On the Metaphysics of Least Action

Benjamin T. H. Smart*[1] and Karim P. Y. Thébault†[2]

[1] *Department of Philosophy*, University of Birmingham, Birmingham, United Kingdom
[2] *MCMP*, Ludwig Maximilians Universität, Munich, Germany

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Abstract

When it comes to predicting the evolution of physical systems, there seem to be two mathematically equivalent, but conceptually distinct kinds of what we might call 'fundamental laws': there are those laws we are most used to talking about – Newtonian-style laws whereby we can take the state of a system at a time, t, apply the relevant laws of nature, and predict the state of the system at time $t + 1$. These kinds of laws we refer to as 'equations of motion'. But there is also a fundamental principle of a different nature – a teleological principle telling us that as a physical system evolves from one state to another, the path the system takes through its state space is that which minimizes, or to be more precise, extremizes action. The apparent teleological aspect of the law is conceptually somewhat strange – how does the electron know where it's going to end up, and what route it should take to take there to minimize the action? Nonetheless it is not a principle to be ignored by the metaphysician, purely because it is strange. We consider, from the three most popular metaphysical perspectives, what this principle of least action should be taken to be ontologically, its modal profile, and where it appears in the explanatory hierarchy. We conclude that whereas the dispositional monist and Armstrongian can account for the principle of least action, they can only do so by implementing primitives at a level they would be uncomfortable with; the Humean conception of laws, without amendment or discomfort, is happily committed to the PLA being the most fundamental law of nature.

*email: ben@jerseyserve.com
†email: karim.thebault@gmail.com

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1 Introduction

Although it is not uncommon for metaphysicians to put the fundamental laws to one side and discuss, say, causation in terms of macroscopic objects like vases, matches and so on (Mumford and Anjum 2011), in this paper we are concerned with our most fundamental physical principles. When it comes to predicting the evolution of physical systems, there seem to be two mathematically equivalent, but conceptually distinct kinds of what we might call 'fundamental laws': there are those laws we are most used to talking about – Newtonian-style laws whereby we can take the state of a system at a time, t, apply the relevant laws of nature, and predict the state of the system at time $t + 1$. These kinds of laws we refer to as 'equations of motion'. But there is also a fundamental principle of a different nature – a *teleological* principle telling us that as a physical system evolves from one state to another, the path the system takes through its state space is that which minimizes, or to be more precise, extremizes action.[1] The apparent teleological aspect of the law is conceptually somewhat strange – how does the electron *know* where it's going to end up, and what route it should take to take there to minimize the action? Nonetheless it is not a principle to be ignored by the metaphysician, purely because it is strange. We consider, from the three most popular metaphysical perspectives, what this principle of least action (PLA) should be taken to be ontologically, its modal profile, and assuming that laws must have explanatory value, where it

[1] This distinction with regard to laws of nature corresponds precisely to the difference between the 'Newtonian schema' and 'Lagrangian schema' discussed in (Wharton 2012). There the author argues in favour of the latter.

appears in the explanatory hierarchy. We conclude that whereas, the dispositional monist and Armstrongian can account for the PLA only by implementing primitives at a level they would be uncomfortable with; the Humean conception of laws, without amendment or discomfort, is happily committed to the PLA being the most fundamental law of nature.

2 The Lazy Plumber and The Principle of Least Effort

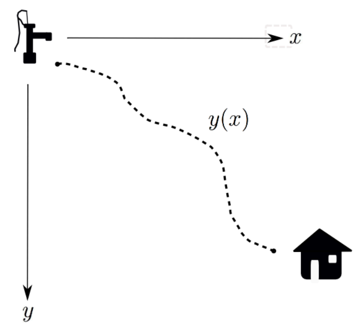
Since action principles and state space are rather unintuitive things, we will provide a metaphorical introduction to them via the *Fable of the Lazy Plumber*.

A man wants to pump water from a well at one end of a steep, smooth valley to his house at the other end using a long piece of flexible rubber tubing. Being a lazy man (who would rather spend his time reading philosophy) he decides he only wants to spend at the very most half an hour a day pumping water. Starting at his house, he rides to the well on his donkey laying the tubing along the valley as he goes in a haphazard manner, without any concern for which path will allow the water to run most easily. This initial path between the two fixed endpoints (i.e. the well and his house) proves inadequate since the energy it takes for the water to run the distance of the piping (sometimes against the gradient sometimes with it) is such that only a very small amount makes it to his house after his half hour of pumping. To remedy this, the next day he adjusts the path of the tubing slightly, and observes that evening that a little more water has come through – but still not enough for his daily household tasks. After spending several weeks adjusting the path of the pipe each day and observing that sometimes more and sometimes less water comes through, he finds a path where enough water comes out. However, by this time, maddened by the tedium of his task, he has decided that he must establish not simply a good path, but the best possible path.

Although our lazy plumber knows only a very little physics, he does know some mathematics, and rather than spending the rest of his life simply trying out every single one of the infinite number of paths, he thinks up a methodology for proving that a given path either is or is not the best. He reasons that how much effort[2] it takes for the water to get through the entire pipe (and therefore the total amount of water that makes it through after his half hour of pumping) can be represented as follows:

[2]The plumbers notion of effort is, of course, not a real concept used in physics – rather we use it here to provide a intuitively helpful analogy to the fundamental physical idea of action that will be introduced in the next section.

Figure 1.



Each point in the valley can be labeled by the horizontal distance from the well, x, and the vertical distance from the well, y. A given path for the piping can then be represented in terms of a curve of the form $y(x)$.

The characteristics of each point in the valley allow him to associate it with a number indicating the amount of effort it takes for the water to pass through the pipe when laid across it. This association between numbers and points in the valley constitutes an effort function $f(x, y)$. Knowing some elementary calculus, the man realizes that for a given path the total effort to pass the length of the pipe, E, is going to be given by integrating the effort function along that path.

Each point in the valley is labelled by co-ordinates x and y. The function f(x,y) gives the 'effort' required for water to flow though a given point. The total effort required for a path $y(x)$ is given by the equation:

$$E = \int_{y(x)} f(x, y)dx$$

In order to find the best path he must find the $y(x)$ which makes E is as small as possible. Now of course, he can't just try out every single possible path since they are infinite! Rather he needs a methodology for evaluating a given path as the best simply from the knowledge that the effort will be at its minimum value for such a path. Again drawing on his knowledge of calculus (specifically the calculus of variations) he reasons that the least effort path will be the one for which very small variations in the path produce no change in E. This is because finding such an extremal path indicates that the quantity E is stationary (either maximum, minimum or an inflection) and given the physical structure of the problem only the minimum option is possible. So the problem of our lazy plumber is solved by insisting that for variations between paths infinitesimal close to his test path there is no change in the amount of effort ($\delta E = 0$), and so we have arrived at a principle of least effort: the path which extremizes the effort is the best possible path. Having thus discovered the principle of least effort our lazy plumber can then rest easy in the knowledge that he is obtaining his daily water supply in the most efficient possible manner and go back to reading his philosophy books.

3 The Principle of Least Action

Consider two uncharged massive point particles whose motion in three dimensional space is described by Newtonian mechanics. At a given time we can represent the state of this system in terms of six configuration variables (three for each particle) and six velocity variables (again three for each particle). Thus, for a two particle system we could describe the state at a given times by a point in a 12 dimensional space. Extending this to N particles, since we still have three position and three velocities per particle, we would need a point in a 6N dimensional space to represent a moment in time. Such a state space is know as velocity-configuration space.

Now, a path through this space is a sequence of possible states, and therefore represents a history of the system. Different paths (different histories) involve the system passing through more or less *energetically demanding* sequences of possible states. It is found in nature that the path that the system actually takes (that which is instantiated in reality) is the one which is in, a specific sense, least demanding. Thus it seems nature must implement a principle for choosing the most efficient path similar to that used by our lazy plumber, only the valley that must be spanned is the space of possible instantaneous states, and the paths represent entire histories of the system in question.

For a given physical system there is a special function particular to it called the Lagrangian, L, which associates each point in velocity-configuration space with a number. The Lagrangian is a functional of all the degrees of freedom of a classical system and may be interpreted as a 'function of all the...intrinsic properties ascribed to the objects in the system by classical mechanics' (Katzav 2004, p. 208). Typically, the explicit form of L is given simply by the kinetic energy of the system minus the potential energy.[3] For our two particle Newtonian system the Lagrangian can be written using two position vectors, $\mathbf{q}_1$ and $\mathbf{q}_2$, as:

$$\begin{aligned} L &= T - V \\ &= \frac{1}{2}m_1\left[\frac{d\mathbf{q}_1}{dt}\right]^2 + \frac{1}{2}m_2\left[\frac{d\mathbf{q}_2}{dt}\right]^2 - G\frac{m_1 m_2}{|\mathbf{q}_1 - \mathbf{q}_2|} \end{aligned}$$

The action, I, of a physical system is then defined between two points in velocity-configuration space for any given path, γ, between those two points. A direct physical interpretation of action is not generally given in physical theory, but its SI units of joules-seconds indicate a close connection with both energy and time (and justifies our analogy with the plumbers concept of effort). The action can be explicitly calculated by the integral of the Lagrangian

[3] The Lagrangian of General Relativity is an important exception to this.

with respect to time along a path, γ.

$$I[\gamma] = \int_{\gamma} L dt$$

The principle of least action (PLA) is then that the actual path taken by a real physical system in velocity-configuration space is that which extremizes the action (where the variation of action upon infinitesimal variation in the path is equal to zero - $\delta I = 0$). Significantly, one can derive all the equations of motion for a system using purely its Lagrangian and the PLA.[4]

Figure 2.

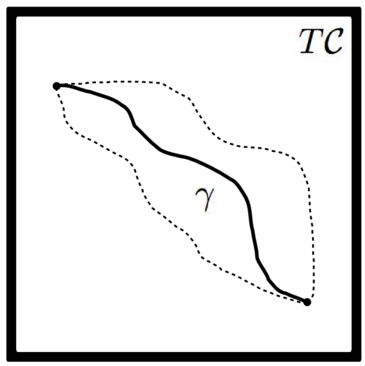


Figure 2. shows the Velocity-Configuration space, $T\mathcal{C}$, of a four-dimensional world, where each point represents the state of the world (including velocities, mass, charge etc) at a particular moment. The dotted lines represent possible paths that could have been taken to get from the first point to the second (sets of possible states the world could have taken en route to the second point). The path of least action – which will always be the actual path – is shown in black.

The PLA and its formulation in terms of the system's Lagrangian is not restricted to Newtonian mechanics. It is found in all classical theories of mechanics including electromagnetism, statistical mechanics, special relativity and general relativity. Moreover, through the path integral formalism, essentially the same structure is also utilised within quantum theories also. So quantum field theory and even string theory in fact manifest a quantum version of the PLA.[5] the appropriate classical limit it is the correct expression for the quantum amplitude of a path.So in an important sense, quantum mechanics, quantum field theory and

[4]More precisely, the PLA allows us to derive the Euler-Lagrange equations which in turn provides us with the equations of motion for any given Lagrangian.

[5]We can, for instance, consider Feynman's second postulate in his original derivation of the path integral formulation (Feynman 1948) (which partially derives its origin from remarks due to (Dirac 1933)). The postulate is that the function $e^{\frac{i}{\hbar}\int_{\gamma} L dt}$ will be proportional to the quantum amplitude of a classical path γ. The nature of the exponential functional form means that the amplitude will peak upon the path of the least action path (i.e. when $\delta I = 0$). It is this feature that allows us to recover the classical theory in the limit of $\hbar \to 0$. Thus, the PLA is still, in a sense, fundamental within the quantum realm.

even string theory manifest a quantum version PLA. Thus, one has a solid physical basis to consider the PLA to be a universal principle.

This being so, we hope it is apparent why we feel it to be crucially important that any viable theory of laws provides an adequate metaphysical account of the principle, and where it should stand in our ontology.[6]

4 Metaphysical Desiderata

The Many Possible Paths Problem (MPPP)

Katzav makes the non-trivial assumption that the PLA presupposes the contingency of the equations of motion within physical systems. The principle, he claims, suggests that 'the action of any given physical system could have taken various values, and thus that any such system could have been correctly described by different equations of motion, even once the objects that comprise it, along with their intrinsic properties and initial distribution, are determined.' (Katzav 2004, p.210). Any metaphysical theory with this implication, it is argued, must be incompatible with the PLA.

Explanatory Force

It is generally agreed that laws must have explanatory force. What the conditions are for having explanatory force is of course hotly disputed, and often an account of laws is rejected because, or at least partly because the so called laws, as conceived by their opponent, have no explanatory power.[7] But if the PLA is to be law, and intuitively it should be, it must have explanatory power as – and a great deal of explanatory power at that. It is, after all, a principle from which *all* the equations of motion can be derived. In the following discussion we do not presuppose any conditions for explanatory value independent of the metaphysical view - rather, we shall consider the PLA's explanatory force in the context of the metaphysical position being discussed; that is, if x requires property p for x to have explanatory power under the conditions specified by those holding metaphysical theory M, and the PLA has property p, then in the context of M the PLA has explanatory power.

There is then the further question of where in the explanatory chain the PLA sits. Given the universality of the PLA, and the fact that the many distinct equations of motion can be derived from the principle (along with the system's Lagrangian), on the face of it we might expect the PLA to be the ultimate explanands. In fact, Katzav even argues that we can

[6] Arguably the PLA should not be thought of as a law itself, but as a principle that describes the way our physical system evolves. In these terms an adequate account of laws should explain the PLA but need not include it as one of its laws.

[7] Armstrong rejects Humeanism about laws partly for this reason.

rule out any metaphysical position that appeals to anything other than, or in addition to the PLA, to explain the dispositions of things (in the sense of how things are disposed to behave under certain conditions). We dispute this claim, but the PLA's explanatory role is nevertheless an important issue to be discussed.

In the following sections we look at the three metaphysical accounts of laws individually, initially to show where the PLA will fit into their respective ontologies, and ultimately to see how each of these conceptions of natural laws will deal with the MPP and explanatory value issues described above.

5 Dispositional Monism

Undoubtedly the kernel of any metaphysic of laws and causation is the nature of the properties and relations it advocates. Take categorical properties to be all and only those properties that are essentially non-dispositional. As there is no tie to their causal/nomological role, the identity of categorical properties is antecedent to modal facts. As such, their identity is primitive – they are quiddities. In contrast, dispositional properties are modally active, with their identity fixed by the stimulus/manifestation-relations (SM-relations) they bear to one another.

Dispositional essentialists hold (DE_p), '(that) at least some sparse, fundamental properties have dispositional essences' (Bird 2007, p.45), where any object that possesses the dispositional essence of some potency P is disposed to manifest the corresponding disposition M under stimulus conditions S, in any possible world:

$$(DE_p) : \Box(Px \rightarrow D_{(S,M)}x)$$

Bird shows that from (DE_p) and the conditional analysis of dispositions as a necessary equivalence[8] that we can derive (I) $\Box(Px \rightarrow Sx \Box\!\!\rightarrow Mx)$, and furthermore that when we 'consider any world w and any case where some x in w possesses the potency P, where x acquires the stimulus S (that is, (II) (Px & Sx), we can ultimately derive a universal generalization of the form $\forall x((Px \& Sx) \rightarrow Mx)$.[9] The laws of motion, we take it, will be derived from the dispositionalist's natural properties in this way.[10] It follows that according to the dispositionalist's thesis of physical modality, the equations of motion are necessary, as

[8] $\Box(D_{(S,M)}x \leftrightarrow Sx \Box\!\!\rightarrow Mx)$

[9] From (I) and (II) we have (III) Mx, discharging (II) we have (IV) $(Px \& Sx) \rightarrow Mx$, and since x is arbitrary we may generalize $Ax((Px \& Sx) \rightarrow Mx)$ (Bird 2007, p.46)

[10] It is perhaps worth noting that some dispositional essentialists, notably (Mumford 2004), deny that there are laws in nature. But properly understood what Mumford is denying is the existence of extrinsically governing laws. It seems to us that Mumford's 2004 leaves the possibility of internally governing laws an open question.

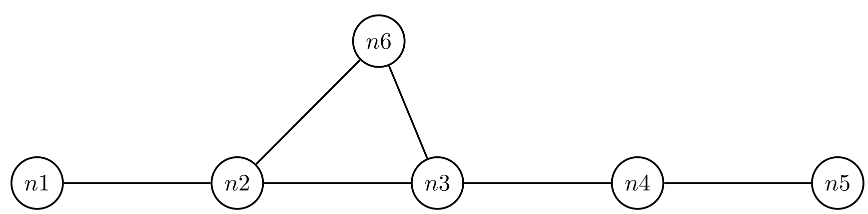
the SM-relations properties stand in to one another is fixed (the properties' very identity is fixed by these relations).

Applying dispositional essentialism to the PLA, we can translate the principle as follows:

Each point in velocity-configuration space represents an instantaneous pattern of dispositional, or dispositional and categorical property instantiations,[11] and the curve (as shown in Figure 2.) represents the actual evolution of the system through various states. According to (DE), then, the curve is entirely determined by the nature of the properties of the system (and the initial distribution of property instantiations). This is, it would seem, at odds with the 'many possible paths' claim.

The dispositional monist (unlike those who hold a 'mixed view' of both categorical and dispositional properties) takes all properties to be powers, where the identities of these properties are fixed relationally; that is, the first order natural properties have their identities fixed by the relations they stand in to their stimuli and manifestations. These stimuli and manifestations are natural properties in their own right, and so these, too, have their identities fixed by the stimulus-manifestation relations (SM-relations) they stand in to other natural properties. On the face of it a regress, or 'infinite complexity' arises here, as every natural property has its identity fixed by another natural property, which has its identity fixed by a third property, and so on. But it need not be a regress. The pattern of relations may turn back on itself, creating a network. It might seem as if we've simply turned the regress into circularity, but using 'graph theory', the monist claims the kind of circularity involved in the fixing of properties' identities is virtuous.

Figure 3.



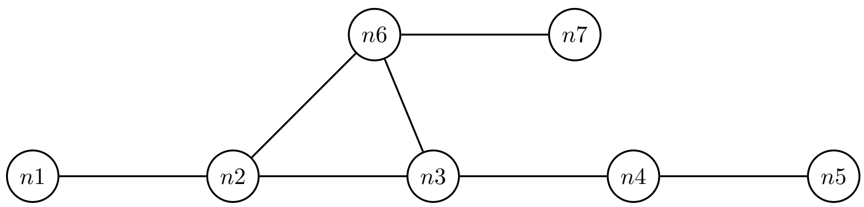
Take the graph above to represent the natural properties (the nodes) and the relations fixing their identities. Aside from the graph with just a single vertex, the graph in figure 3 is the simplest non-trivial asymmetric structure that can fix the identity of properties, but it is possible to include loops and digraphs (directed graphs) to add further asymmetries (it seems there needs to be some directedness when using graph theory, as a representation of the relations between properties and their manifestations are, in most cases, irreversible). Once we allow directedness into the graph theory the graphs look to provide an acceptable

[11] There are a number of dispositional essentialist accounts, some for which accept and some of which deny the existence of categorical properties.

representation of the property networks, and the identity regress problem seems to be have been dealt with, as the identity of the vertices is uniquely fixed by the structure of the graph.

Suppose figure 3 represents the property network of the actual world, w, with only 6 fundamental properties, and where $n4$ is the natural property, 'charge $-e$'.[12] Inhabitants of w would identify 'charge $-e$' primarily with its SM-relations to $n3$ and $n5$, but more accurately its identity is fixed by its place in the entire network. Indeed, any world with 'charge' would have to be one with a property structure identical to that of w.

Figure 4.



Now suppose there is a world, $w1$, with the graph structure outlined in Figure 4. Here we see an asymmetric graph structure very similar to Figure 3., but with an additional natural property, $n7$. It's possible for $n7$ never to manifest (suppose that whenever its stimulus condition is met, there is some 'fink' preventing the manifestation). Arguably, $w1$ could look identical to w, but it would share none of the same properties. $n4$ in $w1$ would not be 'charge', but, say, 'schmarge'.

For the dispositional monists, the structure of these property networks determines the nature of physical systems' natural properties, and as such how these systems evolve through velocity-configuration space. Thus the PLA, and where it holds, seems to supervene upon these property structures. One could not change the nature of the actual world such that it is no longer a PLA world without changing all its properties – there would be no 'charge' in any non-PLA world. However, it seems that the PLA might well be multiply realisable; that is, it's conceivable that, in principle, different graph structures in different possible worlds could give rise to the PLA. This should not be surprising. After all, even in the actual world we have seen the PLA to hold for many different physical theories (Newtonian Mechanics, General Relativity...).

But if the PLA supervenes upon the graph structure, the dispositional monists still owe us an explanation as to why it holds; that is, why does the graph structure lead to the system following the path that extremises action, rather than any other path? The path itself is of course necessitated by the network of properties, but is it really a cosmic coincidence that the actual network is one that extremises action? Given that ours is a world with charge-instantiations, ours must be a PLA world – but why must ours be a world with charge-instantiations? Why couldn't it have been a non-PLA world with schmarge-instantiations?

[12] Where the unit of charge e, is approximately equal to $1.6 \times 10^{?19}$ Coulombs

6 Laws as Extrinsic Governing Relations between Universals

We have now seen the problems the dispositional essentialists face when dealing with the PLA. Bird's dispositional monism struggles to show why the least action path is followed. Joel Katzav clearly expresses his belief that, as the PLA is fundamental, including a dispositional account of the evolution of the physical system can only lead to unwarranted causal overdetermination, and as (he asserts) the PLA is more fundamental, we should deny dispositions any explanatory value. This is all very well, but he does not provide any positive metaphysical account of the PLA – he rather just states that it must be more fundamental than the equations of motion. Neither does he give a metaphysical account of the instantaneous equations of motion. What he *does* say is:

> that some quantity is an extremum seems to imply that, if it is actual, its actuality is not an accident. Moreover, that something is not an accident enables appealing to it in explanations. The dispositionalist assumes that the dispositions of objects alone are the ultimate explanatory ground for events. However, if we accept the PLA, we accept that there is an additional ground, and hence explanation, for why all the objects that make up the physical system [behave the way they do in certain circumstances]' (Katzav 2004, p.212)

The implication is that the PLA is something over and above dispositional properties; that it is not merely a regularity, that the path the physical system takes through velocity-configuration space is contingent, and that it is something in the world. This rules out Humeanism (to which we return in Section 7) because of the apparent cosmic coincidence of the regularity, and dispositional monism (in a sense) for the same reason. The only option left from the four main contenders for a metaphysics of laws is the Armstrongian view that laws are extrinsic governing relations between universals.

'Armstrongianism', we take it, is motivated by a conceptual assumption: namely that the spatiotemporal distribution of properties should be determined by *governing* laws, rather than laws supervening on their instances or on dispositional properties. Armstrongians make the following claims:

1. That all natural properties are universals, and universals are quiddistic;

2. Immanent realism about universals; that is, one and the same universal is wholly present in each instantiation of it, but it exists only in its instances. (The universal 'charge' therefore exists only where there is an instance of charge; this clearly contrasts with the view that they exist transcendentally.);

3. That there is a hierarchy of universals. Examples of first order universals would be 'red' and 'charge $-e$', but there are also first order relations (second order universals) between first order universals, and so on;

4. There are natural necessitation relations, N[13], that hold between two (or more) first order universals, denoted $N(F, G)$. Where $N(F, G)$ holds in a world, it is metaphysically necessary that all Fs are Gs - $N(F, G)$ denotes a law of nature.

Although N, F and G are all universals whose causal/nomological roles are contingent (as $N(F, G)$ holds only contingently in any world), in all possible worlds in which '$N(F, G)$ and x is F' is true, 'x is also G' [14]. Therefore, if, in the actual world, the PLA is a law of the form $N(F, G)$, then this is the reason action is extremised.[15] The fact that action is extremised surely can't be coincidental, and so we should assume the PLA to be a metaphysically substantial law rather than (merely) a description of what happens. There is of course the issue of what $N(F, G)$ would look like in the case of the PLA. In cases like charge and electrons, it's simple to picture what the universals in the law would be: N(Electrons, charge $-e$). Things aren't quite so simple with the PLA. In principle, though, we see no reason why the proponent of this view could not claim it to take the N(F,G) form. If, for example, we allow states of physical systems to be universals, and quantities of action through velocity configuration space to be universals, the PLA could take a form resembling the following:

$$Arm(PLA) : N(\text{Initial state, Final state, Extremal path})$$

This formulation involves highly structured universals, but nonetheless, it seems possible that a law resembling this structure could be formalised by expressing a particular state of a system (a property we have invented that could, in principle, be instantiated by many different systems, or by the same system on multiple occasions) in terms of the fundamental universals.

[13] The relation itself itself is sui generis

[14] There are thus necessary connections between distinct existences according to this view

[15] It has been argued (Bird 2005) that there is a serious problem with the natural necessitation relations between universals view, in that Armstrong would have to account for the relation between $N(F, G)$ and the holding of the regularity 'all Fs are Gs' – he would, Bird argues, have to posit a further necessitation relation linking $N(F, G)$ and 'all Fs are Gs', but again this would require a similar explanation, generating a regress. As (Authors 2012) have shown, firstly this objection is equally applicable to Bird's own dispositional monist account, and secondly, so long as Armstrong is willing to accept that the kind of necessitation implicit in his view is brute, there is no regress. We shall not, therefore, rule Armstrong's view out tout court.

Laws as Natural Necessitation Relations between Universals and the Many Possible Paths Problem

The MPPP is not problematic for the Armstrongian. The fact, $N(F, G)$, holds contingently at a world, and thus so does the type of necessitation it affords. In every possible world where $N(F, G)$ is true, 'all Fs are Gs' is true, but there are many more possible worlds where $N(F, G)$ is false. The fact that the path is extremised therefore comes out as contingent for Armstrong (in the sense that $N(F, G)$ holds contingently at a world).

Laws as Natural Necessitation Relations between Universals and the Explanatory Problem

On the face of it there is no problem concerning explanation with this view, either. The laws are states of affairs residing in the world. They govern the motion of the particulars in the physical system, ensuring that the system as a whole follows the path of least action. The theory does not posit anything in addition to the natural necessitation relations between universals to explain the evolution of the physical system, and so the objection Katzav raised against the dispositionalist holds no weight. So where does this leave the equations of motion? Suppose two massive objects attract one another. We would have two explanations for this: an instantaneous 'equations of motion' explanation, $Arm(Gravity)$, and a teleological PLA explanation, $Arm(PLA)$ – both expressible in $N(F, G)$ form:

$$Arm(Gravity) \quad : N(\text{Massive Objects, Gravitational Pull})$$
$$Arm(PLA) \quad : N(\text{Initial state of system, Final state of system, Extremised path})$$

Both seem to be laws that reside in the world, and both are supposed to govern the way the massive objects behave, but they appear to be distinct existences. So if we allow the metaphysical interpretations of both the equations of motion and the PLA to be of the form $N(F, G)$, then there is causal overdetermination, a most unwelcome consequence in the context of Armstrong's view. If $Arm(PLA)$ and $Arm(Grav)$ really *are* distinct existences, this is bad news for Armstrong.

We think there are two options to consider. Firstly we could deny that one of the two laws is not, in fact, a law, and thus cannot be appealed to in explanations. If we chose to reject the PLA, then the one principle that seems to hold universally turns out not to be a law of nature, and perhaps, even, that the fact that physical systems in the actual world always follow paths with extremised action is purely accidental. This is the same conclusion the dispositional monist is committed to, and one that, if possible, should be avoided. But it also seems strange to accept that we cannot appeal to the laws corresponding to the equations of motion in explanation. But that would be the consequence of discarding $Arm(Gravity)$.

Rejection of either law is unappealing.

A second option might be to consider $Arm(Gravity)$ and $Arm(PLA)$ not to be distinct existences. But if they are not distinct existences, what is the relation between them? One *prima facie* possibility is that $Arm(Gravity)$ is a determinate of the determinable, $Arm(PLA)$. But this can't be right. It may be true that the equation of motion corresponding to $Arm(Gravity)$ must be true of our world for the PLA to hold, but it cannot be a determinate of $Arm(PLA)$, as $Arm(PLA)$ is a teleological and not an instantaneous principle.

Perhaps another possibility is to see the relation between $Arm(Gravity)$ and $Arm(PLA)$ as one of identity, in the same vein as we have seen in the philosophy of mind. Perhaps when we give explanations in terms of $Arm(Grav)$and $Arm(PLA)$we are referring to the same thing: the quality of the actual world governing the evolution of its physical systems, but with a different meaning (just as in the case of 'morning star' and 'evening star' having the same referent but different meaning). But this won't work, either. The mathematical formalism of $Arm(Grav)$ is not equivalent to the mathematical formalism of $Arm(PLA)$ - you can't take Newton's law of gravity and derive that electrons follow the principle of least action. There are no other plausible options, so either we take all the explanatory value away from the equations of motion, or we take it from the PLA.

When Katzav criticises dispositional essentialism, he does so partly because he feels the essentialist attributes explanatory value to the dispositions of things, when the explanation of causal interactions should be exclusively given in terms of the PLA. Katzav would therefore welcome the conclusion reached above - although $Arm(Gravity)$ in not a dispositionalist state of affairs, the same objection would be raised against the Armstrongian who wished to keep it, so Katzav would be happy to dispose of $Arm(Gravity)$ altogether, leaving $Arm(PLA)$ to do all the work. If physical systems follow extremised paths, we take this to be non-accidental and in need of explanation. For the Armstrongian, the extrinsically governing $Arm(PLA)$ looks to be the option to go for. This conclusion might be counterintuitive, after all, we do tend to use the laws mathematically expressed by the equations of motion in explanation more regularly than the action principle formalisms, but the fact that it's counterintuitive is not enough to disregard the position from a metaphysical perspective, where we require our explanantia be real and in the world.

7 Humeanism

Take the Humean account of laws to be the view that laws are mere regularities, expressed by the universal quantifications that form part of a best system of law-statements (Lewis 1986). Laws, according to this view, are contingent truths, where contingent truths are true in virtue of patterns of fundamental property instantiations and the fundamental relations between these instantiations, and in virtue of these alone - or as Bigelow puts it, 'truth supervenes on being' (Bigelow 1988, p.132).

Not only do the fundamental laws or axioms of the best system count as laws, but so do any propositions that logically follow from these axioms. *Prima facie* this is a good result, as many of the statements we accept as law-statements do not express the basic laws, but something less fundamental. The statement 'all electrons have charge $-e$', for example, does not seem to express one of the fundamental laws in our world (since it enters our current fundamental theories merely as a measured parameter), but it is widely accepted as a law of nature nonetheless.

So where does this leave the PLA? In the actual world, the natural property 'electron' is always co-instantiated with the natural property 'charge $-e$'; in other words, in the actual world the members of the set of electrons are all members of the set of things with charge $-e$. We know that the proposition 'all electrons have charge $-e$' is a law, but not a fundamental one, and so according to this view it must be derivable from the fundamental axioms of the best-systems account. Take the fundamental laws to be the equations of motion we usually take them to be – the laws of electromagnetism, gravity, the conservation of momentum, the conservation of energy. For the Humean, from these are derivable all other laws of nature. As the equations of motion are mathematically equivalent to the PLA plus its corresponding Lagrangian, in knowing the equations of motion we can derive the PLA. It is thus a law.

But of course the equations of motion are also derivable from just the PLA and the Lagrangian, so we have no prima facie reason to think the equations of motion are fundamental. The equations of motion are distinct from one another - the law of gravity makes no reference to charge, or magnetic fields. The PLA, on the other hand, is not like this. It is a single principle that encompasses all motion within a physical system. It would make more sense, it seems, to take the PLA as our most fundamental law, and the equations of motion to be laws in virtue of them being derivable from the PLA. The equations of motion retain their status as laws of nature, but they are no longer the most fundamental. Despite the apparent oddness of this teleological law, there is a nice parsimony to their being a single principle from which all other laws can be derived.

The Humean, then, will have the following to say about the PLA:

1. It is our most fundamental law, from which all other laws can be derived.

2. As a law, it is a mere regularity, expressed by a universal quantification forming part of a best system of law-statements

3. The properties the law refers to are categorical – and in being just primitively perfectly natural sets of particulars, they have no intrinsically powerful nature.

Many Possible Paths and Humeanism

For the Humean there simply is no problem when it comes to the inherent contingency of the path the physical system follows. Yes, 'the action of any given physical system could have take various values, and thus that any such system could have been correctly described by different equations of motion' (Katzav 2004, p. 210), but rather than being problematic, the Humean will embrace this. According to Humean Supervenience, the PLA is a contingent truth that supervenes on the 4-dimensional mosaic of property and relation instantiations. For the Humean, the PLA is a law *in virtue of the path the physical system follows being that which extremises action.*

Humeanism and The Explanatory Problem

For the Humean, ontologically speaking, laws are nothing more than regularities. This is problematic, so say the anti-Humeans, because the Humean has no good explanation for why the natural uniformities we observed every day have held, and we have no good reason to suppose they shall continue to do so. To see where the Humean believes explanatory power might lie, consider the following example provided in (Smart 2012, p.321-325): the results of the exit poll come through, and Obama has 52% of the vote. The exit poll provides a roughly random sample of the total number of voters, and it is reasonable to suppose that results of the polls will be indicative of the election results. Sure enough, Obama gets roughly 52% of the vote. So what best explains the results of the poll? The answer, surely, is that 52% of the total population voted for Obama. Similarly, a phenomenon such as 'all spherical balls with mass are drawn to all other objects with mass' might be explained by the higher order regularity, and moreover the *law*, that 'all *objects* with mass are drawn to all other objects with mass', and so on. Higher order regularities can, it seems, have explanatory value. An objector might object that we need to know about Obama's policies and the needs of the voter to explain the polls – there is a reason they voted the way they did, and the way the total population voted doesn't give us this, but the Humean doesn't need to give a reason for his regularities. There are just the patterns, and all truths supervene on these patterns. But, as we have just seen, some of these truths seem to be perfectly good explanations of others. So does the PLA have explanatory value? We claim that, from the Humean perspective, it does. It is the highest-order regularity there is – the regularity from which all the other, lower order law-statements logically follow, and arguably thus the regularity with the most explanatory value.

8 Concluding Remarks

For any state space, there are many possible curves between any two points, so a physical system can in principle evolve in many different ways and end up in the same final state. And yet in our world (presuming a classical mechanics) the path of least action is always followed. It seems right to say this is no accident, and it also seems right to say that if the PLA is the reason the path followed is always that which minimizes action, it is something we can appeal to in explanations. In these respects we can agree with Katzav.

If one makes the assumption that the curve must be contingent, however, one automatically rules out dispositionalist ontologies - but this is not a trivial matter, and needs to be justified. All that has been demonstrated is that there are many logically possible curves between the two points in the velocity-configuration space, not that there are many *metaphysically* possible ways in which the physical system could have evolved. It has been shown that for dispositional monism there is only one metaphysically possible path, but this still allows for there to be many logically possible paths, so the MPPP does not seem overly problematic. But the dispositional essentialist is left to explain why the only metaphysically possible path is that which minimizes action, and this looks far more troublesome. The fact that the PLA holds, for the dispositional monist, supervenes on the fundamental dispositional properties and on them alone, so the only option seems to be to appeal to brute metaphysical necessity. The graph structure of the dispositions just has to be such that it fixes the 'right' curve, but there is no explanation as to why. The instantaneous laws are explicable by the dispositional properties of things, yet there seems to be no reason as to why this is mathematically equivalent to the teleological description. The dispositional monist, it seems, can give a complete explanation of the evolution of the system from one moment to the next in terms of the instantaneous laws of motion, but has no teleological metaphysical interpretation explaining the important and surely non-accidental PLA.

The Armstrongian position, with its sui generis natural necessitation relation, successfully adheres to Katzav's requirements. The necessity in this conception is contingent, so although it is no accident that the PLA holds as it is governed by a law, the fact that the law holds is contingent. There are many possible worlds in which $Arm(PLA)$ is not a law, so the curve is not only logically, but also metaphysically contingent. $Arm(PLA)$ also seems to serve us well in explanation. It explains the evolution of the physical system in accordance with the PLA, and from this alone can all the equations of motion be derived. The equations of motion will not be laws, but this is a welcome consequence for the Armstrongian, not a worry. Again, though, a considerable primitive fact must be accepted. It just happens to be the case that in all metaphysically possible worlds where $N(F, G)$ is a fact, 'All Fs are Gs' is a fact. The position remains perfectly coherent, but perhaps the Armstrongian is again committed to unwelcome primitives..

It is our contention that the Humean, too, has a position that satisfies the criteria set by Katzav. Humean Supervenience dictates that laws supervene upon the contingent distribution of local particular matters of fact, so in our world the PLA comes out as a law from which all the equations of motion can be derived. Furthermore, unlike with Armstrongian view, the equations of motion also come out as laws just because they are derivable from this most fundamental regularity. Arguably this is more in line with our intuitions than the Armstrongian view. Necessitarians will complain that the laws, for the Humean, are accidental. But the Humean has a means of judging between accidental and non-accidental uniformities, and the PLA is non-accidental under these conditions. We should judge the Humean account of the PLA by Humean standards, not by the intuitions of an Armstrongian, and as we saw, for the Humean the PLA comes out as a non-accidental and contingent regularity that's part of our best system of law-statements, and one that can be appealed to in explanation.

Dispositional monism, Armstrongianism and Humeanism are all consistent with the PLA - they just have to appeal to brute facts in different places. The dispositional monist has to do this at a level unpalatable to a necessitarian. To admit that the graph structure just primitively gives rise to the PLA would be somewhat strange, particularly given their commitment to the necessity of 'metaphysical glue' to avoid laws being accidental regularities. Despite the quiddistic character of all the other universals in their ontology, the Armstrongian is always, without explanation, committed to the N-relation playing a fixed and particular role in their laws of nature. They must rely on it being a primitive fact that all the possible worlds where $Arm(PLA)$ is true are worlds where the path of least action is followed, and furthermore, with no metaphysical account of why there is this relation between the higher order law and the first order property instantiations. For the Humean, though, as is the case with all laws of nature, the PLA is a law because across the 4-dimensional spacetime, the regularity holds, and forms a part of the best system of law-statements. The PLA should be seen as a paradigm case of a law of nature grounded in Humean Supervenience, and should be embraced by regularity theorists. It is the Humeans, then, who can most readily endorse the intuitive conclusion that the PLA is our most fundamental law of nature.

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