54(1):3–16.