Is Howard's Separability Principle a sufficient condition for Outcome Independence?

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This essay contains, in the opinion of the author, substantial errors in Sec 2.1 and 2.2. A revised version is in work.

Howard [1985, 1989, 1992] has argued that the, experimentally confirmed, violation of the Bell inequalities forces us to to reject at least one of two *physical* principles, which he terms locality and separability principle. To this end, he provides a proof [Howard, 1992] of the equivalence of the separability condition, a formal condition to which the separability principle gives rise, with the condition of "outcome independence". If this proof is sound, then Howard's claim would gain strong support in that "outcome independence" and "parameter independence", where the latter arises from Howard's locality principle, have been shown by [Jarrett, 1984] to conjunctively constitute a necessary condition for the derivation of the Bell inequalities [Clauser and Horne, 1974]. However, Howard's proof has been contested in a number of ways.

In this essay I will discuss several criticisms of Howard's equivalence proof that focus on the sufficiency of the separability principle for outcome independence. I will argue that, while none of these criticisms succeeds, they do constrain the possible form of Howard's argument. To do so, I will first introduce both the separability principle and outcome independence in the context of EPR-like experiments before I discuss the individual arguments.

1 Howard's equivalence proof

Set-up

Following [Laudisa, 1995],[1] a typical (Bohm type-) experimental set-up to produce Bell inequalities is that of two particles (named 1 and 2) being fired simultaneously in opposite directions from a common source. On these particles some binary quantity is then measured by placing detectors with adjustable parameters along their trajectories. This procedure can be formalised in the framework

[1]For the sake of clarity, throughout the essay I will stick to the terminology and formal representations of Laudisa's paper and discuss or even mention differing naming only where necessary.

of stochastic hidden variable theories in the following way: With respect to particles 1 and 2, if we denote the detector parameters a and b, outcomes m and n and the complete state λ of the two particles, then we read read $p^i_\lambda(x|\mathbf{a},\mathbf{b})$ as the conditional probability that for a given λ outcome x will be measured on particle i under the condition that detector parameters are set to a and b. Analogous probabilities are construed in the obvious fashion. By further specifying the measurement events to be space-like separated this set-up allows one to derive Bell's inequality (or the CHSH-inequality in the more general form).

Now while the set-up described can give rise to different sets of premises to derive the Bell inequalities, Jarrett [1984] demonstrated that all of them include a factorizability condition, which is expressed as:

$$p^{12}_\lambda(m,n|\mathbf{a},\mathbf{b}) = p^1_\lambda(m|\mathbf{a})p^2_\lambda(n|\mathbf{b})$$

Factorizability can further be decomposed into two independent conditions, the conjunct of which is equivalent to the latter. These are **Parameter Independence (PI)**

$$p^1_\lambda(m|\mathbf{a},\mathbf{b}) = p^1_\lambda(m|\mathbf{a})$$
$$p^2_\lambda(n|\mathbf{a},\mathbf{b}) = p^2_\lambda(n|\mathbf{a})$$

and **Outcome Independence (OI)**

$$p^1_\lambda(m|\mathbf{a},\mathbf{b},n) = p^1_\lambda(m|\mathbf{a},\mathbf{b})$$
$$p^2_\lambda(n|\mathbf{a},\mathbf{b},m) = p^2_\lambda(n|\mathbf{a},\mathbf{b})$$

While the question what physical principles underlie these conditions lacks canonical agreement, their stochastic meaning is clear: PI implies that the probability of measuring an outcome at one detector does not depend on the parameter setting of the other. OI implies that the probability of measuring an outcome at one detector does not depend on the outcome measured at the other.

The argument

Initially motivated by Einstein's attempts to construct an argument to the effect that quantum mechanics (QM) was incomplete, Howard argues that the physical content behind factorizability is given by identifying the PI with a locality principle and OI with a separability principle. But the separability principle, in Howard's [1989, 226] formulation, asserts that

> [**SP**]: the contents of any two regions of space-time separated by a non-vanishing spatio-temporal interval constitute separable physical systems, in the sense that each possesses its own, distinct physical state [**SP(1)**], and the joint state of the two systems is wholly determined by these separate states [**SP(2)**].

Hence, Howard concludes, unless we are willing to give up locality and with it the validity of all special relativistic theories, we are forced into accepting non-separability, from which a kind of "ontological holism" follows (ibid.225).

To motivate his identification of outcome independence with the separability principle, Howard defines a "state separability condition"[2], stating that two systems 1 and 2 are state separable if there exist separate states α and β for these systems such that

$$p_{\lambda}^{12}(m, n|\mathbf{a}, \mathbf{b}) = p_{\alpha}^{1}(m|\mathbf{a})p_{\beta}^{2}(n|\mathbf{b}) \tag{SEP}$$

Together with the two identifications

$$p_{\alpha}^{1}(m|\mathbf{a},\mathbf{b}) = p_{\lambda}^{1}(m|\mathbf{a},\mathbf{b}) \tag{ID I}$$

$$p_{\beta}^{2}(n|\mathbf{a},\mathbf{b}) = p_{\lambda}^{2}(n|\mathbf{a},\mathbf{b}) \tag{ID II}$$

it can then easily be proven that OI and SEP are equivalent [Laudisa, 1995, 317]. This equivalence implies that

[**SIOI**]: SP entails OI

SIOI then grounds, in Howard's view, a physical interpretation of OI and the implications that he ascribes to the latter's falsehood.

2 Discussion

After this brief overview of Howard's argument, let me now introduce and assess the most note-worthy criticisms.

Unsoundness of the proof

Laudisa (1995) argues that Howard's equivalence proof is unsound, because ID I/II, necessary for the proof's validity, are not generally true. This, Laudisa argues, arises from the fact that it is possible in general to conceive of probabilities for measuring an outcome on either system being different depending on whether the complete state or only the separate state for that system are given. However, in order to be convincing state separability needs to be defined for all possible complete states. Therefore, state separability is either ill-defined or not equivalent to outcome independence.

Of course, Laudisa needs to support the claim that there can be cases, at least one, where p_α and p_λ differ. To do so, he employs a distinction between two state concepts in quantum mechanics, initially proposed by van Fraassen [1991] in a modality context: States have two components, where the *value* component is "fully specified by stating which observables have values and what

[2] As Howard is aware, there is the possibility of defining system separability, according to which not only do the states have to be distinct, but also the systems on which the states are defined. System separability will not be discussed here, see Laudisa (1995:319) for more.

they are", while the *dynamic* component is "fully specified by stating how the system will develop if isolated, and how if acted upon in any definite, given fashion" (ibid.:274). Importantly, since measurement is an interaction, the outcome probabilities are comprised in the dynamic component of a state. Hence,

> [g]iven this distinction, it is perfectly possible that p_α differs from p_λ. The probability p_λ is actually related to the dynamic component of λ, which is the (complete) state of the composite system at the source. This probability, being induced by λ, could well be different from p_α: The latter is the probability induced by α and the dynamic component of α, which p_α is related to, is in general different from the dynamic component of λ (Laudisa, 1995:318).

While unfortunately this is not made quite clear by Laudisa himself,[3] the way in which Laudisa takes van Fraassen's distinction to support his argument is apparently that, if acted upon, the joint state will in general develop differently from either of the separate states and that hence the probabilities which represent these dynamics may differ. It is, then, for this reason that (identifications) are not generally true and hence, SEP and OI are not equivalent with the consequence that stochastic hidden variable theories that violate SEP but are factorizable are possible, according to Laudisa.

Laudisa's argument, I want to suggest, is invalid. This is because he fails to recognise the reference to systems in the definition of states, as they figure in the separability principle. To show this, it is necessary to have a closer look at the way Howard motivates ID I/II from the separability principle SP.

From the above definition it follows that, if the separability principle is true, then given that the space-time regions, on the contents of which the detector measurements are performed, are separated by a non-vanishing spatio-temporal interval, these contents constitute two systems with distinct states α and β (SP(1)). But states are defined by Howard as "a conditional probability measure assigning probabilities to outcomes conditional upon the presence of global measurement contexts" (Howard, 1989: 230). In this case, these probabilities are p_α, p_β resp. for global measurement context represented by full knowledge of both parameter settings i,j. Further, these states wholly determine the joint state of the two systems (SP(2)). Here, the "wholly determine" says that the joint state can be completely mapped on α and β and the "distinct" assures that such a mapping will be single-valued, $f : \lambda \to \alpha \vee \beta$.

But this is just formally summarised in ID I/II: The conditioned outcome probability for measuring an outcome at some-space-time region is completely determined by the state of the system in that region (which is defined against the

[3] Belousek [1999, 6f.] takes Laudisa's argument to be that neither SEP nor the identifications, both of which refer to the dynamic components, are motivated by the separability principle, which itself "should be characterised generally in terms of value, rather than dynamical states (i.e., in terms of definite properties rather than probabilities)". But not only is this not what Laudisa says, from my reading, it also appears to involve invalid reasoning in that there is no obvious reason why space-time individuation should not be accomplishable by means of probabilities rather than properties.

global context). But *by Howard's definition of state and SP(2)* all and exclusive reference to 1 in λ is comprised in α.[4] This is important because it is here that Laudisa's argument goes wrong: Although his employment of van Fraassen's distinction seems valid in that it is in accord with Howard's definition of states, Laudisa is wrong in asserting that the dynamic component of the complete state *for a given system* generally differs from that of the separate state *for this given system.* This is because p_α^1, *by SP(2),* captures exclusively and exhaustively all the information that λ contains *about system 1.*

Therefore, while the dynamics comprised in p_λ in general will be different from that in p_α, if we are concerned solely with the dynamics with respect to system 1 - and if the separability principle is true -, then the identification of p_λ^1 with p_α^1 is correct. Thus Laudisa'a argument is found lacking.

Formalization of SEP-the minimalist *non sequitur*

Fine and Winsberg [2003](FW) put forward another counter-argument worth discussing, yet advancing from a different vantage point. While sympathetic to Howard's definition of separability, they do disagree with Howard's formalization of it. This is because the definition of separability, while requiring a complete determination of the joint state from the distinct states, does not specify the formal representation of this determination. Howard's choice of the arithmetic product function in SEP is thus only one of many functions that provides a "wholly determinate" mapping of the separate states to their joint. Consequently, while factorizability implies separability, this is not the case vice versa.

To support this argument, the authors develop an algorithm that, given a joint state λ, first identifies *bijectively* the "marginal distributions" $p_\lambda(m|\mathbf{a}), p_\lambda(n|\mathbf{b})$ at either wing, which they understand as the local states, and in a second step finds a two-placed function $F(..)$ that completely reproduces the joint state distribution. Crucially, the gained freedom in being able to assign different $F(..)$ in the second step means that the decomposition in the first step is not unique. Instead the local states need only satisfy a number of inequalities. Together with parameter independence constraints which the authors include in their formalism, all of this effectively allows for certain choices of the local states (or, more precisely, the probabilities that constitute these states) that satisfy the separability principle while at the same time violating the Bell inequalities (and thus both factorizability and outcome independence).

The underlying position that motivates FW's analysis is known as minimalism, which asserts that the violation of the Bell inequalities by theories like quantum mechanics (QM) arises from the way joint probabilities are defined in it, rather than from some physical principle. Muller and Placek [2001](MP)

[4]In quantum mechanics, this operation of disregarding information in a joint state that does not contribute to some designated compound state, is, of course, represented by the partial trace operator, which, in the case of being applied to joint state that can be expressed as the tensor product of its substates, results exactly in that substate.

provide a rigorous criticism of this position which, if correct, undermines the above reasoning and restores Howard's argument.

In order to see their point, it is essential to understand that in the theory of joint probabilities, it is always possible to define a measure of the joint probability p^{12} for some binary observable on a corresponding Boolean algebra $\mathscr{B}_{12} := \mathscr{B}_1 \times \mathscr{B}_2$ (where these measures are the $F(..)$ in Winsberg and Fine) such that it satisfies a "marginal property" which states that the joint probability can be reduced to the local probabilities, i.e.

$$p^{12}(m, \mathbf{1}_n) = p^1(m); p^{12}(\mathbf{1}_m, n) = p^2(n)$$

where $\mathbf{1}_{m/n}$ is the probability identity.

The specific motivation for minimalism is an asserted inability in QM to rigorously define joint probabilities for non-commutative observables. For the minimalist it is then this feature that, by definition, dooms QM to violate the Bell inequalities, where non-commutative class of observables may be involved, on grounds of mathematics and not the physics. However, MP show that joint probabilities can actually be well-defined for all observables including non-commutative ones *as long as* outcome independence (among others) is assumed to hold true.

This is important for the assessment of FW's counter-argument in that, if we accept MP's proof, the set of admissible joint probability measures $F(..)$ is constrained by the outcome independence condition. More specifically, any $F(..)$ can serve to define joint probability in general (i.e. including joint probabilities for non-commutative observables) only if it satisfies OI (among other premises). In other words, if we take A to be the set that contains all functions that produce joint probabilities satisfying separability (and parameter independence), that is $F(..) \in A$, all $F_{QM}(..) \in B$, where B is the set of $F(..)$ consistent with quantum mechanics (and probably most other stochastic hidden variable theories involving non-commutative algebra) have to lie within the subset C of functions $F_{OI}(..)$ that satisfy outcome independence, giving $B \subset C \subset A$. Since the $F_{QM}(..)$ are the only $F(..)$ we are concerned with in the separability condition, no admissible function $F_{QM}(..)$ can produce joint probabilities consistent with separability *and* be inconsistent with outcome independence. But these are the only counter-examples against the statement that separability implies outcome independence.

It should be emphasized that MP's work does not establish direct support for SIOI. That is, the above argument does not assert that only subset C is admissible *because* SIOI is true. Instead, they show only that no F(...) that can both be produced by FW's algorithm and that is adequate to define joint probabilities for all possible observables, can serve to *falsify* SIOI.

Well localized particles and superluminal communication

Two more arguments worth note have been proposed: One by Berkovitz [1998] and one by Maudlin [2011], which I will discuss in this order.

Berkovitz (op.cit:1998:213) asserts that Howard's equivalence proof "presupposes that particles in Bell-type experiments are well localized during the measurements." and that, since the separability principle does not imply this latter fact, it follows that SIOI is false. To asses this assertion we need to check whether (a) Howard's proof requires the above presupposition and (b) in case that it does, under which, if any, conditions it may be implied by the separability principle.

To do so, let us first get clear about the notion of "being well localized". Berkovitz gives the only hint for his understanding of it as

> ...systems involved [in a Bell-type experiment] are well localized in space-time, so that the particle and the apparatus in the L-wing are space-like separated from the particle and the apparatus in the R-wing (ibid., 210).

Space-like separation of the wing contents here appears as a necessary condition for the truth of being well-localized (in a Bell-type experiment). Berkovitz's statement, however, leaves us uncertain about exactly which aspect of a systems' being well localized suffices for their space-like separation. Le me, therefore, adopt a working definition sufficiently general to cover this uncertainty:

> [**WL**]: A system A is well localized in space-time iff for any space-time region the question whether A is constituted by the contents (not necessarily completely) of this region has a well-defined answer and this answer is either "yes" or "no".[5]

As can easily be seen, if WL holds, then *in a working Bell-type experimental context* we know the marginal systems to be space-like separated; the working definition is consistent with Berkovitz' statement.

Now, if Berkovitz is right regarding (a), SIOI becomes invalid if WL is not satisfied during the measurements. Berkovitz does not expand on Howard's requirement of the particles' being well localised. The easiest way to allocate it is by seeing that (SEP) is defined only for systems that I already know to be distinct (however, not necessarily separable!). And the decidability of a particle's being distinct from another would then, it seems crucially depend on its being well-localised. In this sense, at least, Berkovitz assertion (a) is true.

We thus have to look at (b). If SP implies WL, then it must be impossible for SP to hold and WL to be false, i.e. in no case where distinct contents of two regions of space-time constitute distinct systems such that the marginal states together wholly determine the joint state, is it unclear whether a system stretches over a space-time region or not. But this is true, because the concept of distinctness used in SP necessarily requires WL in order to be well defined. To see this, suppose that there are two space-time regions the contents of which

[5]Note that this last part is necessary because otherwise one may conceive of a system's being well localised as a superposition of "yes" and "no", which, unless expansion coefficients are equal, I take to be a kind of localisation which is possible but not consistent with what Berkovitz has in mind.

constitute distinct systems (i.e. if SP is true). This distinctness can only be meaningful if I can ask whether two space-time regions contain the same system or not (with the answer being “no”). Thus, when we understand distinctness as well defined to be a sufficient condition for the satisfaction of WL and WL as sufficient for something to be well localized, then since the use of distinctness in SP implies the latter to be well defined (besides its truth), SP implies WL and consequently SIOI holds, thus refuting Berkovitz' criticism.

However, other ways of understanding something as well localized or distinct are probably defendable. Thus, we can only say that SIOI holds for the specific conditions given above. The value of Berkovitz' criticism lies in its highlighting the difference between state and system separability, together with the question to what extent their corresponding separability conditions can be constured independently and brought into relation: Even if the falsehood of a state separability principle can be defended in this essay, the stronger version concerning systems is left untouched here. In fact, if, following the above formulation of the separability principle, systems are constituted by the contents of space-time regions, then non-separability of systems could imply the indistinguishability of these contents and require a complete reformulation of the principle in a less substantivalist way.

Maudlin's argument applies to the case where particles are actually well localized. It bases on the premise that we cannot exclude the possibility of superluminal signaling: Maudlin (2011) opens for the possibility of communication between the two particles by means of superluminal and massless tachyons as signal carriers, where the particles alter their state depending on whether or not they receive a message from their partner. In such a model, according to Maudlin, all particles “have perfectly determinate intrinsic states at all times,and the joint state of two distinct regions of systems is just the sum of their individual systems” (ibid., 89f.), hence satisfying SEP but violating OI.
Berkovitz (1998, 214) raises worries about this argument: Such a model would be separable, only if the communication instructions “could be realized by the qualitative, intrinsic properties of each of these particle [not the tachyons]”, requiring a highly problematic “open line of communication” between them. With this criticism Berkovitz seems to anticipate a by now established result of recent quantum information theory: The no-communication theorem, according to which “no instantaneous information transfer can result from a distant intervention” [Peres and Terno, 2004, 8]. Now while Maudlin (op.cit: 90) argues that his tachyons do not even require energy to constitute a counter-case to Howard's separability, it seems clear they have to carry extractable information to do so. But since this is ruled out by the no-communication theorem, at least in quantum mechanics, Maudlin's argument stands little chance of being empirically sound.

3 Conclusion

To conclude, I have shown that neither of the criticisms advanced succeeds in showing that state separability is not a sufficient condition for outcome independence. However, for this to hold true our freedom of interpreting separability has to be significantly reduced: Only state separability was discussed. We require that distinctness of states implies that they are well localized in the sense defined above and these states to be dynamical states as discussed by van Fraassen. Further, future generalizations of the separability principle in other formalisms, can be guaranteed to maintain the truth of SIOI so far only for stochastic hidden variable theories involving non-commutative algebra and obeying the no-communications theorem. Since, however, all of these restrictions seem to have a good chance of enduring, Howard's claim that giving up on outcome independence forces us to accept some kind of separability continues to be an option for interpreting the physical implications of the violation of Bell inequalities, an option that should be taken seriously.

word count 3000

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