Individuality, quantum physics, and a metaphysics of non-individuals: the role of the formal

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The notion of an individual and the related issues on individuation are topics that appear in the philosophical discussion ever since the antiquity. The idea of an individual thing is intuitively clear: an individual is something of a specific kind that is a unity, having its own identity, and being so that it is possible at least in principle to discern it from any other individual, even of similar species. But when we try to leave the intuitive realm and push this idea to a logical analysis, we find a cluster of problems that are difficult to overcome within standard logico-mathematical contexts. In this work, we shall be concerned with some aspects of this intuitive concept of an individual and on some related facts about individuation taken from recent discussions that arose ever since the inception of quantum theory, pushing the discussion to a "logical" view, which in our opinion is still lacking in the usual debates on such issues. In the final part of the paper, we propose a metaphysics where the notion of identity is substituted, for some objects, by a weaker notion of indiscernibility, and we try to justify such a move. In most of the uses of the expression "quantum theory", we shall not make explicit the distinction between the non-relativistic and the relativistic approaches, although they of course are quite different, for we think that the problems as we shall present them appear in both versions. But, as the text goes, the context will distinguish between them and these questions will become clear to the reader.

Individuals and individuation

Intuitively speaking, by an individual we understand something which in some context is considered as *one*, distinct from any other individual even of similar species, and which can at least in principle be re-identified in a different situation (within the same context) as being *that* same item. Some authors leave re-identification out of the account, claiming that since it is an *epistemological notion,* it does not make part of a legitimate definition of the *ontological notion* of individual. Here we shall not discuss which approach is preferable, but we shall point to the fact that with or without re-identification, some interesting metaphysical consequences may be drawn when we turn to quantum entities (see Wiggins [2012] for a defence of re-identification as an important feature of individuality). In expressing the concept the way we have done does not amount to providing a *definition* of an individual, due to the redundancies and vagueness of the proposed "definition", since most of the notions employed remain undefined, such as "distinct", "same", "re-identification", and so on. But we can continue with this informal description, saying that an individual (whenever it remains so) has an *identity*, that it has *individuality*, being *different* from any *other* individual (we shall take for granted that an individual is something of a kind). And this is in principle so. In our view, individuals may be entities (for the lack of a better word) of whatever sort, provided that they fall under the above characterization.

Notice that some of the intuitive conditions for individuality may be fail in some cases, making it clear that the item in question is not an individual. Thus, consider the example of an isolated cloud. Taken in isolation, it seems to be an individual: it has a definite identity, and it is one. However, the illusion is dissipated when we consider that our cloud sometime merges with other clouds to form a bigger one. In that case, it loses its identity, and we cannot claim that there is a determinate matter of fact as to whether there are, say, *n* clouds which happen to form the bigger one. There is no such thing, and so, there is no individuality for clouds. The same can be said of the content of a portion of water, say, which at first can be thought as an individual until merging the other portions in a lake, when its "individuality" is lost forever. So, our notion of individual is for now a very intuitive one, which applies to some things and does not applies to others, leaving some cases open. And this is due to the fact that we intend to address some questions concerning the quantum realm; we claim that despite the controversy reigning nowadays, we have different ways of seeing quantum objects other than "objects" (substances) of a kind. Anyway, our examples and main argumentation will deal with "standard individuals", that is, as "objects" of our intuition.

Thus, considering identification and the apparent individuality of some items, even if we don't have the actual means for pointing to a certain individual, for instance, a planet in some distant galaxy not seen yet, we still believe that it is an individual, and that its individuality is preserved to some extend, which apparently does not happen with clouds of portions of water (and perhaps with quantum objects). For instance, suppose we are following an ant entering an anthill. Once in the anthill, we cannot follow her any more (by hypothesis) and if some ants now leave the anthill, we may be in doubt if some particular ant is *our* ant. Even so, we believe that in some sense she *has* her individuality, and that the problem is ours of being incapable of distinguishing her from the other ants. But we can mark the ant before she enters the anthill, say with a little mark of paint and so, when some ants leave the anthill, we can verify whether a particular ant is or is not our ant by searching for the paint mark. Contrariwise, we cannot mark neither a cloud nor the content of a portion of water, nor a quantum object (here and in other parts of the text, when we speak of *quantum objects* we use "object" in a metaphysically neutral sense, as "thing" or "entity", and not in the traditional sense of the word which usually commits it to some form of identity condition for the items to which it applies); long time ago, Schrödinger made the remark that "we cannot mark an electron; we cannot paint it red" (Schrödinger [1953]). According to the standards of quantum theory, he is right. Of course, this further fact is not once again grounded in limitations of our abilities or of our technology, that is, it is not the case that we cannot mark an electron with a red paint because we are unable to so with our current technology: we cannot do that because an electron is an entity such that this kind of identification is rendered impossible.

The search for a Principle of Individuation is well documented in the philosophical literature (see Quinton [1973] and Lowe [2003] for an overview), and refers to the question: "what is it that confers to an individual its individuality?". One may take a great variety of approaches to this problem, which may include understanding individuality as a brute undefined fact, or as a consequence of some primitive undefined identity every individual is supposedly endowed with. In the same vein, we may hold that there is no such thing as individuality, so that no principle grounding it needs to be sought. In this line of reasoning, items may be taken as non-individuals, understood as a brute undefined fact. We shall not pursue those kinds of approaches here (but see our discussion of a version of primitive individuality related to quantum mechanics in Arenhart and Krause [2012]). Here we stick with the idea that individuality may be a defined concept, because this line of approach allows us to link identity, individuality and indiscernibility in a very profitable way from a metaphysical point of view as well as from a logical point of view; also, taking those notions as primitive does not seem to help us understanding them unless further explanations are provided. A third kind of answer which deserves to be mentioned could perhaps be summed up in terms of structures, lying within the realm of the ontological structural realism (Rickles and French [2006]). Some ontological structural realists, the ones pursuing eliminativism concerning objects, dismiss the discussion as to the metaphysical nature of particular objects as ill-grounded, since they take particular objects to be eliminable in terms of relations (more about this later). The relevance of ontic structuralism for us comes from the claim by its proposers of making an *Aufhebung* of the two poles of the discussion individuality/non-individuality, leaving only structures in its place.

Seen as a defined concept, there are two basic standard answers to the problem of a Principle of Individuality; the first group can be unified by the term *transcendental individuality*, while the second group falls under the common denomination of *bundle theories*. The first group assumes that the individuality of a thing is provided by something lying *behind* (or *transcending*) its qualities or attributes, something that recalls the Lockean concept of substratum. Thus, despite the fact that two distinct individuals may share all their qualitative properties, the underlying substratum works for the purpose of granting numerical identity and individuality. Also, there is an easy answer to property change and transtemporal identity: an individual can change all its qualities, but it remains the same since it has that particular substratum, something that does not change and which makes the individual *that* individual. This seems to be closer to the man-in-the-street view of the existence of the soul; although we grow old with time, and change most of our features from the childhood to the adult age, we are still ourselves due to some kind of quid we have, in the case, our souls. Obviously, we have employed the label transcendental individuality since it allows other kinds of approach to the problem, not only the substratum theory. To mention other cases of such approach, there are *haecceities* and *primitive thisness*. In general, a *haecceity* may be taken to be a *non-qualitative* property each item possesses and which serves for the purpose of individuation. In this sense, it is a non-shareable property which lies beyond the items qualities that grants individuality (there are distinct versions of haecceitism, and the doctrine is in general related to the philosophical problem of identity in distinct possible worlds; even though this is an interesting problem, we shall not deal with it here). The problems with those kinds of approaches are well known in the literature: we need to explain the nature the underlying substratum (or quid, or haecceity), a difficult task. Besides, the literature has presented good argumentation against its existence both in general as well as in the particular case of quantum entities (for instance, see Teller [1998]).

Bundle theories, on the other hand, do not suppose any kind of underlying substratum or non-qualitative properties; they rather see a particular object as bundle of properties instantiated together, so that only properties are needed to account for the nature of a particular object. Distinct versions of the theory arise accordingly as we specify how we should understand the "bundle" metaphor and also as we specify what it is that ties together the properties. Just as in the case of the substratum theory, however, the bundle theory may be seen as both a theory about the composition of particular objects as well as a theory about their individuality. In this last sense, it is agreed that according to bundle theorists the individuation of an individual is provided by some of its qualities or by a group of qualities. But this view has also problems, such as: can *two* distinct individuals be characterized by exactly the *same* bundle of properties? If we claim that they can, there will be difficulties in explaining how numerical distinction is to be accounted for, and, some have claimed, some resource to spatial location will have to be made (for a defence of such a view, see Demirli [2010]). If they cannot, why is it so? Here too we encounter difficulties, for we need to assume some metaphysical hypotheses. Leibniz did it by means of his famous Principle of the Identity of the Indiscernibles (henceforth, PII): indiscernible entities, that is, those which share all of their qualities, are in fact not distinct entities, but the very same one. According to this view, there are no entities that differ *solo numero.* Apparently, this idea fits quite well with the objects of our surroundings, for we never find two objects exactly alike; they always present a difference, some peculiar scratch or marc. In empirical science, the objects treated by classical mechanics are also possibly of this kind; they may share most of their properties, but they never share spatio-temporal location; thus space and time act as individualizing qualities in classical physics. Really, each object has a unique spatial position at any given time at which it exists, assuming as it is usual in these contexts that classical physical objects are *impenetrable*. Of course, in order to maintain this position, we need to explain the nature of space and time without mention of the objects inhabiting it, which is acknowledged to be a difficult task. Furthermore, one must provide good grounds for the claim that spatial location is a legitimate individuating property. This is part of a general debate of what counts as a legitimate property that may do the job when we wish to individuate particular objects, a debate the bundle theorist will surely have to face.

In classical logic (and mathematics), this view was captured for instance by Whitehead and Russell's definition of identity in their *Principia Mathematica* (Whitehead and Russell [1999], p.168), and standard logic and mathematics are Leibnizian in some way. In updated language, their definition, which links identity (x=y, meaning "the same", "not two", etc.) with indistinguishability (agreement with respect to the attributes) can be put as follows: $x=y := \forall F(Fx \leftrightarrow Fy))$, where F is a variable ranging over the collection of the properties of individuals and x and y are individual terms (this definition can be extended for other levels on the type hierarchy). Part of the trick is to allow that the property "is identical with y" be in the scope of the properties quantifier in the definition. In that case, the notions of numerical identity and qualitative identity are sure to collapse into one and the same. However, as we mentioned in the previous paragraph the case of spatio-temporal properties, it is metaphysically doubtful whether such properties as "to be identical with y" may be legitimately employed in the individuation role, since they already presuppose that the item denoted by *y* is already individuated and available to somehow individuate *x*.

But, logically, it is possible to suppose the existence of entities differing *solo numero*, that is, non-identical objects which are qualitatively indistinguishable. By the way, this was precisely the argumentation presented by Ramsey against Whitehead and Russell's definition, qualifying it as a "serious defect in *Principia Mathematica* in the treatment of identity" (Ramsey [1968], p.28]; Wittgenstein made similar remarks in the *Tractatus*, proposition 5.5302). Classical logic and standard mathematics are typically Leibnizian theories; within their scope, there is no place for indiscernible (indistinguishable) distinct entities. In order to consider indiscernible things within a "standard" mathematical context, we need to make some mathematical tricks we shall see below (but let us remark that these moves do not conduce to truly indiscernible entities, but just mimic the concept). Surprisingly enough, this fact is not obvious for most philosophers.

For completeness, we shall mention ontic structural realism, considered here in its most metaphysically revisionary form, that is, as a form of eliminativism concerning particular objects. This is not a theory about individuation, since it is not a theory about particular objects, but is a supposed way to overcome the problem of individuation. The idea is that objects should be reconceived as placeholders in structures, as the nodes of the relations composing the structure. However, if we try to push those heuristic ideas to a rigorous form we begin to see the trouble. Firstly, the general thesis says (roughly speaking) that the world is a structure, and that everything there is, is structure. We agree that the world or, better, the portions of the world we are interested in, can be "structured" generally by mathematical means; this is what we do when we construct our theories. But the defender of the structuralist view wishes more: as we mentioned, the underlying ontology is to be changed from an ontology of objects (of some kind) to an ontology of structures, "typically conceived in terms of relations" (Ladyman [1998], Rickles and French [2006], p.25). The problem is that the defenders of such a view don't explain what a structure is. Since the secondary existence of objects is to be explained by the relations they enter in, the relations must explain the relata (the reference to objects related by the relations from whom the structure is "composed"), not vice-versa, so that they claim we should eliminate the relata from these relations. But it is difficult to see how they can do that in a standard framework such as the Zermelo-Fraenkel set theory (ZF), which can be taken (and it is usually taken by philosophers too) as our paradigm of a mathematical basic logical theory. Elsewhere, one of us (DK) suggested that perhaps Tarski's set theory based on his calculus of relations (a set theory without variables – Tarski and Givant [1987]) may be useful in this issue, but as far as we know, no one has pushed such a view to something interesting for the structuralist yet. Furthermore, if the world is a structure, which kind of structure is the world? Where can we construct such a structure, conceived in terms of relations, out of relata? By these reasons, we regard this view as (still) ill funded, yet philosophically interesting.

Some terminology and questionings

Let us fix some terminology. We say that two objects *a* and *b* are *numerically distinct* if they differ numerically, irrespective of whether or not there are any differences in their qualities. Thus, if *a* and *b* are numerically distinct objects, then they are two, and it is a matter of debate whether this difference always is (or can or should be) grounded in qualitative differences, or whether a bare numerical difference is enough (the reader should acknowledge the lack of precision in this definition). We say that two objects are *indiscernible*, or *indistinguishable* relatively to a certain set of properties (which may involve relations) if they agree in all these properties and are related by the same relations to all the same items. This definition is better than the previous one, once we know what properties and relations are. Identity, written $a=b$, means that *a* and *b* are *the very same* object, that is, there is no more than *one* entity, which can be referred to differently as either *a* or *b*. As we shall see soon, definitions such as these cannot be given in precise terms without reference to an underlying logico-mathematical (and perhaps physical) framework. Now, let us turn to the standard (or classical) theory of identity (henceforth, STI), so that some of those intuitive notions may be put more clearly.

We should note, as indicated above, that we have a problem with the notion of "property". What is a property? In quantum physics the notion of property is a very problematical one, with the issue acquiring different meanings in distinct interpretations of the theory (and we shall not try to explore the problem in its whole extent here). The main problem seems to be that we cannot take a naïve realist position that quantum entities are property bearers in the same sense as everyday objects are. If we assume that a quantum object, say an electron, "has the property of having a value of spin" in a certain direction, even though we admit that we can know its value only after measurement, we shall be committed to a realistic position similar to Einstein's in the infamous EPR experiment. Thus, if we adopt, say, some variant of the Copenhagen interpretation, we surely cannot say that the electron has a pre-existing spin attributed to it, unless a measurement is made. Worst, we cannot even speak with significance of "the electron". In a certain sense, the electron exists as an entity bearing properties definitely only in the measurement processes. In this particular case, how can we say that it *has* identity? To have an identity is, in some sense, similar as to have a value of spin out of a measurement. To assume such a view entails a metaphysical hypothesis we aim to criticize below. Thus, we need to distinguish between "property" in a logical setting and "property" in a physical context.

Informally, a property is an attribute of a thing, something we can ascribe to it or deny of it. But in this case we need to have the thing first, except if we adopt a bundle view. But anyway we need to sharpen the concept. In the same vein, within the formalism of quantum mechanics, a property is something we can attribute to a physical system in a certain state described by its wave-function ψ. Depending on the interpretation of the theory we adopt, and in particular on how we see the eigenvalue-eigenvector link, the issue will take a distinct form. Some believe a system has a property represented by the eigenvalue of a Hermitian operator *if and only if* the system is in its corresponding eigenstate. Others believe that the link is weaker, that is, that *if* a system is in the eigenstate of a Hermitian operator, then it has the property represented by the corresponding eigenvalue of the operator. This second option leaves open the matter as to whether a system may bear properties even if it is not in an eigenstate, as some modal interpretations of the theory seem to hold. Of course, even modal interpretations and others with a more realistic flavour than the Copenhagen interpretation will have to deal with issues like non-commuting observables, contextuality, and the like, which impose serious restrictions on a naïve understanding of properties in quantum mechanics. Anyway, the theory allows us to calculate the probability that the value (an eingenvalue) of some observable (a Hermitian operator) for the system in a certain state lies inside a certain Borelian. And this is given by Born's rule, as is well known (and alternative interpretations will have to reproduce this probability attribution of the standard mathematical formalism).

But in logic, for instance in stating the above Leibniz Law, we regard a property simply as a formula with just one free variable. Assuming classical logic, as we are here, given such a property denoted by P each object *a* either has it or not. It is difficult to merge this view with the above (very brief) discussion on properties in the quantum world. Worst, the issue becomes complicated if we aim at to consider spatio-temporal "properties". Which kind of space and time are we speaking of? Would it be the standard absolute view, typical of non-relativistic quantum theory, or perhaps the space-time of (special) relativistic quantum mechanics? Whatever move we chose to do we tend to conform ourselves with the standard mathematical (and logical!) framework, and this framework encompasses Leibniz's identity in some way or another. So, let us turn to consider it a little bit.

The standard theory of identity (STI) in a nutshell

By the classical (or standard) theory of identity we understand the theory of identity of standard classical logic and mathematics, and by this we understand that portion of present day mathematics that can be built within the first order Zermelo-Fraenkel set theory. Of course we could admit alternatives, say by using another set theory, a higher-order logic, or even category theory. In all these alternatives, the particular STI could be a little bit different, but at the bottom all of them say the same: these frameworks are *Leibnizian*. Depending on the framework we are working with, STI is formulated with its peculiarities, and Leibnizianism manifests itself accordingly. Let us see in what sense that happens in the most usual frameworks.

To consider the case of a first-order setting first, we usually regard the binary predicate of identity "=" as a primitive concept subjected to the following postulates: (Reflexivity) every object is identical to itself; in symbols, $\forall x(x=x)$, and (Substitutivity) identical objects may be substituted one each other *salva veritate*; in symbols, $\forall x \forall y(x=y \rightarrow (\alpha(x) \rightarrow \alpha(y)))$, where $\alpha(x)$ is a formula where x appears free and $\alpha(y)$ results from $\alpha(x)$ by substituting y in some free occurrences of x, being y a variable distinct from x. It results from those two postulates that the relation "=" is also symmetric and transitive. In an extensional set theory, such as ZF, these axioms are supplemented by the Axiom of Extensionality, which is a kind of converse of the substitutivity law, namely, $\forall x \forall y(\forall z(z \in x \leftrightarrow z \in y) \rightarrow x=y)$, which says that sets with the same elements are the very same set. If the set theory involves ur-elements, that is, objects which are not sets but can be members of sets, then the axiom of extensionality reads $\forall_S x \forall_S y(\forall z(z \in x \leftrightarrow z \in y) \rightarrow x=y)$, where S is a predicate saying that something is a set (and not an ur-element), and $\forall_S w\, \alpha(w)$ stands for $\forall w(Sw \rightarrow \alpha(w))$.

In higher order logic, identity can be defined. The definition is Leibniz Law stated earlier, namely, $x=y := \forall F(Fx \leftrightarrow Fy)$. It is important to remark about some consequences of these postulates and definitions.

(I) Sometimes the first-order language involves a finite number of predicate constant symbols only (and here we shall be concerned with predicate symbols and neither with individual constants nor functional symbols). In this case, we can define identity by the exhaustion of the predicates as follows: let P and Q be the only predicates, a unary and a binary one, respectively. Thus, we can give the following definition:

$$x=y := (Px \leftrightarrow Py) \wedge \forall z((Q(x,z) \leftrightarrow Q(y,z)) \wedge (Q(z,x) \leftrightarrow Q(z,y)))$$

This is essentially Quine's strategy (Quine [1986], see also Ketland [2006]), but it is reputed to Hilbert and Bernays. It is important to note that even though this definition is reflexive and a version of the substitution law holds, it provides only a definition of indiscernibility with respect to the primitive predicates of the language, and not identity properly speaking (in the sense of our informal characterization given above). In fact, it is easy to present a structure with two distinct objects that obey this definition, that is, that are "identical" according to this definition. Just think of a domain composed of straight lines in the Cartesian plan and let P stand for "to have null slope", while Q stands for "to be parallel to". Thus any two horizontal lines are "identical" according to this definition, even though it is easy to see that they may be *distinct* straight lines. So, although the language cannot discern then, this fact does not make then *identical* in the sense exemplified above (to be "the same" object). This definition is perfectly consistent with the theory of identity given above, in the sense of satisfying the reflexivity law and the substitution law, but we prefer to call the defined relation "indiscernibility with respect to the primitive predicates of the language".

(II) Even Leibniz Law can also be valid in certain interpretations, although the involved objects are not identical. In fact, think of a second order language, endowed with Henkin style semantics. As is well known, in this kind of semantics, monadic predicate variables do not have as their possible range of values all the subsets of the elements of the domain; rather, we restrict the range to only some of such subsets. Thus, we may envisage situations in which we have two (seen from the outside) distinct objects that agree in all the chosen predicates, for they may belong to all the subsets of the domain taken in consideration in our semantics (for details, see French and Krause [2006], p.257). From the semantic point of view, the only way to vindicate Leibniz Law is to take into account all the subsets of the domain, for in this case we need to consider all singletons, and they make the difference (see below). In that case, however, some important results, such as completeness, no longer hold.

(III) Some philosophers say that even within a logical framework containing the above theory of identity it is possible to speak of *weakly discriminable* objects, that is, objects that obey an irreflexive and symmetric relation (see Muller and Saunders [2008]). Furthermore, those philosophers claim that some objects, as for example quantum entities, may be such that they are only weakly discriminable, so that we may not speak about absolutely discriminable objects, in the sense that there are no monadic predicates that distinguish them. A typical example taken from quantum physics would be the two electrons of a Helium atom in its fundamental state. The relation could be "to have spin opposite to". It is a physical fact that no electron has spin opposite to itself, and that the two considered electrons have opposite spins. So, they are weakly discriminable. But can we assure that there are no monadic predicates involved? That is, can we grant that they are not also absolutely discriminable? As our previous discussion shows, it depends on what we call a "property". Within the framework of ZF, and by hypothesis we may assume it as our underlying logic, we can reason as follows. If we have a finite number of objects, say the two electrons (apparently, physics does not need more than a finite number of physical objects), we can always name them, so enlarging the language of ZF with new constants *a* and *b* to name the two electrons. Thus, we can define the "properties" (formulae with just one free variable) $I_a(x) := x \in \{a\}$, $I_b(x) := x \in \{b\}$, and so on, depending on the objects we have (in a finite number). So, it is a consequence of the axioms of ZF that in being *a* and *b* two objects, $I_a(a)$ but $\neg I_a(b)$. Thus, they are discerned by a monadic property, and this happens for any object in ZF. In other words, the objects of ZF are individuals.

Some philosophers could argue that the defined property is not a "property" at all, for membership is a relation. But once we accept our above characterization of a property as a formula (of the language of ZF) with one free variable, we have no reasons to refuse the given definitions. We shall turn to this point below (see also Krause [2010] and Arenhart [2012] for further discussion on weak discriminability).

(IV) The above conclusions have the following consequence. In a recent paper, Jonathan Lowe defined an individual as an *x* such that: "(1) *x* determinately counts as *one* entity and (2) *x* has a determinate identity." (Lowe [2012]). We shall not discuss Lowe's interesting paper in full here, but just remark that we can (so we suppose) interpret his claims as follows, again within the ZF setting (we insist that we need to consider the logical framework for certain discussions). For an entity to be a *one* may mean that we can always form the unitary set of this entity and such a set has cardinal 1. This is a way to say that the object which is the (only) member of this set is a *one.* And this is so with every object in ZF due to the pair axiom and the definition of cardinal (but see the next section). The second clause can be read as indicating that *x* obeys the standard theory of identity exposed above. Thus, it results that all ZF objects are individuals in Lowe's sense. This conclusion matters, for Lowe is just trying to say that it is possible that non-individuals exist, that is, objects the fail to obey either (1) or (2) or both. But this hypothesis cannot be performed in whatever theory encompassing the standard theory of identity. In order to have "legitimate" non-individuals (and we shall see soon what this means), we need to change the logical framework, although in ZF we can have *mock* non-individuals.

V-- The problem with a framework such as ZF is that there may be individuals that cannot be defined by a formula of the language and, in particular, cannot be described in the language; that is, they cannot be named. But even so they are individuals in the above sense due to the underlying logic: they are one and have a well defined identity as objects of ZF. Let us give an example. Due to the axiom of choice, we can prove in ZF that there is a well-ordering over any set *A*. A well-ordering is a partial order such that any non-empty subset of *A* has a first element relative to the well ordering, that is, an element less than any other element of the set (in the given well-ordering). So, in particular there is a well-ordering on the set $\mathfrak{R}$ of the real numbers (in fact, there are infinitely many of them). Thus, the interval (0,1), described in the usual order, has a first element, which is different from any other real number. The problem is that we cannot point to the difference, for the well-ordering (really, any well-ordering) on $\mathfrak{R}$ cannot be described by a formula of the language of ZF, nor the first element of any non-empty set can be named. Really, we could "name" it, say, *t.* But, what is *t*? Without the well ordering, we cannot say that with sense, for we would need to stress that $\forall x(x \in X \rightarrow tRx)$, being X the non-empty subset, and being R the well ordering; but, without a definition of R, this expression cannot be a theorem of ZF. Even so, as we have said, the first elements of the non-empty subsets X of $\mathfrak{R}$ are individuals, having an identity as all real numbers have.

Thus, in considering the Max Black's widely famous example of the two spheres a mile apart (Black [1952]; see Lowe op.cit.), we may say that if we model them within a framework such as ZF, there is no way out: they become individuals, and cannot be simply weakly discernible, as some want to say, independently either we can (qualitatively or not) discern them or not (Lowe ibid.). Logic forces this conclusion.

Thus, Lowe's argumentation, according to which "[i]f there are indeed *two* spheres and *two* electrons [he is mentioning the two electrons of an Helium atom], then the spheres must be *distinct* spheres and the electrons *distinct* electrons." This is true due to STI and the standard definitions of cardinal numbers, but, as we shall see, the mere fact that two objects are *two,* this does not entail that they must be distinct!

Summing up, due to the argumentation we shall still present below, in order to correctly take into account non-individuals, we need to change the logical framework. But, some may say, the language of ZF is so powerful that we can do the job within ZF in some way. This is correct, but the road is full of mathematical tricks we regard are far from the philosophical and metaphysical interest in considering *legitimate* non individuals. Let us see why.

Non-individuality in a standard setting, or the existence of "ersatz" non-individuals in ZF

Let us fix some criteria for something to be a non-individual. Lowe says that it suffices that they fail to obey at least one of his conditions. But, as we have seen, Lowe also considers that given any two objects, by the simple fact that they are two, they are necessarily distinct. This is in accordance, for instance, with the approach proposed by Dorato and Morganti [2012] concerning quantum ontology, and perhaps also by other people. But this claim is simply not necessarily true. The best we can say is that this claim holds good in ZF or in any theory of a standard fashion, encompassing STI and the standard notion of cardinal. However, we may still ask, why should we be working in ZF? In a forthcoming paper, we have shown that by using *quasi-set theory* we can define collections (quasi-sets) having cardinality greater that one but whose elements cannot be discerned one from each other in any way (Arenhart and Krause [2012], forthcoming). As we are insisting, the issue depends heavily on the logico-mathematical framework we are working in, and for certain purposes –as of course the ones concerning ontological disputes on identity and individuality-- this framework must be rigorously specified.

To make clear what the proposal of rigorous specification of the underlying logic amounts to and to show how it bears on the issue of identity and individuality, let us proceed as follows. Firstly, we shall see how we can "simulate" non-individuals within ZF, so defining what we can call *ersatz non-individuals*. Then we turn to a characterization of *truly non-individuals*.

Suppose we are working in first-order ZF. By a structure we mean an ordered pair $E = \langle D, R_i \rangle$ $(i \in I)$ comprising a non-empty set D (the domain, which can be the reunion of other sets whose elements may be subsets of some considered sets), and a collection of n-ary relations on D. This definition is quite general, and does not encompass just relations whose relata are individuals of some set. Our relations may be of higher-order (by the way, this is another mistake made by philosophers, for the structures which are relevant for science and even for mathematics –topological spaces, well-ordered sets— are not "first-order" structures), but we do not lose generality if we consider just these "first-order" structures, which we prefer to call "order-1" structures. An automorphism of E is a bijection function h from D into D such that for any $x,y \in D$, xR_iy iff $h(x)R_ih(y)$. If the only automorphism of E is the identity function (which of course is an automorphism of E), then E is a *rigid structure*. Two individuals a and b of D are E-

indiscernible if there is an automorphism h of E such that h(a)=b. Otherwise, they are E-discernible. For instance, consider the additive group of the integers, A = ⟨Z, +⟩, where Z is the set of integers and + is a ternary relation on Z (the binary operation of addition). Thus A has two automorphisms, namely, the identity function and $h: Z \rightarrow Z$ defined by $h(x) = -x$, as it is easy to see. Then, for any $n \in Z$, we have that n and –n are A-indiscernible. In other words, inside the structure or, as we may say, from the *internal* point of view, nothing distinguish between n and –n, for instance, 2 and –2 are A-indiscernible. But of course they are distinct! The problem is that their distinction cannot be seen *from the inside* of the structure A. If we leave A, we can see the difference. But, what does it mean "to leave" a structure? In the sample case, we can add to A one new relation <, getting an extended structure A'= ⟨Z, +, <⟩, and in this structure, the integers $n \neq 0$ and –n are no more A'-indiscernible, for $-2<2$ but not the other way around. The structure A' is rigid. The fact is that it is a theorem of ZF that *any structure build in ZF can be extended to a rigid structure.* In other words, we can always "leave" a ZF structure E in order to discern the E-indiscernible elements. If we call the E-indiscernible elements non-individuals (better to say E-non-individuals), we find a way to treat this concept within ZF, vindicating the above claim about the expressive power of its language. So, by means of some mathematical trick, we can indeed simulate non-individuality within standard logical frameworks, but in our opinion this is a philosophical sin if we strongly believe that *right*, *true* non-individuals exist (see below).

The above technique of simulating indiscernibility (and non-individuality) is exactly that used in the standard quantum formalism. And of course we could not do anything different, for our standard languages are *objectual*, made to speak of individuals. Let us exemplify with quantum physics. The two electrons in a Helium atom are firstly named *a* and *b*, and then we make a trick to veil this distinction, namely, using anti-symmetric functions (and symmetric functions in the case of bosons), such as (being the spin the property being considered),

$$|\psi_{ab}\rangle = 1/\sqrt{2}\ (|\psi_a{}^{\mathrm{up}}\rangle|\psi_b{}^{\mathrm{down}}\rangle - |\psi_{\mathrm{b}}{}^{\mathrm{up}}\rangle|\psi_a{}^{\mathrm{down}}\rangle)$$

This function changes sign if we permute the labels, but its square, namely, $||\psi_{ab}\rangle|^2$, that gives the relevant probabilities, remains the same. Done! The indiscernibility of the electrons is born to light. But, recall that, within ZF, being two, they are discernible. Thus, in order to speak otherwise, we need either to introduce some new kind of hidden variables, perhaps of a "logical nature", or to leave the standard formalism. Due to the difficulty with the former, which resembles the need of specifying the substratum in the substratum theories, we prefer to take the second option very seriously into account.

Non-individuals

Our intuitive view of non-individuals can be summed up as follows. They would be such as the Smiths in the film Matrix, when Mr.Smith, really a program computer, multiply himself in hundreds to capture the good guy, Neo. During the childhood of one of us (DK), there was a cartoon called The Impossibles in a TV series where one of the good guys was the Multi-Man, who could multiply himself at will.

The Multi-Man, by Hanna-Barbera.

But, you could say, two replicas of the Multi-Man, or of Mr. Smith, occupy distinct locations in space and time, so they can be distinguished. Independently of the notions of space and time, we can say that this is right. But the problem is that two duplicates of the Multi-Man are to be indiscernible, so if they change position, nothing changes at all (at least this is the idea). The same happens with electrons or with quantum objects in general, yet they sometimes (depending on the interpretation or the theory) cannot be considered as little balls as the Multi-Man can, at least from a topological point of view). The very nature of quantum objects is not in the discussion now; the only fact we want to fix concerning them is that they are invariant under permutations, in addition to the fact that they may be (sometimes) considered as *ones* and *distinguishable* (instead of Lowe's "distinct") from others of similar species. Important to remark that there is a theory of *multisets* where the collections (the multisets) may have several copies of *the same* object. For instance, the multiset {1,1,2,3,3,3} has cardinal 6, and not 3 as if it were an object of ZF. But in this theory STI also holds, so the two '1s' are really *the same* mathematical object, although counted twice in this case. But we do not regard a collection of indiscernible quanta as composed of replicas of *the same* entity, yet we can also find an interpretation of them in terms of multisets (for a discussion involving quasi-sets and multisets, see Krause [1991]).

Thus, let us characterize (metaphysically) non-individuals as entities/ objects/ particulars having the following characteristics (partially based on Lowe op.cit.): a non-individual is an x such that (i) it counts as *one* entity, (ii) is does not have a definite identity in the sense that x does not obey STI, (iii) but x may be said to be *discernible* from non-individual y in certain circumstances and *indiscernible* from y in others, and (iv) any permutation of non-individuals of the same kind does not conduce to a different state of affairs.

Indistinguishable quantum objects, in the sense of quantum theory, fit these conditions, either if they are described by non-relativistic quantum mechanics or by the quantum field theories. Obedience to quantum statistics means exactly this, if we regard "state of affairs" as the relevant probabilities. But note that in (iii) we have said that non-individuals may be *discerned* from another non-individual, and not that they may be *different* or *distinct* from another non-individual. This is consonant with item (ii) for otherwise we would be committed to STI. So, we prefer not to speak of *identity*, *difference*, *distinctness* or whatever expression that connotes commitment to STI.

Can non-individuals be counted? As usually understood, to count a collection with, say, five objects (again we shall be restricted to the finite case) is to define a bijection from the collection in question to the ordinal 6={0,1,2,3,4,5}. But this entails that the considered collection must be a set of, say, ZF, that is, as put long time ago by Cantor, "a collection into a whole of definite, *distinct* objects of our intuition or of our thought" (italics added) (Cantor [1958], p.85). In fact, if we cannot distinguish among the objects being counted, we cannot define the bijection (as Russell [1940] had already informally pointed out; see pp. 102-103). But, at least for the purposes of quantum theory, we do not need *to count* electrons, say. We need to know *how many* there are in a certain collection in some circumstances, say in the shell 2p of a Sodium atom $1s^2 2s^2 2p^6 3s^1$. The six electrons are indistinguishable, yet they obey Pauli's principle. Despite the differences in their quantum numbers, nothing can tell us which is which. This does not matter. The important thing is the *cardinal* of the collection. But sometimes we have a sense in saying that we can count them. For instance, suppose a neutral Helium atom. By ionization, we can make the atom lose one of its electrons, and we can name it a. Thus, the electron that remains is electron b. So, some would say, $a \neq b$, and this again commits us with STI. What is the problem? The problem is that by ionization, we can make the atom neutral again, and the two electrons, b and the new one (is it a?) will be entangled and, according to quantum theory, nothing can tell which is which. But, if they obey STI, they would retain their individuality and the distinction would exist. Since we are not disposed to assume hidden variables, the best thing is to say that there are no differences to be restored: the important thing is that the neutral atom has again *two* electrons, and we don't need to be worried with their individuality.

A positive proposal

Identity and its metaphysical consequences could be suppressed from the philosophical discussion about quantum objects, and perhaps about objects in general, remaining only at a metatheoretical level. Before we comment these issues at the quantum level, let us comment on "standard" objects first. We think that Hume is right in saying that we attribute identity to an object by habit (Hume [1986]; see Krause and Becker [2006]). The person we are looking at just now and who we have met for the last time years ago is of course "different" from the person that she was, having modified her traces and acquired new experiences. But she is indeed "the same" friend of ours. There is no contradiction here. The fact is that if we accept that our friend (and we too) obey STI, we *must* say that we both have changed and consequently we are distinct from ourselves of some years ago: the two persons that meet today are not the same ones that have met some years ago for they have different properties than before. Remember that classical logic does not involve time, but this is not necessary: suffice it to consider our "properties". For the relationship, it is enough to acknowledge that we and our friend are indiscernible in many aspects from who we were some years ago. Identity is of course of a fundamental importance in mathematics; if *my* number 2 is not the same as *yours*, we cannot discuss mathematics. But concerning people, chairs, clouds, and quantum objects, indiscernibility is enough. People are discernible from themselves of some time ago, and when time is short, such as a few nanoseconds (the time of a blink), they look as indiscernible from themselves, and all happens as if they are identical from the person of a time before (time, here, is a way of speech). Quantum objects fall under this category of objects: suffice to say that, in the above case of the Helium atom, the electron released from the atom by ionization and the electron that remains in the atom are discernible, and not necessarily distinct, which would commit us with STI (or with some other theory of identity). Concerning the electron that was released and that which was captured by the ion, since the state of affairs does not change at all between the "first" neutral atom and the "second", we may say that they are indiscernible (and not that they are "identical").

Of course we do not need to change our terminology and ways of speaking; we can continue to say that our friend we meet today is the same person we have met years ago for the last time. But we should acknowledge that this is an abuse of language. If we apply STI, we will be in trouble. Concerning non-individuals, they can be discernible or indiscernible, even if they are of a same kind, say electrons. Bosons in a BEC could perhaps exemplify better the indiscernibility of non-individuals, but entangled electrons do the job as well. Summing up, we think that the philosophical discourse would be simplified in much if we simply drop the notion of identity (as linked to STI) from our logic and assume a weaker notion of indiscernibility instead. In this sense, all objects turn to be non-individuals according to the given definitions, but still "objects of discourse".

How can we deal with non-individuals, either discernible or indiscernible? How can we put together our proposal with common language and reference demanding uniqueness in cases where we speak about "the" electron released by ionization? Is it not the case that definite descriptions, naming and reference to particular objects in general need identity, at least self-identity (see, for instance, Jacquette [2011])? This is not the place for the development of a full account of how reference should work, but we believe that uniqueness conditions may be freed from identity by the very notion of cardinal. Really, in quantum mechanics we may be able to say that there are *n* items without needing to count them, or to establish their identities (for the counting relation and cardinal attribution, see Arenhart and Krause [2012], forthcoming). That would do the job quite well, but as we mentioned, this is only a sketch of how things may be adapted to non-individuals.

Still concerning individuals and cardinals, it is important to say that, once non-individuals should count as one, they may also form collections with cardinal 1, but in this case we cannot distinguish between two unitary collections of indistinguishable non-individuals of the same species. Such collections (which by their turn also count as two) can have a cardinal, but not an associated ordinal. These collections, which we term *quasi-sets* may be also indiscernible, namely, when they have the same cardinal and the quantity (given by cardinals) of elements of one kind are the same. Furthermore, these collections must still remain indiscernible from an original one when one of its elements is exchanged (in some way) by an indiscernible one, as it happens with the atoms in the ionization processes mentioned above. Collections with such properties (*quasi-sets*) can be considered within *quasi-set theory*, but we shall not develop it here (see French and Krause [2006], [2010]).

Conclusions

Identity is one of the invariants we use to construct a view of the world (Schrödinger [1964]). Things seem to be individuals. Thus, when we face objects of some kind that appear to violate this condition, we become suspicious that something wrong is happening and try to accommodate what we face within our previous frameworks. Physicists are making these moves when they use standard mathematics and classical logic (encompassing STI) in what respects the indiscernibility of quantum objects (recall the use of the anti-symmetric function as exemplified above), even if they are not aware of that. We propose something different. In believing that our logic and our mathematics should be compatible with our metaphysics, and in accepting a metaphysics where non-individuals are possibilities, it seems that we have strong arguments favouring the idea of looking for a different formalism to cope not only with our intuitions, but also with observed quantum facts, a formalism which could cope with discernible non-individuals (playing the role of individuals in certain circumstances), individuals properly speaking (such as mathematical objects like numbers), and of course indiscernible non-individuals. We can do it by eliminating identity for some objects of our discourse, although it can be maintained for certain objects. Thus, as a primitive concept, we may use a weaker notion of indiscernibility, or indistinguishability, so exchanging "=" by "≡" (an equivalence relation), but contrary to identity, the substitutivity rule doesn't hold. That is, indiscernible objects cannot be substituted one each other *salva veritate* in whatever context (but just in some of them).

But, you may say, in refusing substitutivity, you are just eliminating the very intuition regarding quantum objects, namely, that the expectation value (roughly, the probabilities) do not change when a quantum object, say an electron, is substituted by "another" one! Of course, this is the right conclusion *by the moment*. In fact, to suppose the failure of substitutivity was the way we have found to make indiscernibility distinct from identity. It is only an equivalence relation. Concerning permutations, we can reason as follows: quasi-set theory encompasses certain quasi-sets of objects which can be indiscernible (no identity criteria exist for them) but that have a certain cardinal. It is a theorem of the theory that if we "exchange" (by the set-theoretical operations) an element of the collection by another one which is not in the collection (it is not a *different* one, but a *distinguishable* one), the new collection remains indistinguishable from the original one (yes, the indistinguishability relation applies also for collections) ---see French and Krause [2010], Th.3.1). This way of expressing the permutation invariance, anyway, seems to be in closer connection with quantum mechanics. If we regard the collection of the electrons of the outer shell of a certain neutral atom as being represented by a quasi-set, the exchanging of electrons by ionization may be represented by the (quasi)set-theoretical operations in accordance with quasi-set theory.

Furthermore, in eliminating identity, we can maintain Lowe's first condition, namely, that the considered object (a non-individual) is a unity. Really, we wish to make reference, and speak, of *the electron* being released from an atom by ionization. The problem is that we cannot keep its identity after he has merged with other electrons, say in the environment. If we regard it as an individual, we need to conform it to the STI and then all the above undesirable consequences enter again by the back door.

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