Non-probabilistic chance?

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May 20, 2013

"Chance" crops up all over philosophy, and in many other areas. It is often assumed – without argument – that chances are probabilities. I explore the extent to which this assumption is really sanctioned by what we understand by the concept of chance. I argue that indeterminacy and chance-probabilism are in conflict.

"Chance" or "chancy mechanisms" crop up all over science, philosophy and elsewhere. It is often taken for granted that the proper representation of chance is probability theory. I call this claim *chance-probabilism*. There's nothing in the pretheoretic notion of chance that makes this conceptual link so tight as to block discussions of alternative theories. What arguments are offered for chance-probabilism are made in passing and none is convincing.

First I discuss the motivations for the project. I talk about why we might be interested in the structure of chance. I next discuss various properties people have attributed to the chance concept. These give us a way to triangulate the concept I am dealing with. I argue that chance is a cluster concept, and thus that none (or at least not many) of the properties are necessary conditions on being a chance.

There is some nontrivial groundwork that needs to be done before one can even properly *articulate* the claim that chances are probabilistic. I do this by discussing length. I claim that the two cases are analogous in certain ways. Length, like chance, is a quantity that admits of a certain kind of structure. Discussing the less controversial length case gives us an easy warm-up for the hard case of chance. In particular, I discuss what one needs to say about the relation "is longer than" in order for there to be a function that represents length and for this function to have the requisite structure. The idea is that if we see chance as some sort of quantity that attaches itself to events, then a similar measurement theory analysis can take place as takes place in the case of length. I argue that there could be nonprobabilistic chances. That is, I argue that the requirements one would need to place on the "is more likely than" relation in order to have a probabilistic representation are too strong to apply universally. Through doing this we shall discuss a direct argument for chance-probabilism. I will argue that the argument fails. Showing that chance-probabilism is not analytic or platitudinous does not show that it is false. It does, however, at least show that it could be false, and thus that there might be alternative possibilities that are underexplored.

Further, I argue that if chancy events can be vague or indeterminate, then chance-probabilism is false: that is, I give an example of nonprobabilistic chances for indeterminate chancy events. I will then offer an example of nonprobabilistic chances. This example will rely on there being indeterminate, vague chancy events. The example appeals to vagueness, and thus shows that there is an interesting connection between chance-probabilism and determinacy. I then discuss another argument for chance-probabilism, one that exploits the relationship between chances and statistics. I show that vagueness undermines this argument as well. Finally, I argue that even some of those who deny that there is any genuine vagueness will also struggle to argue for chance-probabilism.

What's at stake?

First, let's just ask what's at stake. Why should we care about the structure of chances? Chances come up in various places:

- Discussions of determinism/indeterminism
- The related topics of randomness and unpredictability
- The debate about Humean chance

- The debate about deterministic chance
- Discussions of risk
- Darwin's appeal to "chance variation" as part of natural selection
- Probabilities in quantum mechanics
- Measures of phase space in statistical mechanics
- The use of mixed acts in game theory
- Von Neumann–Morgenstern and Anscombe–Aumann representation theorems in decision theory and economics
- "Probabilistic causation"
- As a method of making some allocation process fair
- Constraining credence via the principal principle

On top of its role in these debates, chance is an important concept in and of itself. First because we have this category of "chance" and it's worth making clear what sort of structure it has. In the same way that philosophical analyses of important concepts – causation, mind, scientific theory – are just of inherent interest, I think chance is important. Second because a proper analysis of chance makes clear some aspects of the relationship and difference between determinism and determinacy.

To take one example of where this unexamined assumption of chance-probabilism could well make a difference, let's consider Lewis' famous Principal Principle (Lewis 1986). This says that your degrees of belief ought to conform to your knowledge of the objective chances. Without wanting to wade into the details of this tricky discussion, we can summarise the PP as saying that "If you know that the chance of X is x then you ought to believe X to degree x". Now if chances are nonprobabilistic, then your credences ought to be so too. However, there are other norms that govern credence. One important one is, awkwardly, *credence-probabilism*: your degrees of belief ought to conform to the calculus of probabilities.

So, credence probabilism, the Principal Principle and nonprobabilistic chance are mutually incompatible. The question is which should you give up? Lewis seems to favour abandonning nonprobabilistic chance. Lewis (1986) suggested that the Principal Principle "seems to… capture all we know about chance" (p. 86). Since he also took credence-probabilism for granted, he would presumably endorse some argument of the following form: “My credences are necessarily structured in a certain way, and my credences must track chances. Thus chances must be structured the same way”. But this seems backwards. My beliefs should conform to how the world is, not the world to my beliefs in it. I shouldn’t be able to learn about the structure of the world merely by reflecting on what structure my beliefs ought to have.[1]

It seems to me that conflict with putative norms shouldn’t be enough to adjudicate on the truth of a claim about the structure of the world. That is, conflict with putative norms like PP shouldn’t be enough to guarantee the impossibility of nonprobabilistic chances. The incompatibility of the above three claims, plus the argument I am about to give that nonprobabilistic chance is possible, yield a good reason to deny one of PP or credence-probabilism. Hájek and Smithson (2012) also use the possibility of nonprobabilistic chance as an argument against probabilistic credences.

As another example, consider the project to give a reductive account of causation in terms of “probability raising” (Hitchcock 2011). What seems to really be at stake is not probability raising, but *chance raising*. So if some chances failed to be probabilities, this would have consequences for the scope of arguments made in this field.

What are chances?

There are, I think, two main ways to understand what sort of thing chances are. You could take chances to be features of a chance set up that has certain dispositions to behave in certain ways. So a flipped coin has certain dispositional features that make it the case that the outcomes of the chance set-up – the events – have the chances that they do. Call these *dispositionalist* understandings of chance. The alternative is to understand chance as a *relational* property of events. Relative to a reference class, the chance of X is the relative frequency of Xs in the reference class. Chance is a relational property of, for example, reference classes. Call these sorts of view *relationalist* views. More subtle forms of relationalism can be found in Lewis (1986) and Hoefer (2007). For the most part I am going to be speaking as if I am taking a dispositionalist understanding of chance, but I

[1] I encourage someone with more patience for Kant to turn the above into a Transcendental argument for chance-probabilism.

think everything I say will translate straightforwardly into the relationalist understanding.

I claim there is some pretheoretic concept of chance. This is, after all, what probability theory was invented to deal with. I think the current technical usage of the term is connected to this pretheoretic usage in the same way that "force" in physics or "continuity" in mathematics are connected to folk uses of those terms.[2]

Probability theory began as the study of games of chance. So it seems that probability theory should be the right formalism to discuss chance. Early works on the theory of probability describe themselves as works about chance. John Venn's influential book on probability is called "The Logic of Chance". Thomas Bayes' contribution to the study of probability is entitled "An Essay towards solving a Problem in the Doctrine of Chances". So it seems that chance and probability are intimately connected. And indeed I don't deny that there are cases of chances that are appropriately modelled with probability theory. The "games of chance" that early probabilists studied – cards, dice, casino games – do seem to admit of a reasonable probabilistic interpretation. However, we must remember that Euclidean geometry began as a study of real space. It does not follow that actual space is Euclidean. The claim I am making here is that the pretheoretic use of the term "chance" does not necessarily line up with *probability theory*. The pretheoretic use of the term "probability" arguably is closer to pretheoretic use of "chance" than it is to the theory of probability.

Is there a single chance concept? Probably not. But the chance concepts are related, I think that there is enough of a common core of shared properties among the various chance concepts that I can discuss all of them together. In any case, an unexamined assumption of most uses of chance concepts is that chances are probabilities. Ellis (1966, pp. 34–8) suggests that all quantities are cluster concepts.

Our next task is to work out what sort of thing a "chance" is. What are chances for? What role does something have to play to be considered as a chance? I shall first list several plausible platitudes about chance, and then discuss them. In what follows, we will understand "$\mathbf{ch}(X)$" as giving some sort of numerical description of the chance of X. We want to find out what it takes to play the *chance role*. Schaffer (2007) offers several criteria for

[2]Thanks to an anonymous referee for suggesting this analogy.

playing the chance role.[3] Various other authors have discussed properties of chance. What follows is a brief survey of these discussions. I don't take this to be an exhaustive survey, nor to be providing necessary and sufficient conditions for the chance role.

As I said, Lewis (1986) thinks that everything we know about chance is captured by its role in constraining credence. Roberts (2013) questions the role that credence plays in Lewis' account of chance. Even if we don't agree with Lewis' position, *part* of the role of chances is certainly to constrain credence.

I am understanding chances as being an objective feature of the world. That is, for the purposes of this paper, I am denying the possibility that chance talk is just elliptical for credence talk.[4] I want to understand chance as a quantity: a property that admits of degrees and attaches itself to things in the world. When I say "the chance of X is x" I am making a claim about the feature of the world X. A chance is a quantity that attaches itself to an event, in the same way that length is a quantity that attaches itself to a rigid body. That is, it is a property of events that admits of degrees. There can be more or less chance of some event happening, just like there can be more or less length of an object. Events for the propensity theorist are to be understood as outcomes of "chance set ups". For the relationalist, an event is a member of some reference class.[5] For example, Hoefer (2007) says "chances are constituted by the existence of patterns in the mosaic of events in the world". I am trying to be theory neutral: I want to say as little as possible about what events are. I will ascribe some properties to events that they need to have in order for some property of them to be probabilistic. But other than that, I am being agnostic about what sort of understanding one might have of events.

An important thing to know about a property or quantity is how to recognise when something has it, or to measure how much of it something has. We have various robust ways for measuring length and weight, say, and that's how we know what we're talking about when we talk about those quantities. For the case of chances, we have no such direct measurements available. We do, however, have some slightly more indirect ways of gettting a handle on how much chance certain events have. We have this

[3] My presentation owes much to the discussion of Schaffer in Glynn (2010).

[4] This distinction gets fairly blurry towards the end of this paper, however.

[5] This is, of course, a thoroughly unhelpful definition without an explication of the concept of reference class.

in virtue of the link between chances and observed frequencies. It hardly seems worth saying that the observed relative frequency of a coin's landing heads is part of the evidence we have for the chance of heads of that coin (Eagle 2013).[6] And likewise, it hardly seems worth pointing out that the coin's having a particular chance of heads is what explains that particular frequency. But these two ideas are an important part of the concept of chance.

If we think of logic as assigning 0 to falsehoods and 1 to the truths, then chances seem like an extension of this idea: certain kinds of propositions get intermediate values. The debate about the possibility of deterministic chances turns on whether there can be *non-trivial* deterministic chances. It is accepted that trivial chance functions – that assign only zeroes and ones – do apply to deterministic worlds. The understanding is that trivial chance functions just encode what things are true and what are false of the world. So the trivial chance functions must be bound by the same laws of logic as apply in the world. This insight will give us a way to impute a structure to chances.

Quantum mechanics contains chance-like objects, and these are what prompted to Popper (1959) develop his "Propensity theory". Propensities can be understood as a kind of chance. The debate about Humean chance involves recourse to various other chance-like objects in science for example phase space measures in statistical mechanics. Ismael (2009) argues that chance-like entities are indispensible for physical theories. In any case, chances appear in physical theories: physical theories can give us evidence about the nature and structure of chances.

As well as relating to logic and truth, chances also connect with certain modal notions like possibility. For example, Eagle (2013) suggests that Leibniz considered chance to be a kind of *graded possibility*. A sufficient condition for something to be possible is for it to have non-zero chance (Schaffer 2007). One might want to consider this a necessary condition of possibility; Hájek (ms.) appeals to this understanding of the relationship. This claim is currently underdetermined. There are many kinds of possibility one might think are tied to chance: physical possibility?; meta-

[6]Depending on your view on chance, it might be that coins are not the sort of things that have chances attached to them. Nothing in the current paragraph hangs on this way of thinking. Everything should work just as well here if you replace mentions of the coin with mentions of whatever it is that have chances attached: chance set ups involving flipped coins or whatever.

physical possibility? So there are in fact many different possible properties of chance that differ in what sort of possibility is tied to non-zero chances, and whether the condition is a necessary or a sufficient one. As we shall see, some kinds of possibility conditions are implausible. Eagle (2011) ties chances not to the standard modality of possibility, but to what he calls "CAN-claims". In Eagle's view, "CAN φ" is a kind of relative modality: relative, that is, to certain contextual features. Eagle argues that chances are also thus context sensitive. This does not mean that they are subjective, however.

> I wish to emphasise that even though "CAN" is a relative modality, this does *not* mean that it is in any way epistemic or "subjective"... [T]here is an objective fact of the matter concerning whether a certain contextual restriction is, or is not, in place with respect to a given claim "CAN φ"; and an objective fact concerning whether that restriction is, or is not, compatible with the proposition "φ". Eagle (2011, p. 284)

In any case, it certainly seems that chance claims are related to claims about some sort of modality of possiblity or ability.

Let's summarise the above discussion as follows:

CREDENCE Chances are what constrain credences through the principal principle

WORLD Chance facts are claims about the world

FREQUENCIES ARE EVIDENCE Observed frequencies are evidence of chances

CHANCES EXPLAIN FREQUENCIES How the chances are should explain the frequencies we observe

LOGIC Chances relate to logic and truth

THEORIES Scientific theories tell us about the chances

POSSIBILITY $\mathbf{ch}(X) > 0$ iff X is possible

I don't take this to be an exhaustive list,[7] but I hope that the above properties serve to triangulate the concept of interest.

[7] Among other properties that have been imputed to chances are things like: FUTURITY: If X is an event in the past, its chance is 1 or 0; INTRINSICNESS: If X' is an exact duplicate of X, then $\mathbf{ch}(X) = \mathbf{ch}(X')$; CAUSATION: "Causal chances arise within the causal interval they

A further two properties of chance that it is worth putting on the table now are:

Chance-probabilism Chances obey the axioms of probability theory

Determinacy Chancy events, or the outcomes of chancy events are determinate

The first of these, as we have seen, is the target of the current paper. The second of these is something that, were it true, would make chance-probabilism more defensible. But more of that later. Schaffer took Chance-probabilism to be a basic condition on chances.

Chance is among a bundle of concepts that are often used interchangibly but should perhaps be kept separate. This bundle includes the concepts "chance", "indeterminism", "unpredictibility" and "randomness". Earman (1986), von Plato (1982) and Eagle (2005, 2013) contribute to trying to separate out these similar ideas. Norton (2008b) can also be read as contributing to this project.

This section has given a (partial) picture of the chance concept or concepts I am focusing on. I am going to talk as if there is a single concept for reasons of grammatical simplicity, but I don't expect anything I have to say to rely on there being a unified concept. I take it that one thing users of these concepts have in common is a commitment to chance-probabilism: a commitment I think is generally unwarranted.

Of course, not all ordinary language uses of "chance" are going to be amenable to this analysis. For example, when I say "I had a chance to go swimming with dolphins", this should be understood as referring to some sort of *opportunity*, rather than to some dispositional property with a probabilistic structure. Such uses of the word "chance" aren't connected to frequencies or to scientific theories in the way we consider our chance concept to be.

impact" (Schaffer 2007). These are arguably properties that distinguish a certain kind of *dispositionalist* chance that is of particular interest to Schaffer. Ismael (1996) argues that relationalist chances won't be intrinsic; Hoefer (2007) denies Futurity as a condition on chance.

Chance probabilism

It's interesting to note that no one has really tried to argue for chance-probabilism except in passing. Joyce (2009) says "some have held objective chances are not probabilities. This seems unlikely, but explaining why would take us too far afield." (p. 279, fn. 17).[8] Perhaps it is considered too obvious to be worth commenting on. But many people just take it for granted that chances are probabilities, so it seems like it is something people are committed to.[9] As well as Schaffer (2007), Roberts (2013) and Hoefer (2007) also assume without comment that chances are probabilities. Keller (1986) and Suppes (1987) show that certain kinds of physical systems that one might want to interpret as chancy do indeed have a probabilistic representation, but each representation is *local*: it relies on particular facts about the class of systems at issue. Thus, Suppes' and Keller's arguments do not amount to a general argument for chance-probabilism. Part of the diagnosis for why there is no general argument for chance-probabilism is that views on chance (such as propensity theories) were formulated as *interpretations of probability theory*. So the question of whether there were other chance-like objects that did not satisfy the axioms of probability just did not arise. Colyvan (2008) is a notable exception: he argues that probability isn't the only approach to representing uncertainty. Norton (2008a), while focused on a logic of induction rather than chance, is another example of someone questioning the probabilistic hegemony.

There has been a lot of discussion over exactly what sort of thing chances are. Indeed, just about the only thing that seems to have been agreed on is that they are probabilities. I aim to put even that into doubt.

The plan is to try, as much as possible, to be "theory neutral". This means that I want to argue in such a way that whichever interpretation of chance you subscribe to – dispositional, relational – you can accept my

[8]It is perhaps unfair to single out Joyce here. Joyce is, in fact, doing better than most in even acknowledging that things could be otherwise. The reasons Joyce has for saying what he does here are effectively what I have been calling Frequencies are evidence and Chances explain frequencies (personal communication). Indeed, I owe those ideas to Joyce.

[9]Note that I am talking here about the basic idea of a chancy event. This discussion is orthogonal to that of Humphreys' paradox (Humphreys 1985; Suárez forthcoming). The issue there is to show that some conditional probabilities cannot be understood as propensities. Conditionalisation does not enter into the current discussion. What's really at stake here is the *additivity* of chances, something Humphreys does not discuss.

arguments. The important thing is that chances are a feature of the world, and they have some relation to frequency, logic and scientific theories.

But chance-probabilism, as stated, doesn't really seem to even make sense. Chances are a feature of the world; satisfying the axioms of probability theory is a feature of real-valued functions defined over an algebra. Does it even make sense to say that this feature of the world satisfies this purely formal structure? To show that chance-probabilism makes sense, I want to focus on a simpler and less controversial example first. Before analysing the claim "chances are probabilistic" I analyse the less controversial claim "length is additive". Just like chance-probabilism, length-additivity looks like a category mistake. Length is a feature of the world that certain kinds of things have, additivity is a formal feature that certain kinds of functions can have. So there is some non-trivial groundwork to do to even articulate the claim we want to discuss. I have broken this argument down into numbered claims. The point of this is that when I come to discuss chance-probabilism I shall draw out the same structure in the argument.

1. Length is a quantity – a property that admits of degrees – that attaches itself to some kinds of things.

2. Call "sticks" things that have a length. For my purposes, pencils, arms, book spines, the imaginary line between your outstretched fingers: these are all sticks.

3. Some sticks are longer than others. Say "$X \succeq_{\mathbf{len}} Y$" means "$X$ is at least as long as Y". Define "$\succ_{\mathbf{len}}$" and "$\sim_{\mathbf{len}}$" as the irreflexive and the symmetric parts of the relation respectively.

4. There is an operation you can perform on sticks: composition, or *colinear juxtaposition* (Kyburg 1984). You can lay sticks end to end and parallel. Call the compound of X and Y, $X \oplus Y$. It is also a stick. Any two sticks can be so composed and the composition is commutative: $X \oplus Y = Y \oplus X$.

5. The set of sticks ($\mathbb{S}$) has some structure. For all X, Y, Z we have $X \oplus Z \succeq_{\mathbf{len}} X$. If $X \succeq_{\mathbf{len}} Y$ then $X \oplus Z \succeq_{\mathbf{len}} Y \oplus Z$.

6. There is a privileged stick: the null stick. No stick is shorter than the null stick, $\emptyset$. $X \oplus \emptyset \sim_{\mathbf{len}} X$.

7. Given some technical conditions, there is an additive function $\mathbf{len} : \mathbb{S} \to \mathbb{R}$ that assigns to each stick, a real numbered value: its length. $\mathbf{len}$ represents $\succeq_{\mathbf{len}}$ and is unique up to affine transformation.

8. By "$\mathbf{len}$ *represents* $\succeq_{\mathbf{len}}$" we mean that $X \succeq_{\mathbf{len}} Y$ if and only if $\mathbf{len}(X) \geq \mathbf{len}(Y)$.

9. By "$\mathbf{len}$ is additive" we mean $\mathbf{len}(X) + \mathbf{len}(Y) = \mathbf{len}(X \oplus Y)$.

Measurement theory studies this idea of representing a quantity. You get theorems that look like this: "If $\mathbb{S}$, $\oplus$ and $\succeq_{\mathbf{len}}$ have the right sort of properties, then the function **len** will have certain other properties." The properties of the function that represents the quantity tell us things about that quantity itself. For example, contrast length and temperature. Length has an additive representation. Compose two sticks in the right way – end to end and parallel – and you get a stick that has a length equal to the sum of the lengths of the two component sticks. However, not so for temperature. There's no interesting physical composition procedure such that temperature is additive with respect to that procedure.[10] Put two thermal bodies in contact and the temperature of the composite body (at equilibrium) will be some sort of average of the two temperatures of the composite bodies before composition, not their sum. So this tells us that length and temperature are interestingly different *as quantities*, not just as regards their representations.[11]

There is a mathematical theorem that backs up point 7 on this list. For the technical details, the classic work is Krantz et al. (1971). See also Ellis (1966) and Kyburg (1984) for more philosophical treatment.

I want to take a moment here to emphasise that only some aspects of the representation function can be understood to be telling us things about the world. For example, there exists a particular representation function that measures the particular stick X as having a length $\mathbf{len}(X) = 42$. This is a fact about the function that we don't take seriously as a fact about the world. The X stick has no intrinsic property of "forty-two-ness". The point is that there will be other functions that don't give X that value of length that will represent the quantity just as well. So it is only properties invariant under an appropriate class of transformations that we take to

[10] Ellis (1966) calls length an *extensive* quantity, and temperature *intensive*.

[11] Length and temperature differ in their structural features: $X \oplus Y \succeq_{\mathbf{temp}} X$ is not true for temperature.

tell us something about the world (Ellis 1966; Kyburg 1984; Stevens 1946). Additivity is like this. A ten centimetre stick composed with another ten centimetre stick make a twenty centimetre stick. If we instead think of these sticks as being 3.9 inch sticks, they compose into a 7.8 inch composite stick. So additivity does not depend on the details of the representation used.

One might wonder how important it is that the representation be *unique*, or at least unique up to a certain class of transformations. Krantz et al. (1971, pp. 9–12) suggest that knowing what class of transformations is admissible – what class of transformations give you representations of the same structure – is an important part of knowing what sort of quantity you have. Uniqueness theorems give you that sort of information.

We've seen what it means to claim that length behaves additively. It is a claim about how we can represent an aspect of the world. Arguably, what representations of the world are possible depends on how the world is, so how we represent the world tells us something about how the world is.

Let's look now at what one might want to say about chance in a similar vein. The analogy isn't perfect. Chance, as a quantity, is probably more like velocity or temperature than it is like length. That is, the methods for measuring chance aren't going to be what Ellis (1966) calls "fundamental measurement". Nonetheless, a look at the measurement theory approach to chance-probabilism will be instructive.

1. Chance is a quantity – a property that admits of degrees – that attaches itself to some kinds of things.

2. Call "events" things that have chance.

3. Some events are more likely than others. Say "$X \succeq_{\mathbf{ch}} Y$" means "$X$ is at least as likely as Y".

4. There are operations you can perform on events: conjunction, disjunction, negation... If X and Y are events, then so are

   - $X \vee Y$ (X **OR** Y)
   - $X \wedge Y$ (X **AND** Y)
   - $\neg X$ (**NOT** X)

   Any events give rise to such composite events and the binary operators are commutative: $X \vee Y = Y \vee X$. Events have logical structure.

5. The set of events ($\mathbb{E}$) has some structure: for example, if X, Y are events then $X \vee Y \geq_{\mathbf{ch}} X$.

6. There are two privileged events, $\top$ and $\bot$: the necessary and the impossible event, respectively. $X \wedge \top \sim_{\mathbf{ch}} X$ and $X \vee \bot \sim_{\mathbf{ch}} X$.

7. Given some technical conditions (see the appendix) there is a probability function $\mathbf{ch}\colon \mathbb{E} \to \mathbb{R}$ that assigns to each event, a value: its chance. **ch** represents $\geq_{\mathbf{ch}}$ and is unique.

8. By "**ch** represents $\geq_{\mathbf{ch}}$" we mean $X \geq_{\mathbf{ch}} Y$ if and only if $\mathbf{ch}(X) \geq \mathbf{ch}(Y)$.

9. By "**ch** is a probability function" we mean:
   - $\mathbf{ch}(X) + \mathbf{ch}(Y) = \mathbf{ch}(X \vee Y) + \mathbf{ch}(X \wedge Y)$
   - $\mathbf{ch}(\bot) \leq \mathbf{ch}(X) \leq \mathbf{ch}(\top)$ for all X
   - $\mathbf{ch}(\bot) = 0$ and $\mathbf{ch}(\top) = 1$

These nine points capture what would have to be the case for chance to be probabilistic. The intricacy of the above argument, and the detail required, show that it is not trivial that all chances are probabilistic. Point 1 on the above list captures what we called World. Points 4 and 6 relate to Logic.

Note that the chance-probabilism claim that I am arguing against is a very specific one: the claim is that chances are represented by orthodox probability functions as one would find described in the early chapters of a book on probability theory. One may want to make a weaker claim that chances are represented by some kind of functions in the neighbourhood of probability functions: set valued functions, upper and lower probability functions, possibility functions, hyperreal probability functions and so on (Benci, Horsten, and Wennmackers 2013; Halpern 2003). This is a claim I would tentatively endorse. However, it seems that in the literature the unexamined assumption that is standardly made is the stricter chance-probabilism claim that I am criticising.

There are two places where possibly contentious substantial assumptions are made in the above argument. The first is in the assumptions made about the event structure; second, there are the assumptions made about the "$\geq_{\mathbf{ch}}$" relation structure.

Let's take the event structure first. For **ch** to be a probability function it needs to be defined over a Boolean algebra. That means that it satisfies the structural constraints outlined in the fourth point above. The event structure is such that these "and" and "or" connectives are commutative, and that the compound events are always in the event space. Through the reliance on a basic Boolean structure, chance-probabilism builds in a kind of "classicality" of the event structure.

Typically in formal representations of quantum events, like in quantum logic, you don't have this classicality. It is true that in all observable bases, you do have commutativity, and indeed the mod-squared amplitudes are additive, but commutativity doesn't hold more generally (Rédei 1998; Rédei and Summers 2007). Krantz et al. (1971) discuss "QM-qualitative probabilities", where these differ by not always having conjunctions. That is, it can be that X and Y are in your event structure, but $X \wedge Y$ isn't.[12] Aerts, D'Hondt, and Gabora (2000) suggest another way that quantum systems fail to behave as described by a Boolean algebra. That is, quantum mechanical disjunction and negation do not behave as their classical counterparts do. In any case, note that chance-probabilism commits you to some non-trivial claims about the (classical) nature of the event structure: claims that are not true of quantum mechanical systems, or indeed of other kinds of indeterminate set ups.

There is a second problem with the event structure. In the course of proving the uniqueness of the probability measure, some strong assumptions about the event structure are made. For instance one way to do it is to assume that: if $X \succ_{\mathbf{ch}} Y$ then there is a collection of mutually incompatible but exhaustive events Z_i each of which is so implausible that $X \succ_{\mathbf{ch}} Y \vee Z_i$ for all i (Krantz et al. 1971, p. 206). This forces the event structure to be infinite.[13] This idea that there need to be collections of arbitrarily unlikely events is not something that I can find in my folk concept of an event.[14]

[12] For instance, if X says something very specific about a particle's position and Y something specific about its momentum, then while $\mathbf{ch}(X)$ and $\mathbf{ch}(Y)$ might have values, $\mathbf{ch}(X \wedge Y)$ doesn't make sense, since it would violate the uncertainty principle. See p. 214 of Krantz et al. (1971). On p. 215 they state a theorem to the effect that QM-algebras can still have a probabilistic representation, but this requires a non-standard understanding of a probability space, which allows QM-algebras to be the kind of thing probabilities are defined over.

[13] Krantz et al. (1971) offer an alternative that is satisfied by some finite structures, "although not in most" (p. 207). The axiom is not intuitive, as they admit, and in the interests of space I omit a discussion of it.

[14] The richness assumption is needed for the uniqueness, but not for the existence of a probabilistic representation.

So let's say we are happy with commutativity, existence of conjunctions, and collections of arbitrarily unlikely events. Let's move on to the conditions on the relation structure.

One condition requires that all chances be comparable in terms of their chances. That is, the $\succeq_{\mathbf{ch}}$ relation must be complete. None of the platitudes I listed seem to require this. The argument for length additivity also has a requirement of completeness of the relation. However, in the case of length, we have more of a handle on what "$X \succeq_{\mathbf{len}} Y$" means. We cash this out in terms of comparison procedures. For the case of length, we have some idea what we mean when we say two lengths are always comparable. We have an intuitive idea of procedures we can follow that will establish which of two lengths is the longer.[15] In the case of length, many of the conditions can be given some intuitive weight by talk of sticks, composition and procedures of comparing their lengths. In the case of chance, it isn't so clear what the measurement procedures are supposed to be. That is, we have *some* idea that events can be be ordered by how likely they are – that's what it means to consider chance a quantity – but this nebulous intuition doesn't seem enough to require that events be totally ordered by their chances. Statistics may offer a general procedure for comparing chances. But we must be careful: I can "compare" the height of X centimetres with the weight of Y in kilograms, but this isn't a meaningful comparison. So there must be more to the comparison than merely two procedures that output numbers for each thing to be compared. I discuss the relationship of chances to frequencies later. In the next section I discuss an example that will bolster my argument against completeness.

There is a well-known tension between the demands of probability theory and infinite spaces. Consider throwing a dart at the real unit interval.

Consider the chance of hitting some non-empty X, a subset of that interval. Now, since X is non-empty, one might argue that it is *possible* that the dart hits some member of X. But no probability function is such that every non-empty X has non-zero probability. So it seems that probabilistic chance isn't making some distinctions we might want to make between these sorts of events. Put another way, chance (if probabilistic) doesn't connect with metaphysical possibility (if metaphysical possibility is the kind of possibility in Possibility). There are reasons to think that physical

[15] Comparing lengths of, say coastlines or rivers is far from straightforward, but let's leave that aside. I claim there are extra difficulties in the chance case.

possibility isn't the sort of possibility connected to (probabilistic) chance either. For example, most probability distributions over a state space will give the point representing the actually realised state a probability of zero. Or if there are infinitely many chance events, then the initial chance of the actual course of events will be zero.[16] There has been some back-and-forth on hyperreal probability theory recently[17] that I don't want to get into. I'll just note that hyperreal probability is a significant departure from orthodox probability theory.

In any case, it seems like intuitions or folk conceptions of the "event concept" aren't strong enough to support the heavy duty technical conditions required here.[18]

That chance-probabilism is not obviously or straightforwardly true is a pretty weak claim. Consider the analogous project of demonstrating that water-dihydrogenmonoxidism – the thesis that water is H_2O – is not obvious.[19] One can collect platitudes about water – it is a colourless liquid; it is implicated in rainfall; its freezing and melting points determine certain fixed points of our temperature scale – but they will not force upon you the conclusion that water is H_2O. This does not mean that water is *not* H_2O. Similarly, showing that chance-probabilism is not obviously true does not demonstrate that it is false. In the absence of examples of nonprobabilistic chances, it might still be that despite not being platitudinous, chance-probabilism is true: probability theory is a simple and well understood formal machinery and that might make it the best formal tool for representing chances. The next section will give an example of a nonprobabilistic chance, thus demonstrating that chance-probabilism is false. Or rather, it shows that chance-probabilism is incompatible with the existence of vague chancy events.

Chance and indeterminacy

To recap, a direct argument for chance-probabilism would involve claiming that the space of events had the requisite structure, and that the relational

[16]Thanks to Luke Glynn for both these points.

[17]For example Benci, Horsten, and Wennmackers (2013); Elga (2004); Hájek (ms.); Williamson (2007).

[18]The appendix lists these conditions.

[19]Thanks to an anonymous referee for suggesting this analogy.

structure had certain properties. Neither of these claims seems reasonable in general. One might step back from this and say instead that even though the premises of the direct argument are too strong, there are independent reasons to subscribe to the conclusion. So, one would step back from the measurement theory approach, but continue to maintain that there is a function on events that represents chance; that this function is real valued; that it is bounded; and that it is additive.

In this section, I want to sketch an example that I take to involve chances in the sense we have been discussing, and I want to show that such chances do not satisfy the axioms of probability theory. That is, I want to give an example of chance that is represented by a function that is not additive. The example involves appeal to genuine indeterminacy or vagueness. This might make some people sceptical of the relevance of the example. However, in a later section I argue that some positions that deny the reality of genuine vagueness also should accept nonprobabilistic chance.

An urn contains seventy marbles in a range of hues. Ten marbles are determinately red and twenty are determinately orange. The remaining forty marbles are not determinately red and not determinately orange: they are borderline cases of red or orange. But they are determinately red or orange. The marbles are well mixed, the drawing procedure is suitably fair, and the chance set up has all the other properties you might hope it to have. What is the chance of drawing a red marble from this urn? We can certainly say that the chance is at least one seventh: the determinately red marbles guarantee at least this much chance. And we can say that the chance of red is at most five sevenths: even if we included all the marbles that are such that it is vague whether they are red as well as the determinately red ones, we would only have five sevenths of the marbles in such a collection (because two sevenths are determinately orange, and thus determinately not red). A chance-probabilist confronted with this situation would have to say that **ch**(Red) takes some precise value. But which precise value? How much more than $\frac{1}{7}$ is **ch**(Red)? No answer to this question seems justified. Such a view does not seem to do justice to the vagueness of the situation.[20] One might say that it is vague what value **ch**(Red) takes,

[20]Note that the argument here is stronger than the corresponding argument against credence-probabilism. In the credence case, the probabilist can just say that any particular values for red and orange are allowed: the value is not rationally constrained, but additivity

but that that value is certainly somewhere between $\frac{1}{7}$ and $\frac{5}{7}$. In such a situation, one might want to say that the appropriate model of the chances makes use of, say, interval-valued functions, or sets of proability functions. So $\mathbf{ch}(\text{Red}) = \left[\frac{1}{7}, \frac{5}{7}\right]$, $\mathbf{ch}(\text{Orange}) = \left[\frac{2}{7}, \frac{6}{7}\right]$, and $\mathbf{ch}(\text{Red or Orange}) = 1$. Such a move mimics the move to supervaluationism in logic to accommodate vague predicates.

A somewhat different approach might be to take the view that the chance of red should be understood as the chance of being determinately red. Such a function would be superadditive but not additive, since $\mathbf{ch}(\text{Red}) = \frac{1}{7}$ and $\mathbf{ch}(\text{Orange}) = \frac{2}{7}$, but $\mathbf{ch}(\text{Red or Orange}) = 1$ since all marbles are either red or orange. I don't really want to commit to a particular theory of nonprobabilistic chances, since my main aim is a negative one. However, the sort of alternative formal frameworks I have in mind are those covered by, for example Halpern (2003) or Walley (1991).

In either case, it seems like a move beyond orthodox probability theory is the appropriate response to such examples. It seems that no precise probability should be assigned to the event that a red marble is drawn, since it is vague whether that event occurs.

As in the discussion of quantum mechanics and the event structure, we are driving a wedge between chances and probability theory by appealing to phenomena that are plausibly handled by "going non-classical". Colyvan (2008) makes a similar point.

Let's look at what this example tells us about the measurement theory analysis of the last section. Let's imagine that we're using some sort of interval-valued or set-valued function to represent chances. Now if two such intervals overlap, then which event is likelier than the other? Arguably no relation of "is likelier than" holds between events so represented. This suggests that the requirement that the relation "$\succeq_{\mathbf{ch}}$" be complete is unwarranted. More generally, it seems possible that it can be indeterminate which of two events is more likely. This again suggests incompleteness of that relation.[21]

The above argument works for metaphysical or ontic vagueness. But since chance-probabilism is arguably a claim about how we represent the world, it seems that the same argument holds for linguistic or semantic

is required. Here, since we are dealing with objective chances, the analogous move cannot be made: no determinate probabilistic answer to the question is permissible.

[21] This example also violates the "quasi-additivity" condition on the $\succ_{\mathbf{ch}}$ relation.

varieties of vagueness. It is perhaps less clear whether such an argument works if one believes that all vagueness is epistemic. I argue in the last section that the same argument holds there too.

So it's not just that chance-probabilism is not obviously true, there are good reasons to think it is in fact false. Hájek and Smithson (2012) offer further arguments for the possibility of nonprobabilistic chances. First consider some physical process that doesn't have a limiting frequency but has a frequency that varies, always staying within some interval. It might be that the best description of such a system is to just put bounds on its relative frequency. If we took a Humean perspective on what chances are, this would make it the case that its chance is nonprobabilistic.[22] Note that such a system would be indeterminstic and chancy, but perhaps not random and almost certainly not unpredictable. This drives home the point that such concepts can come apart.

A further example of a possible sort of nonprobabilistic chance is outlined in Hartmann and Suppes (2010). They show that a certain kind of *upper probability* (which is not an orthodox probability measure) turns up in a representation of certain kinds of joint events in quantum mechanics. If one interprets these things as chances, then they are another example of nonprobabilistic chance.

More controversially, consider Norton's dome (Norton 2008b). This is an example of a physical system whose equations admit of multiple solutions. This is a kind of indeterminism which is much less well behaved than the indeterminism we find in quantum mechanics, for example. Norton argues that a probabilistic characterisation of the dome cannot and should not be given (Norton forthcoming). Whether or not there are chances that can be attached to events in the dome example is unclear. What is clear is that indeterminism – a concept closely linked to the concept of chance – is much less well behaved than is typically thought. I take this to be evidence that the ubiquity of probabilistic indeterminism has been overstated, and this in turn gives some sort of indirect support to the possibility of nonprobabilistic chances. Indeterminism is more diverse and less well behaved than commonly assumed.

[22]Dispositionalists need not be moved by this example: they might prefer to say that the system has a determinate chance that varies over time.

Chance and statistics

We are now going to change tack slightly and look at another attempt to give a sort of "representation argument" for chances. The previous attempt focused on the *relational* structure that chance has in virtue of its "is more likely than" relation. The analogy was to the case of length. Now we turn to an attempt to ground chance-probabilism with a *monadic* representation. Instead of trying to derive a quantity *ex nihilo*, we try to relate chance to some antecedently understood quantity. The analogy here is to representing temperature by a column of mercury: this relates temperature to the antecedently understood quantity of length. We relate chances to statistics.

Frequencies are probabilistic, and frequencies are evidence of chances. Hájek (1997, 2009) argues that it should at least be possible for chances to be described by frequencies, and thus that chances should at least be probabilistic. Paris (1994) offers an argument that is similar to Hájek's in that it shows how anything that is measured appropriately by statistics should be probabilistic. Note that these are *not* frequentist arguments. Whatever your attitude to chance, it seems that chances have some relation to frequencies. So even the dispositionalists can take evidence from statistics as evidence for the structure of chancy powers. Two of our platitudes were about the relationship of frequencies to chance.

Unless we can argue that *all* chances must be amenable to statistics, then this argument can't give us what we want from it. That is, this argument can tell us nothing about those dispositional properties that aren't part of a reference class for statistics. If you are of the opinion that chances are relational properties of reference classes, then perhaps you are happy to make this move, and say that all chances must be describable by statistics: they must be determined by the statistics of the reference class. This move has the standard problem that it makes a mystery of various sorts of one-off events that we would otherwise like to assign chances to. If you favour a dispositional account, then this move – tying chances this tightly to frequencies – seems a little less warranted. Why ought it be the case that all such chancy dispositions be amenable to statistical descriptions? Consider the quantum case again. If you accept that the noncommutative algebras that arise there can have chances attached to them, then there's good reason to think that they can't be described by standard statistics: they aren't amenable to measurement in the right way.

I'm not suggesting this position is not tenable: you certainly could be a hardcore operationalist about chances, just as about other quantities. But this seems like an extreme position to adopt in order to be able to claim that chances are probabilistic. Note that in the analogous case of temperature, there are obviously temperatures we can't *actually* connect to the height of a column of mercury: the temperature at the core of the Sun, for example. But we have other *indirect* methods for inferring such temperatures. In the chance case, the problem is more serious: there are certain things that we might want to call chances – one off indeterministic events for instance – that *cannot* belong to any reference class. There is not even any indirect way of ascribing statistics to them.

There is, in any case, a stronger argument against using the fact that statistics are probabilistic to sanction chance-probabilism: statistics aren't necessarily probabilistic. Hájek and Paris' arguments relied on the determinacy of the events. If it can be vague whether X and vague whether Y, but determinate that $X \vee Y$, then the statistics will inherit this vagueness and probabilistic representation will not be guaranteed. Consider the statistics of the vague marbles example discussed earlier. Let's imagine that you draw (with replacement) a large sample from the urn. Some of the time you will draw the marbles of indeterminate, borderline colour. How do you count them? They are unarguably red or orange, and thus should count towards the statistics of that disjunctive category. But should a marble that is not determinately red (but not determinately not red) count towards the statistics of red marbles? If you decided that it should not, then the statistics you would generate would be superadditive, but not additive. That is, the frequency of red or orange marbles would be strictly greater than the frequency of red marbles plus the frequency of orange marbles. The important point to note is that to secure chance-probabilism, the frequentist needs to argue that all chances are amenable to statistics, *and* that all outcomes of chance set ups are determinate. Walley and Fine (1982) also offer a kind of statistics that involves upper and lower probabilities: an importantly different theory from orthodox probability.

To clarify the statistics argument above, let's look at a slightly more general putative argument for probabilism. This relates to another of our platitudes about chance. Chances relate to logic and truth. Trivial chance functions "are" in some sense just the logical truth valuation functions. Non-trivial chances are, in some sense, "between" the trivial ones in the sense that a fair coin flip is somehow *between* the coin's landing heads and the coin's landing tails. Think of this as a kind of generalisation of the intuition that relative frequencies are somehow "averages" of possible outcomes or estimates of possible outcomes. We can formalise this idea: chances are convex combinations of truth value functions, and these are always probabilistic. A theorem due to de Finetti shows that all and only the probability functions are in the convex hull of the set of classical truth valuation functions. $\mathbf{v}$ is a (classical) truth valuation function if it maps the algebra of events into $\{0,1\}$ and:

- $\mathbf{v}(X \vee Y) = \max\{\mathbf{v}(X), \mathbf{v}(Y)\}$
- $\mathbf{v}(X \wedge Y) = \min\{\mathbf{v}(X), \mathbf{v}(Y)\}$
- $\mathbf{v}(\neg X) = 1 - \mathbf{v}(X)$

Call $\mathbf{ch}$ a "convex combination" of a set of valuations $\{\mathbf{v}_i\}$ if:

$$\sum \lambda_i = 1$$
$$\mathbf{ch}(X) = \sum \lambda_i \mathbf{v}_i(X) \text{ for all } X$$

If all the $\mathbf{v}_i$ are classical, then $\mathbf{ch}$ so defined is a probability, and all probabilities can be so characterised (Paris 1994, pp. 13–4).

How intuitive is the claim that chances must be in the convex hull of (a representation of) the truth values? there are good arguments that convex combinations of possible truth values are the right structure for your beliefs. But these arguments don't really translate into the case of chances.[23]

In any case, indeterminacy undermines this argument. That is, if indeterminacy prompts you to revise your logic, then what functions are convex combinations of the nonclassical valuation functions won't necessarily be probabilities. Paris (2005 [2001]) shows that for a particular class of nonclassical valuations you get superadditive functions in the convex hull. Here again we see that chance-probabilism and classicality of the event structure (or of the logic over it) go hand-in-hand.

[23]For example, the argument in Joyce (1998) relies on "epistemic utility" for the agent in question. Nothing seems apt to serve the analogous role in the chance case. Likewise, the Dutch book theorem relies on betting behaviour of the agent. Williams (2012) shows how this theorem relates to convex combinations of truth values.

In this context, it's worth briefly mentioning Cox's theorem (Cox 1946). The idea behind Cox's theorem is to show that if chances are supposed to conform to a logic of plausible entailment, then they must be probabilities. A number of technical criticisms have been raised against Cox's theorem[24] but even putting these aside, Colyvan (2008) argues that Cox's theorem is inherently classical. Thus, again, if you have reason to revise your logic – for example, in the face of genuine indeterminacy – then chance-probabilism cannot be secured this way.

Epistemicism and compatibilism

So far, chance-probabilism has been undermined by appeal to genuine – that is to say metaphysical or semantic – vagueness. One might just deny that such vagueness is part of the world: that all vagueness is epistemic. Williamson (1994) is one such *epistemicist*. Such a position, it seems, would escape from the above refutation of chance-probabilism. I want to show that at least *some* epistemicist positions should accept that there are nonprobabilistic chances. Those epistemicists that should, I claim, accept the reality of nonprobabilistic chances are those who are also *compatibilists* about chance and determinism. That is, those who accept the reality of higher-level chances[25] should not accept chance-probabilism. Put another way, incompatibilist epistemicists can accept chance-probabilism – although I don't think they are rationally compelled to do so. If you think that there are only chances at the most basic level of reality, and that that level of reality is perfectly determinate, then you can accept chance-probabilism. Although, you would have no reason to do so since you have no epistemic access to that level, and no reason to think it is determinate. Also, on such a position, most of the every-day things we consider "chancy" – coins, dice, outcomes of sports events – are not chancy at all.

Let me now turn to showing that if you grant that there are higher-level chances, then there are nonprobabilistic chances; even if you are an epistemicist. I will assume that the world is, at bottom, deterministic. One could avoid my argument by asserting that the world is fundamentally indeterministic, but determinate. Such a position is a little odd since the best reasons to think the world is indeterministic – namely basic physics

[24] See the references in Colyvan (2008) or Van Horn (2003).
[25] Glynn (2010) and Hoefer (2007) accept higher-level chances, for example.

– are also arguably reasons to think it is indeterminate. In any case, I think we should be quietists about whether the world is deterministic or chancy at bottom, and so we should at least accept the *possibility* that it is deterministic.

Let's look at what an epistemicist would say about the example of generating statistics from the vague urns case. There is (for the epistemicist) a fact of the matter for each marble as to what its colour is. Therefore, the failure to determine adequate (i.e. probabilistic) statistics marks a flaw in our knowledge, not a fact about the world. Let's consider the chance that the next marble will be red. Our epistemicist compatibilist will insist that there is a (probabilistic) chance that the next draw will be of a Red marble, but we don't know what that chance is. But then, given that the world is deterministic, there is a fact of the matter about which marble will be drawn and thus about what colour it will be. But the compatibilist doesn't take this to undermine there being non-trivial chances. Now, if there being a fact of the matter about which marble will be drawn doesn't undermine non-trivial chances at the higher level, why should there being a fact of the matter about the marble's colour undermine nonprobabilistic chances? Essentially, accepting genuine higher-level chances, but denying nonprobabilistic chance is an unstable position. Arguments against one are arguments against the other; defenses of one are defenses of the other.

In short, if your account of chance is relative to a state of information so as to allow non-trivial chances at all in a deterministic world, then it won't be able to block nonprobabilistic chance.

Kyburg (1984); Kyburg and Teng (2001) also build a non-probabilistic theory of statistics. Wheeler and Williamson (2011) discuss Kyburg's "evidential probabilities" approach. The basic intuition behind this approach is that statistical inference only happens within certain margins of error. Once we explicitly model those margins of error, we no longer have an orthodox probability measure on events. We have an "interval valued" probability. Again one might worry that the human errors shouldn't be part of our understanding of what chances are. But then, we are happy with the idea that a better knowledge of the precise initial conditions of a coin toss could allow us to predict the result. This doesn't preclude assigning non-trivial chances to it.

Nonprobabilistic chance

My main conclusion is that if you allow events to be vague, then you had better allow the chances of those events to be vague too. Probability theory is not the appropriate formal tool to represent such chances. The "slogan form" version of this conclusion is: "Probabilistic chances only if outcomes of chance events are determinate". That "determinate" can be read ontically, as in metaphysical vagueness; or semantically as in relating to how our language matches up with the world. Indeed, it can also be read epistemically, as referring to something lacking from your state of information; at least by compatibilists.

So chance-probabilism is tricky. Indeed, length-additivity isn't as straightforward as I made it out to be earlier. As Kyburg (1984) discusses, if you interpret "$\succeq_{\mathbf{len}}$" as related to an actual process of measurement, there will be sticks so close in length so as to be indistinguishable. Arguably, at some level, the whole concept of "equal in length" stops making sense. On the scale of the individual atoms at the ends of the sticks, whether one is longer than the other or not doesn't really make sense. The sticks might be indistinguishable in length for all practical purposes. Such indistinguishability is not transitive. Thus strictly speaking, the measurement theory analysis as presented above doesn't hold. Treating length as represented by an additive function is still a useful and productive idealisation, but an idealisation nonetheless. In this paper I have argued that a similar subtle but important readjustment needs to take place with respect to our ideas about chance. Such a readjustment is more important than in the case of length, since much more thought is devoted to the metaphysics of chance than is to the metaphysics of length: the structure of chance is important in many parts of philosophy.[26] Indeed, credence-probabilism has been questioned from a number of angles:[27] I am merely arguing for a similar move in the case of chance. I am not arguing that probability theory and chances have no connection to each other: they do, as Suppes (1987) shows for example. Indeed, the readjustment I am advocating is not all that radical: the nonprobabilistic functions that I claim sometimes represent chances – interval valued functions or sets of functions for example – are not irreconcilably different from probability theory. They are extensions of

[26]That's not to say that length has no philosophical bite, as my colleagues who think about spacetime theories are keen to point out.

[27]Among others: Joyce (2005); Kaplan (1996, 2010); Levi (1974, 1985); Sturgeon (2008).

it. Chance-probabilism is not totally wrong; it is just a special case.

Thanks

Thanks to Luke Glynn, Conor Mayo-Wilson, Lorenzo Casini and John Norton for helpful comments. Thanks also to the audience at the BSPS 2012 in Stirling.

Appendix: Probabilistic representation

Various authors have given theorems to the effect that events with a certain compositional and relational structure are uniquely represented by a probability function (Krantz et al. 1971; Savage 1972 [1954]; Villegas 1964). My presentation follows the treatment of Savage in Joyce (1999).

The theorem states that if the following properties hold:

- Normalisation: $\top \succ_{\mathbf{ch}} \bot$
- Boundedness: $\top \succeq_{\mathbf{ch}} X \succeq_{\mathbf{ch}} \bot$ for all X
- Ranking: $\succeq_{\mathbf{ch}}$ is a partial order
- Completeness: $X \succ_{\mathbf{ch}} Y$ or $Y \succeq_{\mathbf{ch}} X$ for all X, Y
- Quasi-additivity: If $X \wedge Z \equiv \bot \equiv Y \wedge Z$ then
  - $X \succ_{\mathbf{ch}} Y$ iff $X \vee Z \succ_{\mathbf{ch}} Y \vee Z$
  - $X \succeq_{\mathbf{ch}} Y$ iff $X \vee Z \succeq_{\mathbf{ch}} Y \vee Z$
- Richness: If $X \succ_{\mathbf{ch}} Y$ then there exists a partition of the event space: $\{Z_i\}$ such that $X \succ_{\mathbf{ch}} Y \vee Z_i$ for all i

then there exists a unique function **ch** that satisfies the axioms of (finitely additive) probability. See the above cited papers for proof. Securing countable additivity takes a little more work, and is not something I discuss here.

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