Protective measurements and relativity of worlds

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Abstract

It is a fundamental and widely accepted assumption that a measurement result exists universally, and in particular, it exists for every observer, independently of whether the observer makes the measurement or knows the result. In this paper, we will argue that, based on an analysis of protective measurements, this assumption is rejected by the many-worlds interpretation of quantum mechanics, and worlds, if they indeed exist according to the interpretation, can only exist relative to systems which are decoherent with respect to the measurement result.

In standard quantum mechanics, it is postulated that when the wave function of a quantum system is measured by a macroscopic device, it no longer follows the linear Schrödinger equation, but instantaneously collapses to one of the wave functions that correspond to definite measurement results. However, this collapse postulate is ad hoc, and the theory does not tell us why and how a definite measurement result appears. This measurement problem is widely acknowledged as the most difficult problem in the foundations of quantum mechanics. One way to solve the problem is to reject the collapse postulate and assume that the Schrödinger equation completely describes the evolution of the wave function. There are two main alternative theories for avoiding collapse. The first one is the de Broglie-Bohm theory (de Broglie 1928; Bohm 1952), which takes the wave function as an incomplete description and adds some hidden variables to explain the emergence of definite measurement results. The second one is the many-worlds interpretation (Everett 1957; DeWitt and Graham 1973), which assumes the existence of many equally real worlds corresponding to all possible results of quantum experiments and still regards the unitarily evolving wave function as a complete description of the total worlds.

Although the many-worlds interpretation is one of the main alternatives to quantum mechanics, its many fundamental issues, e.g. the preferred basis problem and the interpretation of probability, have not been completely solved yet (see Barrett 1999, 2011; Saunders et al 2010 and references therein). In this paper, we will try to answer a basic question about the stuff of these worlds, namely whether everything in the universe, no matter it interacts with a decoherent measuring device or observer, has a copy in each of the worlds formed

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by the measuring process. According to some authors (e.g. Barrett 2011), the answer seems to be yes, while according to others (e.g. Wallace 2012), the answer may be no. Here we will give a simple proof that worlds, if they indeed exist according to the many-world interpretation, can only exist relative to the systems which are decoherent with respect to the measurement result. In other words, the many-worlds interpretation of quantum mechanics must reject the fundamental assumption that a measurement result, once it has been recorded by a measuring device or an observer, does not merely exist relative to this measuring device or observer, and it also exists for other non-decoherent observers who does not make the measurement or know the result.

According to the many-worlds interpretation, the components of the wave function of a measuring device, each of which represents a definite measurement result, correspond to many worlds (Vaidman 2008; Barrett 2011). What, then, is each of these worlds composed of? It seems natural to assume that everything in the universe has a copy in each world. Obviously, when a system does not interact with the measuring device, its copyies in all worlds are exactly the same. It is unsurprising that the existence of such worlds may be consistent with the results of conventional impulsive measurements[1], as the many-worlds interpretation is just invented to explain the emergence of these results, e.g. the definite measurement result in each world always denotes the result of a conventional impulsive measurement. However, this does not guarantee consistency for all types of measurements. It has been known that there exists another type of measurement, the protective measurement (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996). Like conventional impulsive measurement, protective measurement also uses the standard measuring procedure, but with a weak, adiabatic coupling and an appropriate protection. Its general method is to let the measured system be in a nondegenerate eigenstate of the whole Hamiltonian using a suitable protective interaction, and then make the measurement adiabatically. This permits protective measurement to be able to measure the expectation values of observables on a single quantum system. In particular, the wave function of the system can also be measured by protective measurement as expectation values of certain observables (see the Appendix)[2].

It can be seen that the existence of the above worlds is inconsistent with the results of protective measurements. The reason is that the whole superposed wave function of a measuring device can be directly measured by a series of protective measurements in a single world, namely our world[3]. The results of the protective measurements as predicted by quantum mechanics indicate that all components of the wave function of the measuring device exist in our world. Therefore, according to protective measurements, the superposed wave function of a measuring device do not correspond to many worlds, one of which is our

[1]It should be pointed out that the consistency is still debated due to the controversial interpretation of probability. For more discussions see Saunders et al (2010) and references therein.

[2]Note that the earlier objections to the validity of protective measurements have been answered (Aharonov, Anandan and Vaidman 1996; Dass and Qureshi 1999; Vaidman 2009; Gao 2012).

[3]Protective measurement generally requires that the measured wave function is known beforehand so that an appropriate protective interaction can be added. But this requirement does not influence our argument, as the superposed wave function of a measuring device can be prepared in a known form before the protective measurement.

world.

Several points needs to be clarified regarding this argument. First of all, the above argument does not depend on how many worlds are *precisely* defined in the many-worlds interpretation. In particular, it is independent of whether worlds are fundamental or emergent, e.g. it also applies to the recent formulation of the many-worlds interpretation based on a structuralist view on macro-ontology (Wallace 2003, 2012). The key point is that all components of the superposed wave function of a measuring device can be detected by protective measurements in a single world, namely our world, and thus they all exist in this world. Therefore, it is impossible that the superposed wave function of a measuring device corresponds to many worlds, one of which is our world. Note that this objection is more serious than the problem of approximate decoherence for the many-worlds interpretation (cf. Janssen 2008). Although the interference between the nonorthogonal components of a wave function can be detected in principle due to the unitary dynamics, it cannot be detected for individual states, but only be detected for an ensemble of identical states. Moreover, the presence of tiny interference terms in a (local) wave function in our world does not imply that all components of the wave function wholly exist in this world. For example, it is possible to explain the interference by assuming that each world has most of one component of the wave function that represents a definite measurement result and tiny parts of other components.

Next, the above argument is not influenced by environment-induced decoherence. Even if the superposition state of a measuring device is entangled with the states of other systems, the entangled state of the whole system can also be measured by protective measurement in principle (Anandan 1993). The method is by adding appropriate protection procedure to the whole system so that its entangled state is a nondegenerate eigenstate of the total Hamiltonian of the system together with the added potential. Then the entangled state can be protectively measured. On the other hand, we note that if environment-induced decoherence is an essential element of the many-worlds interpretation, then the theory will be inconsistent with standard quantum mechanics. When a measuring device is isolated from environment, standard quantum mechanics still predicts that the device can obtain a definite result, while the many-worlds theory will predict the opposite due to the lack of environment-induced decoherence.

Thirdly, the above argument does not require protective measurement to be able to distinguish the superposed wave function of a measuring device from one of its components, or whether the superposed wave function collapses or not during an impulsive measurement. Since the determination demands the distinguishability of two non-orthogonal states, which is prohibited by quantum mechanics, no measurements consistent with the theory including protective measurement can do this. Fourthly, we stress again that the principle of protective measurement is independent of the controversial process of wavefunction collapse and only depends on the linear Schrödinger evolution and the Born rule. As a result, protective measurement can (at least) be used to examine the internal consistency of the no-collapse solutions to the measurement problem, e.g. the many-worlds interpretation, before experiments give the last verdict[4].

In order to save the many-worlds interpretation from the above serious ob-

[4]For a more detailed analysis of the implications of protective measurement see Gao (2013).

jection posed by protective measurements, one must drop the initial assumption that everything in the universe has a copy in each world. In particular, one must assume that worlds, if they can indeed be formed by a quantum measurement, can only exist relative to the systems which are decoherent with respect to the measurement result. According to the principle of protective measurement, only observers (or measuring devices) whose states are not entangled with the superposed wave function of a measuring device can make a protective measurement of the wave function, while observers who are decoherent with respect to the result obtained by the device cannot make a protective measurement of its wave function. Then by dropping the initial assumption and assuming the relativity of worlds, the many-worlds interpretation can be saved. For the observers in each world must be already decoherent with respect to the result obtained by the device, and thus they cannot make the protective measurement required by the above argument[5].

To sum up, we have argued that in order to be consistent with quantum mechanics, worlds must be relative in the many-worlds interpretation. This means that a measurement result exists only relative to the systems which are decoherent with respect to the measurement result, and it does not exist for non-decoherent observers who does not make the measurement or know the result. This seems to lead to the so-called relative facts interpretation proposed by Saunders (1995, 1998). Whether such an approach can be ultimately satisfactory deserves further investigation.

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Appendix: Protective measurement of the wave function of a single quantum system

As a typical example of protective measurement, consider a quantum system in a discrete nondegenerate energy eigenstate $|E_n\rangle$. In this case, the system itself supplies the protection of the state due to energy conservation and no artificial protection is needed.

The interaction Hamiltonian for a protective measurement of an observable A in this state involves the same interaction Hamiltonian as the standard measuring procedure:

$$H_I = g(t)PA, \tag{1}$$

where P is the momentum conjugate to the pointer variable X of an appropriate measuring device. The time-dependent coupling strength $g(t)$ is also a smooth function normalized to $\int dtg(t) = 1$. But different from conventional impulse measurements, for which the interaction is very strong and almost instantaneous, protective measurements make use of the opposite limit where the interaction of the measuring device with the system is weak and adiabatic. Concretely speaking, the interaction lasts for a long time T, and $g(t)$ is very small and constant for the most part, and it goes to zero gradually before and after the interaction.

Let the total Hamiltonian of the combined system be

$$H(t) = H_S + H_D + g(t)PA, \tag{2}$$

where H_S and H_D are the Hamiltonians of the measured system and the measuring device, respectively. Let the initial state of the pointer at $t = 0$ be $|\phi(x_0)\rangle$, which is a Gaussian wave packet of eigenstates of X with width w_0, centered around the eigenvalue x_0. Then the state of the combined system after T is

$$|t = T\rangle = e^{-\frac{i}{\hbar}\int_0^T H(t)dt} |E_n\rangle |\phi(x_0)\rangle \, . \tag{3}$$

By ignoring the switching on and switching off processes[6], the full Hamiltonian (with $g(t) = 1/T$) is time-independent and no time-ordering is needed. Then we obtain

$$|t = T\rangle = e^{-\frac{i}{\hbar}HT} |E_n\rangle |\phi(x_0)\rangle \, , \tag{4}$$

where $H = H_S + H_D + \frac{PA}{T}$. We further expand $|\phi(x_0)\rangle$ in the eigenstate of H_D, $\left|E_j^d\right\rangle$, and write

$$|t = T\rangle = e^{-\frac{i}{\hbar}HT} \sum_j c_j |E_n\rangle \left|E_j^d\right\rangle , \tag{5}$$

[6]The change in the total Hamiltonian during these processes is smaller than PA/T, and thus the adiabaticity of the interaction will not be violated and the approximate treatment given below is valid. For a more strict analysis see Dass and Qureshi (1999).

Let the exact eigenstates of H be $|\Psi_{k,m}\rangle$ and the corresponding eigenvalues be $E(k,m)$, we have

$$|t=T\rangle = \sum_j c_j \sum_{k,m} e^{-\frac{i}{\hbar}E(k,m)T} \langle \Psi_{k,m}|E_n, E_j^d\rangle |\Psi_{k,m}\rangle. \tag{6}$$

Since the interaction is very weak, the Hamiltonian H of Eq.(2) can be thought of as $H_0 = H_S + H_D$ perturbed by $\frac{PA}{T}$. Using the fact that $\frac{PA}{T}$ is a small perturbation and that the eigenstates of H_0 are of the form $|E_k\rangle \left|E_m^d\right\rangle$, the perturbation theory gives

$$\begin{aligned} |\Psi_{k,m}\rangle &= |E_k\rangle \left|E_m^d\right\rangle + O(1/T), \\ E(k,m) &= E_k + E_m^d + \frac{1}{T}\langle A\rangle_k \langle P\rangle_m + O(1/T^2). \end{aligned} \tag{7}$$

Note that it is a necessary condition for Eq.(7) to hold that $|E_k\rangle$ is a nondegenerate eigenstate of H_S. Substituting Eq.(7) in Eq.(6) and taking the large T limit yields

$$|t=T\rangle \approx \sum_j e^{-\frac{i}{\hbar}(E_n T + E_j^d T + \langle A\rangle_n \langle P\rangle_j)} c_j \, |E_n\rangle \left|E_j^d\right\rangle. \tag{8}$$

When P commutes with the free Hamiltonian of the device, i.e., $[P, H_D] = 0$, the eigenstates $\left|E_j^d\right\rangle$ of H_D are also the eigenstates of P, and thus the above equation can be rewritten as

$$|t=T\rangle \approx e^{-\frac{i}{\hbar}E_n T - \frac{i}{\hbar}H_D T - \frac{i}{\hbar}\langle A\rangle_n P} \, |E_n\rangle \, |\phi(x_0)\rangle. \tag{9}$$

It can be seen that the third term in the exponent will shift the center of the pointer $|\phi(x_0)\rangle$ by an amount $\langle A\rangle_n$:

$$|t=T\rangle \approx e^{-\frac{i}{\hbar}E_n T - \frac{i}{\hbar}H_D T} \, |E_n\rangle \, |\phi(x_0 + \langle A\rangle_n)\rangle. \tag{10}$$

This shows that the center of the pointer shifts by $\langle A\rangle_n$ at the end of the interaction. For the general case where $[P, H_D] \neq 0$, we can also obtain the similar result. Thus protective measurement can measure the expectation value of the measured observable in the measured state.

Let the explicit form of $|E_n\rangle$ be $\psi(x)$, and the measured observable A be (normalized) projection operators on small spatial regions V_n having volume v_n:

$$A = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \notin V_n. \end{cases} \tag{11}$$

A protective measurement of A then yields

$$\langle A\rangle = \frac{1}{v_n}\int_{V_n} |\psi(x)|^2 dv, \tag{12}$$

which is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region V_n. Similarly, we can measure another observable $B = \frac{\hbar}{2mi}(A\nabla + \nabla A)$. The measurement yields

$$\langle B\rangle = \frac{1}{v_n}\int_{V_n} \frac{\hbar}{2mi}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*)dv = \frac{1}{v_n}\int_{V_n} j(x)dv. \tag{13}$$

This is the average value of the flux density $j(x)$ in the region V_n. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can measure $\rho(x)$ and $j(x)$ everywhere in space.

Since the wave function $\psi(x,t)$ can be uniquely expressed by $\rho(x,t)$ and $j(x,t)$ (except for a constant phase factor):

$$\psi(x,t) = \sqrt{\rho(x,t)} e^{im \int_{-\infty}^{x} \frac{j(x',t)}{\rho(x',t)} dx'/\hbar}, \tag{14}$$

the whole wave function of the measured system can be measured by protective measurement.