§2 波 前

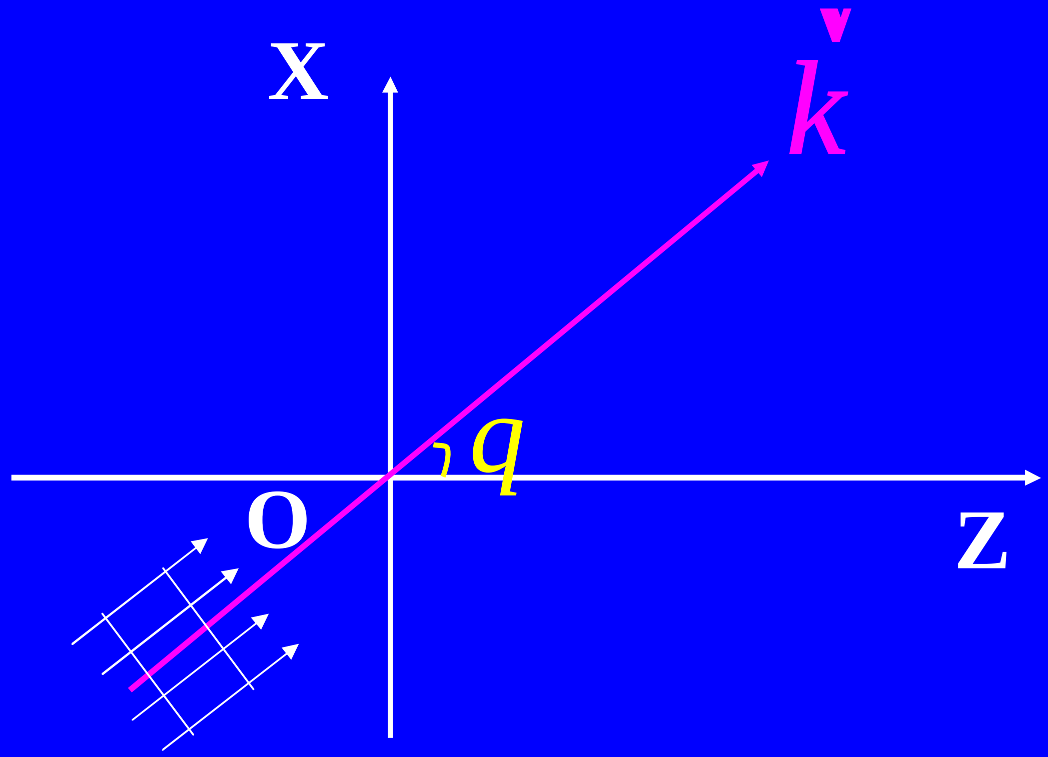
2.1 波前的概念

泛指波场中任一曲面，更多地指一个平面.

不同于波面。

某个波前上的复振幅分布----二维分布函数

例1：一列平面波的传播方向平行于x-z面，与z轴成倾角θ，写出它在波前z=0面上的复振幅分布.



解：

$$\tilde{U}(P) = A(P)e^{i\varphi(P)} = Ae^{i(\vec{k}\cdot\vec{r}+\varphi_0)}$$

$$= Ae^{i(k_x x + k_y y + k_z z + \varphi_0)}$$

z=0 面

$$\varphi = kx\cos\alpha + ky\cos\beta + kz\cos\gamma + \varphi_0$$

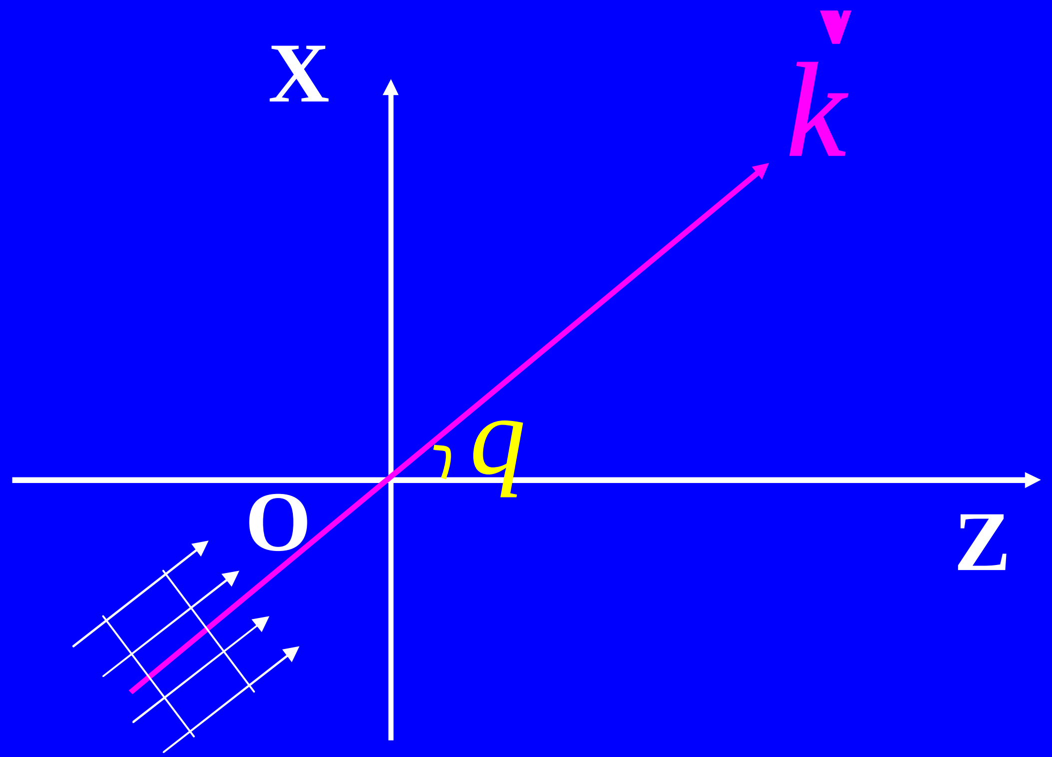
$$\alpha = \pi/2 - \theta, \beta = \pi/2, \gamma = \theta$$

$$z = 0$$

$$\tilde{U}(x, y) = Ae^{i(kx\sin\theta + \varphi_0)}$$

$$\varphi_0 = 0 \qquad \tilde{U}(x, y) = Ae^{ikx\sin\theta}$$

例2：求例1中平面波的共轭波在波前z=0面上的复振幅分布.



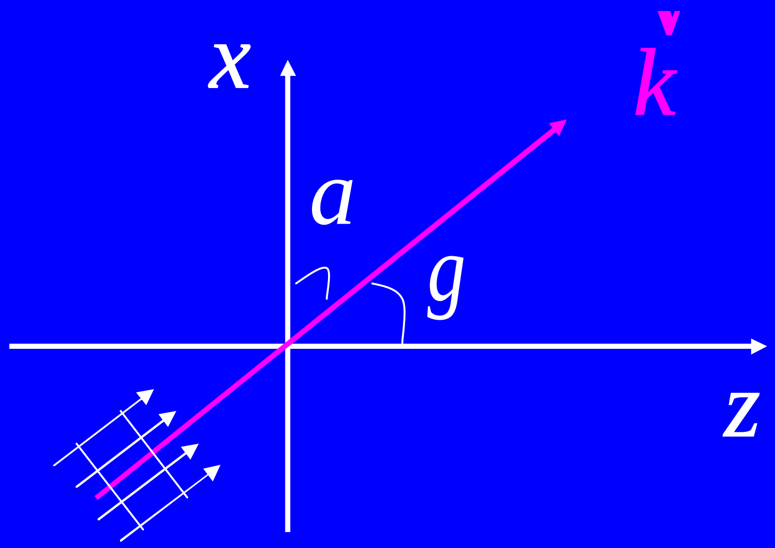
$$\tilde{U}(x,y) = Ae^{ikx\sin q}$$

共轭波： 复振幅互为复数共轭波称为共轭波

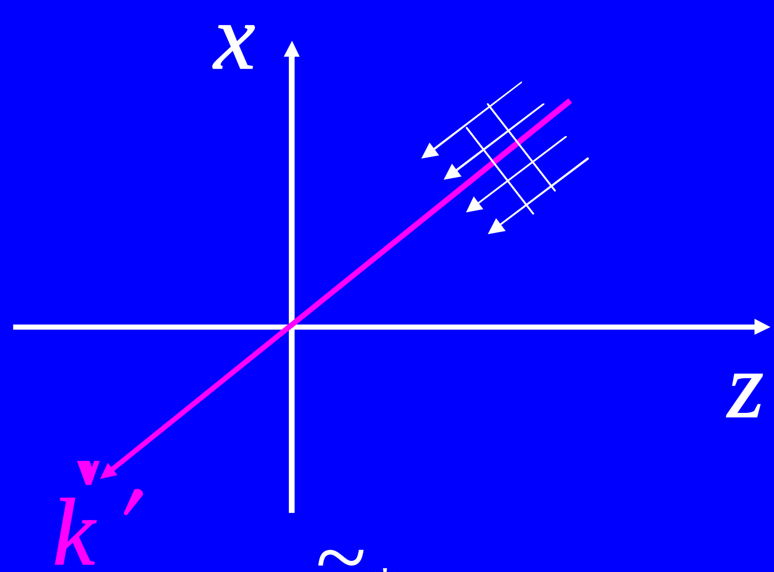
一般地 $\tilde{U}(P) = Ae^{i(kx\cos a + ky\cos b + kz\cos g)}$

$$\tilde{U}^*(P) = Ae^{-i(kx\cos a + ky\cos b + kz\cos g)}$$

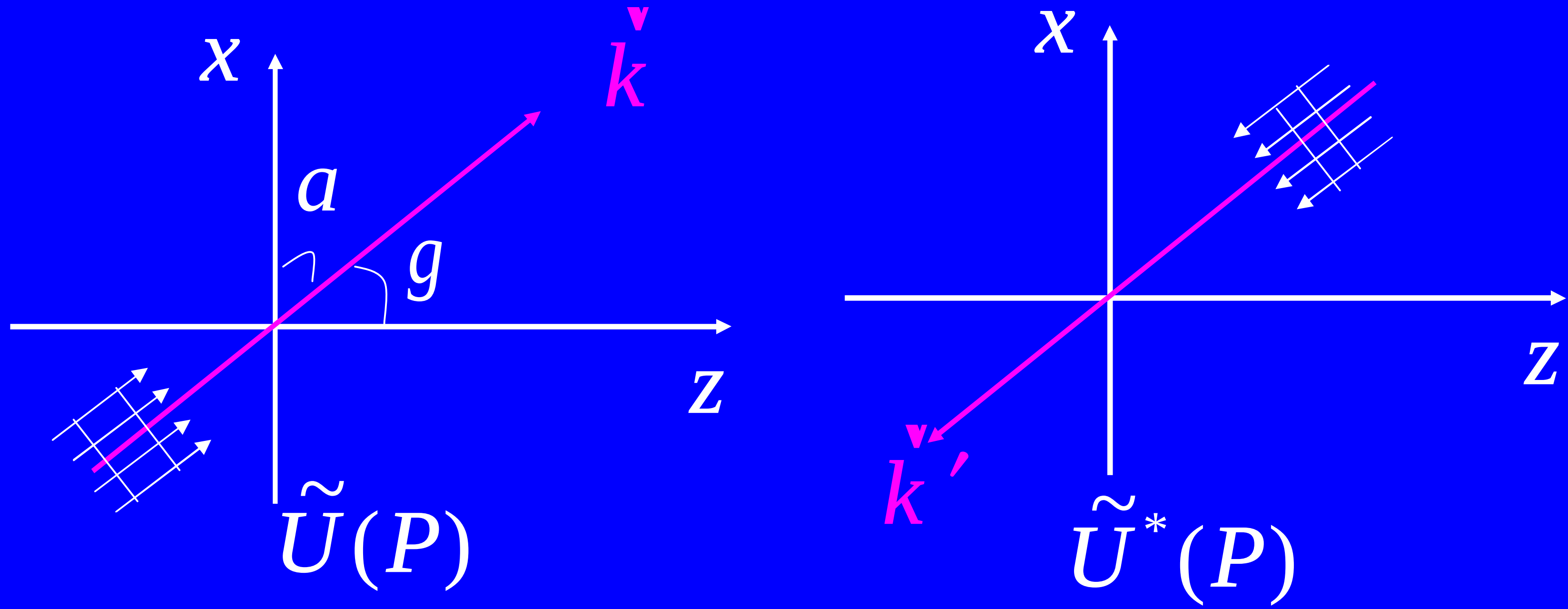
$$= Ae^{i[kx\cos(p - a) + ky\cos(p - b) + kz\cos(p - g)]}$$



$\tilde{U}(P)$



$\tilde{U}^*(P)$



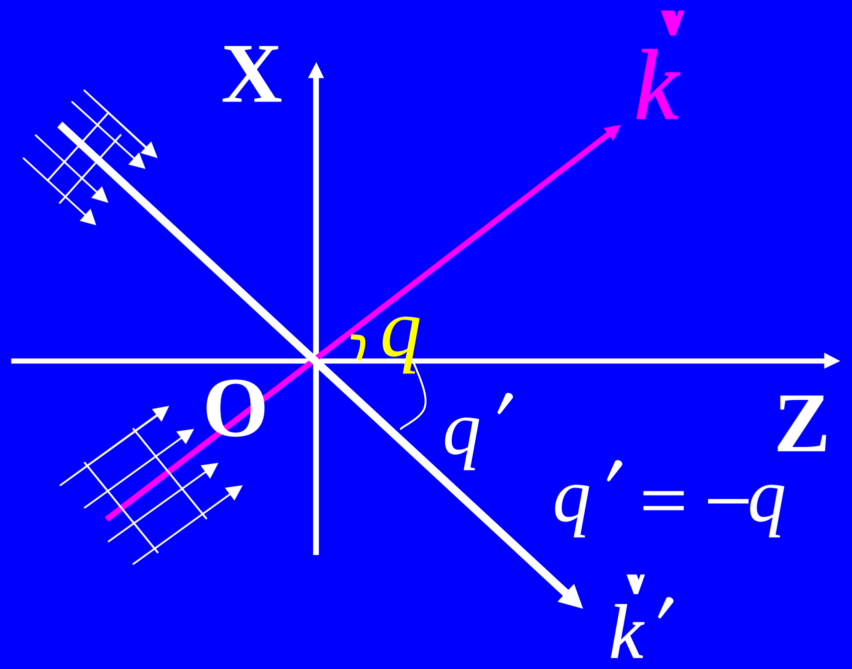
共轭波为反方向传播之波

约定波及其共轭波都来自波前的同一侧

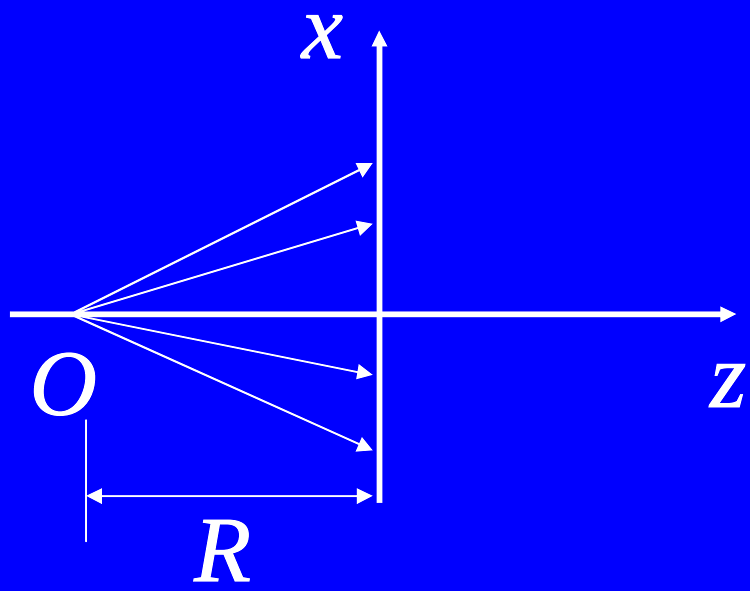
解：$\tilde{U}(x,y) = Ae^{ikx\sin q}$

$$\tilde{U}^{*}(x,y) = Ae^{-ikx\sin q} = Ae^{ikx\cos(p/2+q)}$$

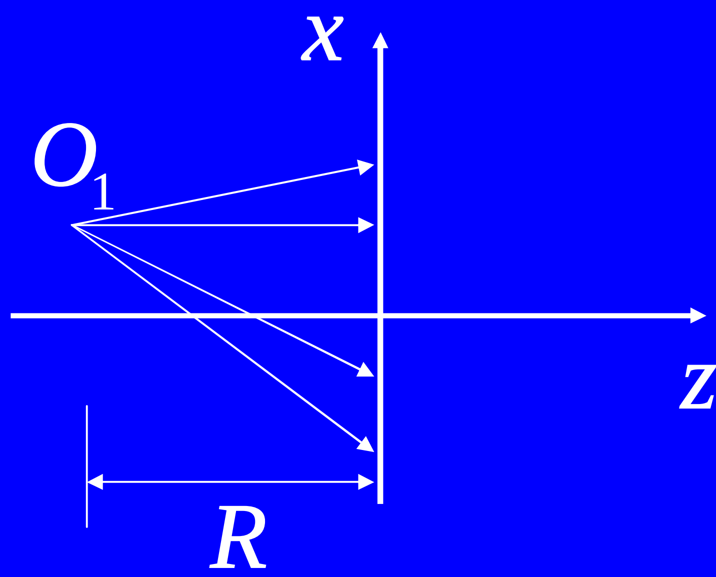
$$= Ae^{ikx\sin(-q)}$$



例3： 分别写出与z=0平面距离为R的两个物点在此平面上产生的复振幅分布。



$O(0,0,-R)$



$O_1(x_0, y_0, -R)$

解：

$$\tilde{U}(P) = A(P)e^{i\varphi(P)} = \frac{a}{|\vec{r}-\vec{r}_0|}e^{i[\vec{k}\cdot(\vec{r}-\vec{r}_0)]}$$

$$|\vec{r}-\vec{r}_0| = \sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}$$

轴上物点 $O(x_0, y_0, z_0) = O(0,0,-R)$

波前z=0

$$\tilde{U}(P) = \frac{a}{\sqrt{x^2+y^2+(z+R)^2}}e^{ik[\sqrt{x^2+y^2+(z+R)^2}]}$$

解：

$$\tilde{U}(P)=A(P)e^{i\varphi(P)}=\frac{a}{|\vec{r}-\vec{r}_0|}e^{i[\vec{k}\cdot(\vec{r}-\vec{r}_0)]}$$

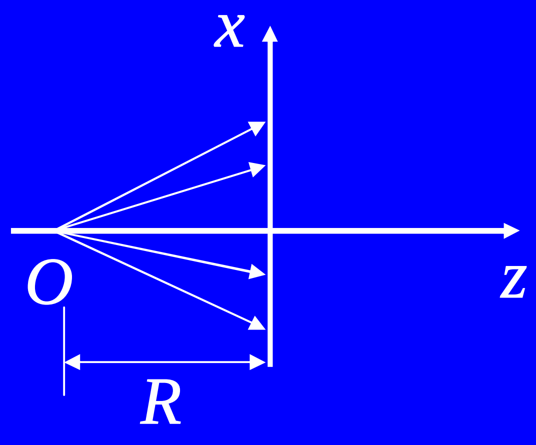
$$|\vec{r}-\vec{r}_0|=\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}$$

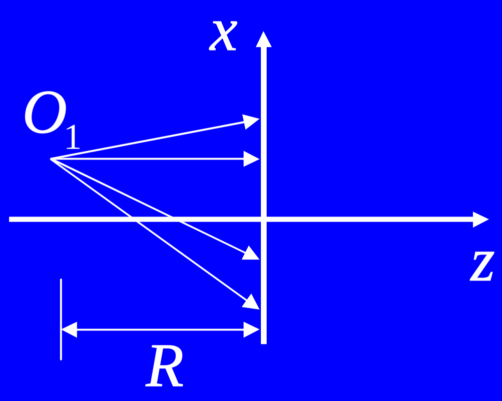
轴外物点 $O_1(x_0,y_0,z_0)=O_1(x_0,y_0,-R)$

波前z=0

$$\tilde{U}(P)=\frac{a}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z+R)^2}}e^{ik[\sqrt{(x-x_0)^2+(y-y_0)^2+(z+R)^2}]}$$

例4：上题中两球面波的共轭波如何？

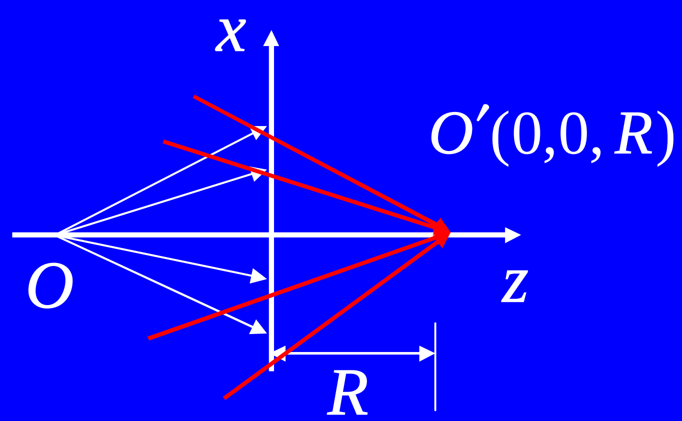


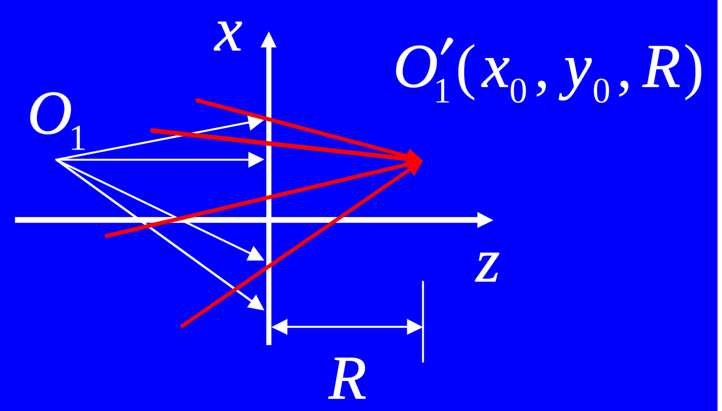


$$\tilde{U}_O(P)=\frac{a}{\sqrt{x^2+y^2+(z+R)^2}}e^{ik\sqrt{x^2+y^2+(z+R)^2}}$$

$$\tilde{U}_{O_1}(P)=\frac{a}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z+R)^2}}e^{ik[\sqrt{(x-x_0)^2+(y-y_0)^2+(z+R)^2}]}$$

解：





$$\tilde{U}_O^*(P)=\frac{a}{\sqrt{x^2+y^2+(z+R)^2}}e^{-ik\sqrt{x^2+y^2+(z+R)^2}}$$

会聚波

$O'(0,0,\pm R)$

$$\tilde{U}_{O_1}^*(P)=\frac{a}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z+R)^2}}e^{-ik[\sqrt{(x-x_0)^2+(y-y_0)^2+(z+R)^2}]}$$

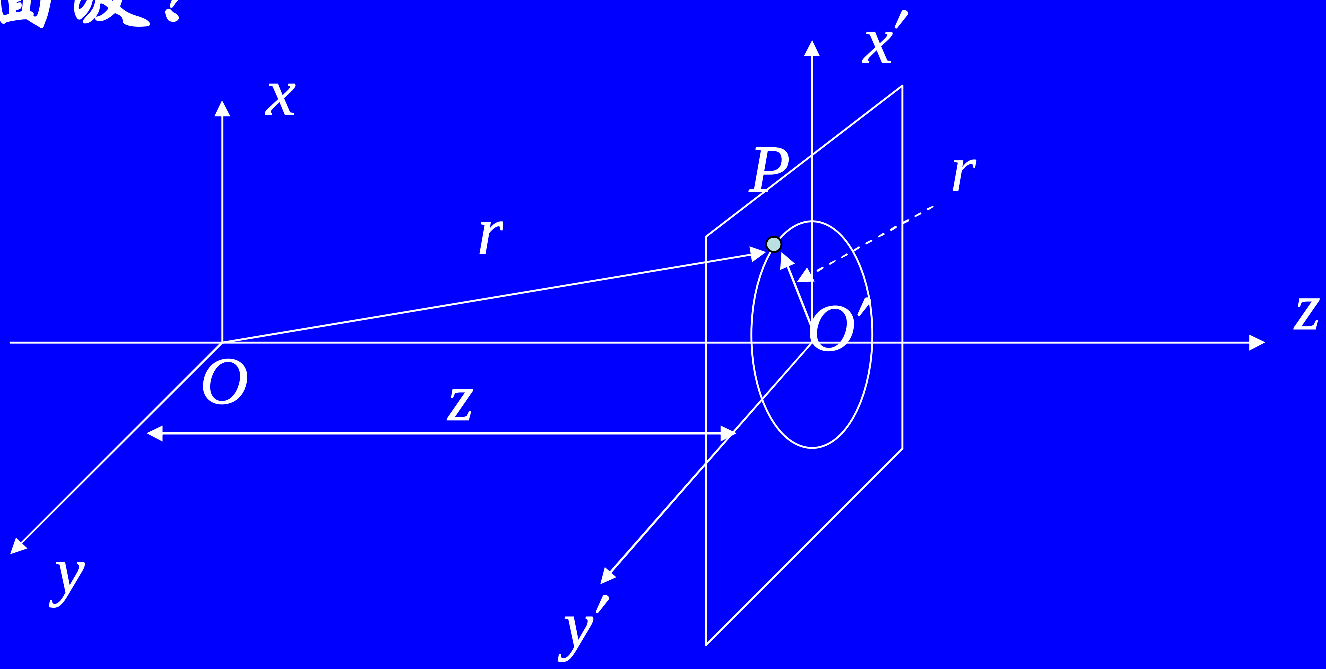
会聚波

$O_1'(x_0, y_0, \pm R)$

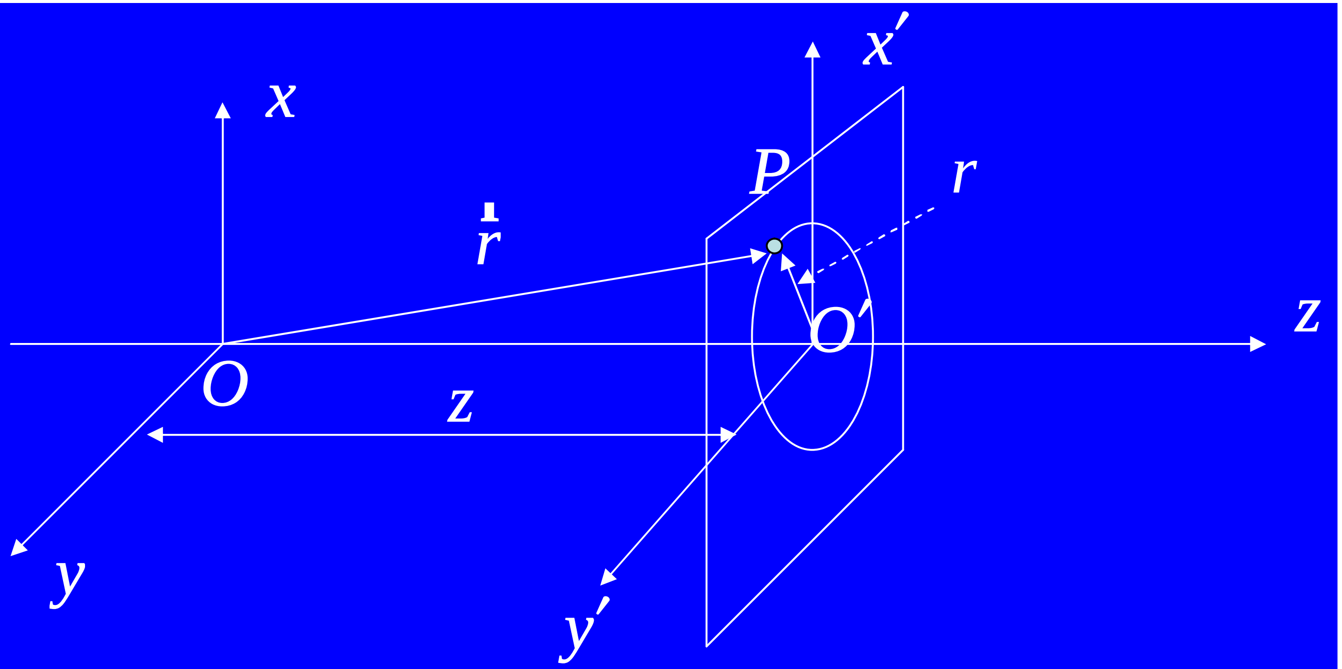
2.2 傍轴条件与远场条件（轴上物点）

1) 问题提出：

物平面x-y上O点发出的球面波，什么条件近似下在接收平面x′-y′上的波前可看作平面波？



2) 分析：



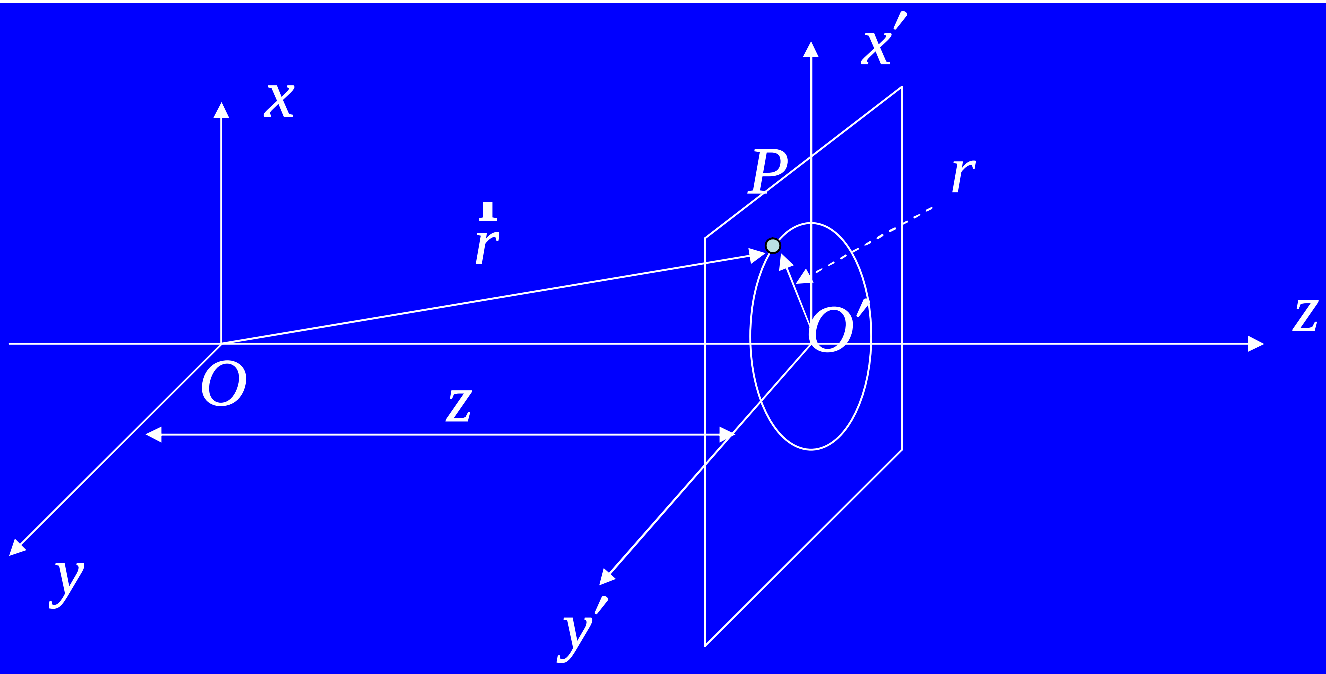
x′-y′平面上的波前函数：

$$\tilde{U}(P)=\frac{a}{|\vec{r}-\vec{r}_0|}e^{i[\vec{k}\cdot(\vec{r}-\vec{r}_0)]}$$

$$r=\sqrt{r^2+z^2}$$

$$r=\sqrt{x'^2+y'^2}$$

$$\tilde{U}(x',y')=\frac{a}{\sqrt{r^2+z^2}}e^{ik\sqrt{r^2+z^2}}=\frac{a}{z\sqrt{1+r^2/z^2}}e^{ikz\sqrt{1+r^2/z^2}}$$

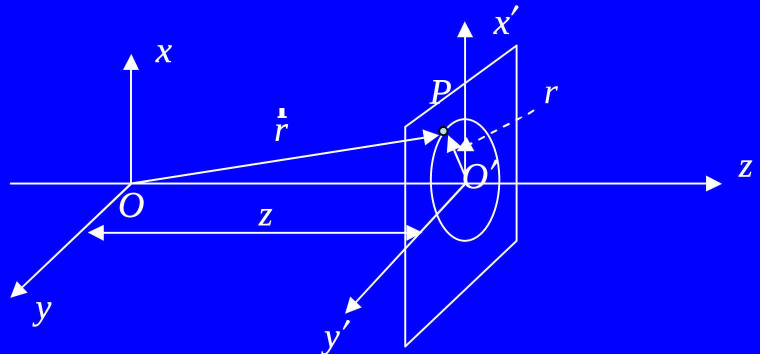


$$\tilde{U}(x',y') = \frac{a}{z\sqrt{1+r^2/z^2}} e^{ikz\sqrt{1+r^2/z^2}}$$

沿z轴传播的x′-y′平面的波前函数：

$$\tilde{U}(x',y') = \frac{a}{z} e^{ikz}$$

$$\tilde{U}(x',y') = \frac{a}{z\sqrt{1+r^2/z^2}} e^{ikz\sqrt{1+r^2/z^2}}$$



$$\tilde{U}(x',y') = \frac{a}{z} e^{ikz}$$

$$\sqrt{1+r^2/z^2} = 1 + \frac{r^2}{2z^2} + \ldots$$

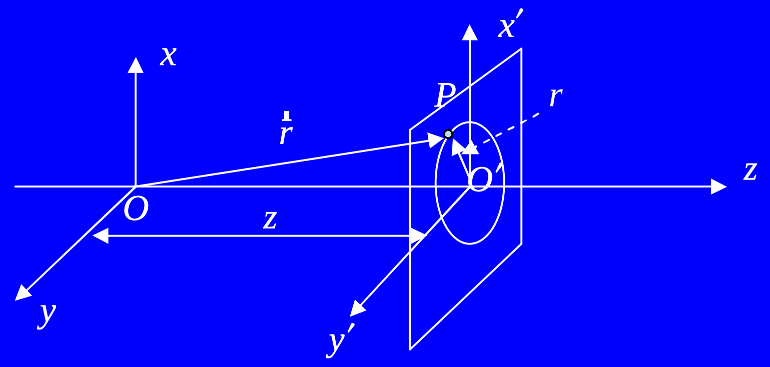
振幅＝常数

傍轴条件： $\frac{r^2}{z^2} << 1$ or $z^2 >> r^2$

位相＝Z的线性函数

远场条件： $kz\frac{r^2}{2z^2} << p$ or $z >> \frac{r^2}{l}$

$$\tilde{U}(x',y')=\frac{a}{z\sqrt{1+r^2/z^2}}e^{ikz\sqrt{1+r^2/z^2}}$$



傍轴条件下：

$$\tilde{U}(x',y')=\frac{a}{z}e^{ik(z+\frac{x'^2+y'^2}{2z})}$$

傍轴和远场条件下：

$$\tilde{U}(x',y')=\frac{a}{z}e^{ikz}$$

光学中往往是远场条件蕴涵傍轴条件.

例5： 单色点光源发射的光波波长为l~0.5mm，横向观测的线度为r~1mm，估算傍轴距离和远场距离。

解： 傍轴和远场条件下：

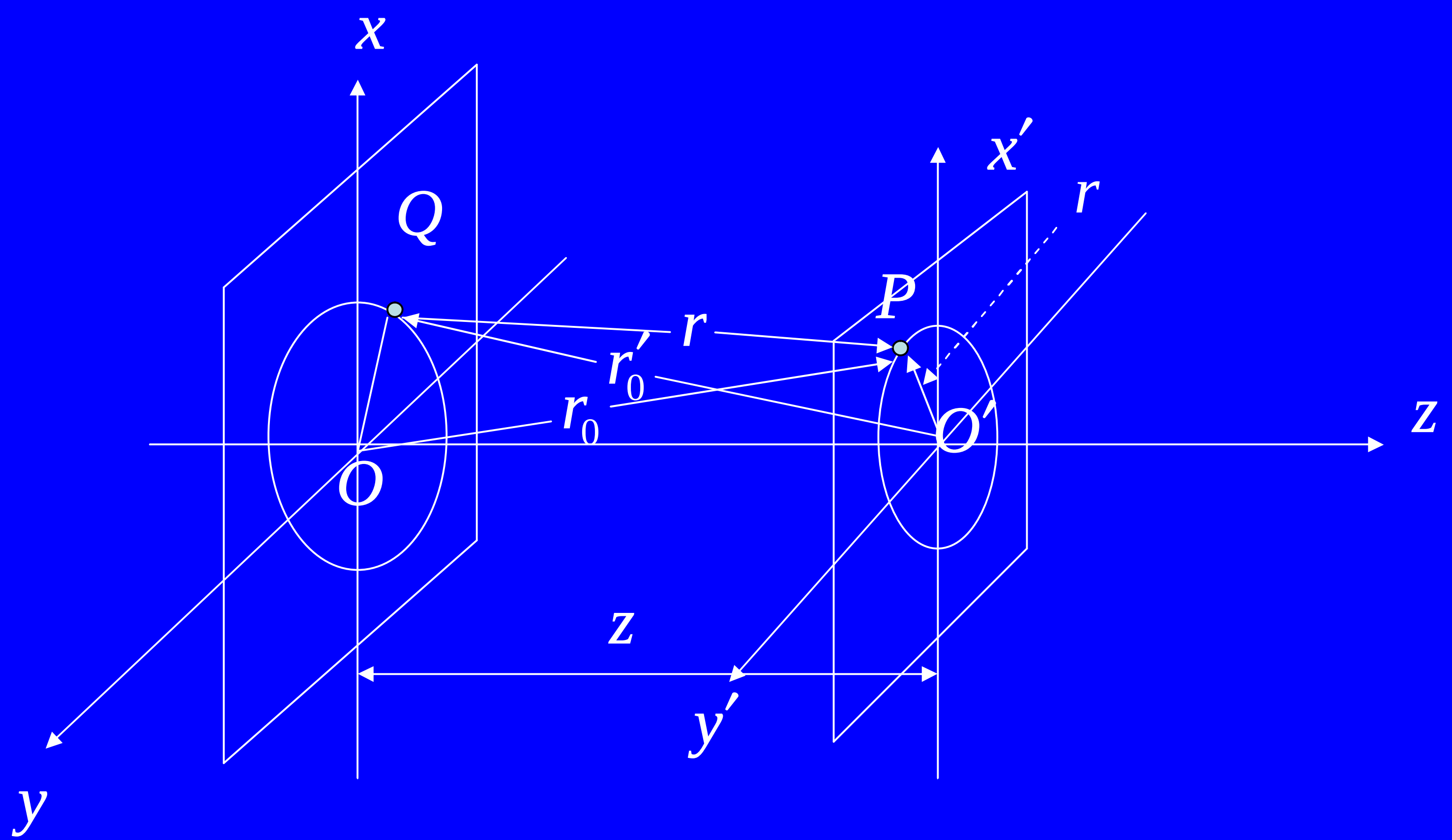
$$z^2 >> r^2 \qquad z >> \frac{r^2}{l}$$

取10~100倍估算，取50倍，有

$$z_1 = \sqrt{50}\, r \approx 7mm$$ 傍轴条件

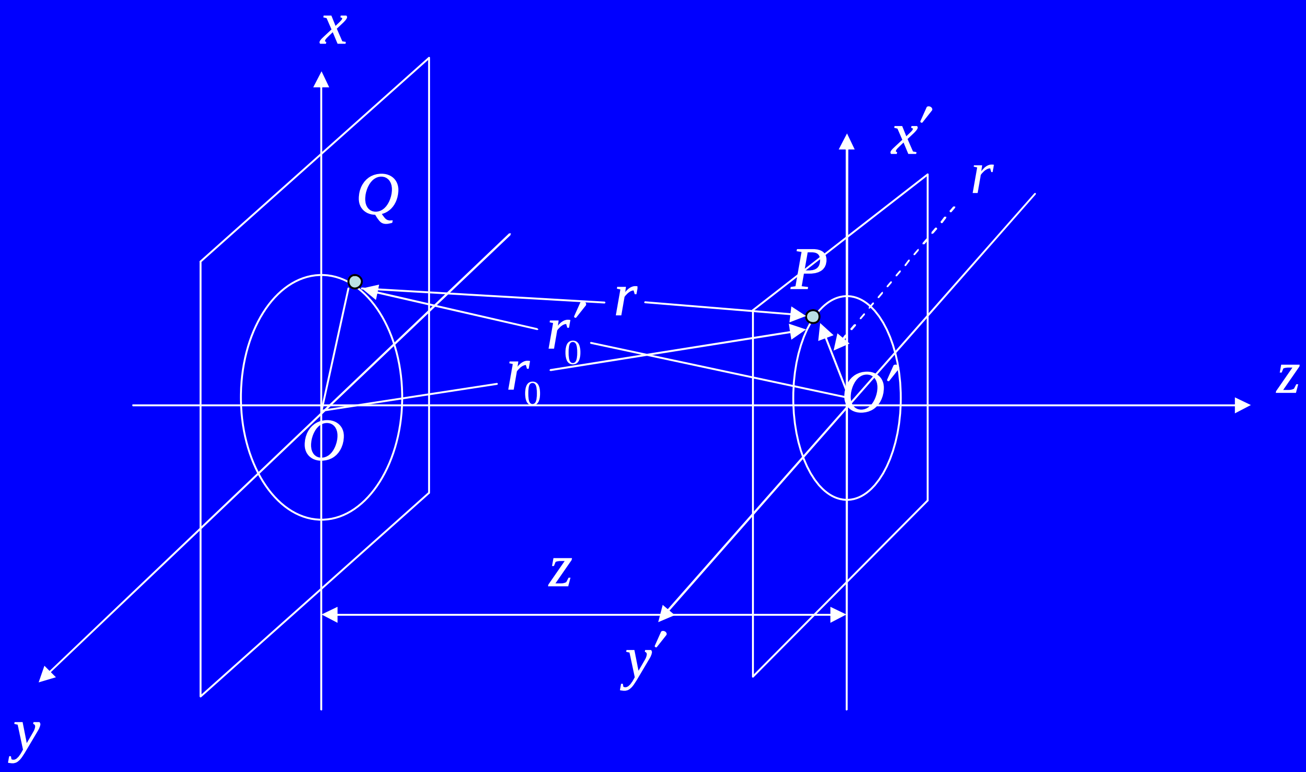
$$z_2 = 50\frac{r^2}{l} \approx 100m$$ 远场条件

2.3 傍轴条件与远场条件（轴外物点）



物点 $Q(x, y)$ 场点 $P(x', y')$

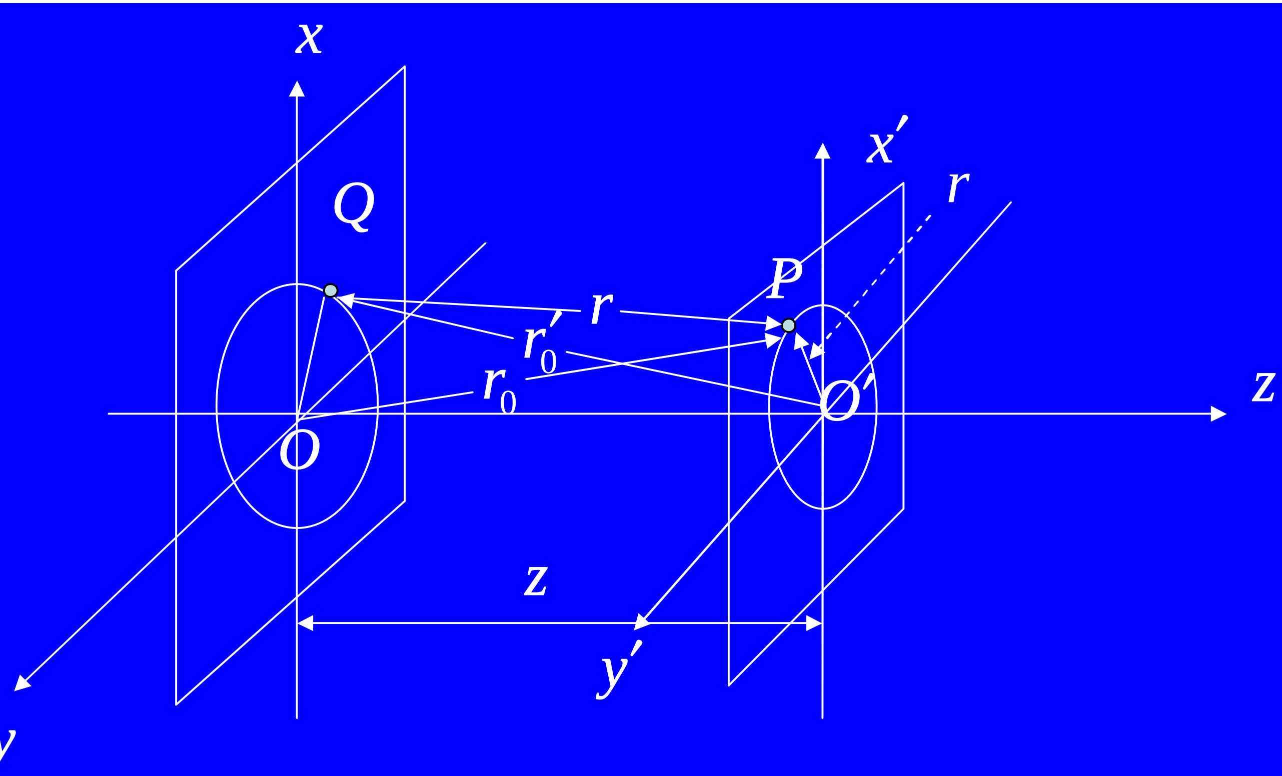
(x’-y’)平面上的波前函数 $\tilde{U}(x', y') = \frac{a}{r} e^{ikr}$



$$r=\sqrt{(x-x')^2+(y-y')^2+(z)^2}$$

$$=z\sqrt{1+\frac{(x-x')^2+(y-y')^2}{z^2}}$$

$$=z[1+\frac{(x-x')^2+(y-y')^2}{2z^2}+\ldots]$$



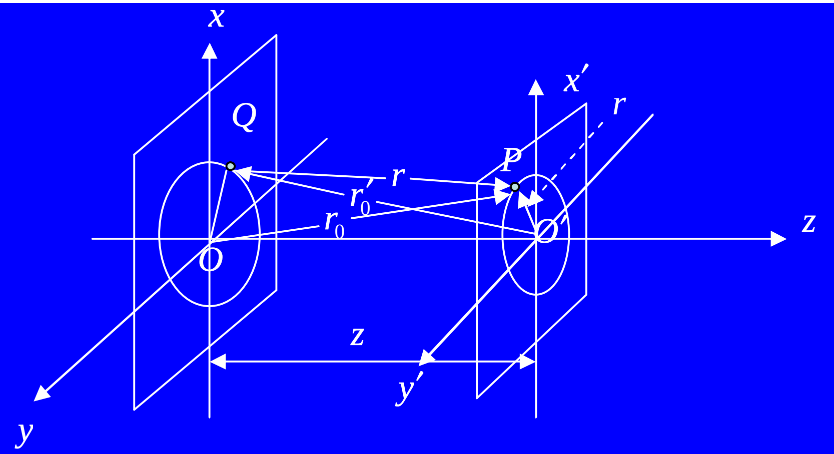
$$r = z[1 + \frac{(x-x')^2 + (y-y')^2}{2z^2} + \ldots]$$

$$\approx z + \frac{x^2 + y^2}{2z} + \frac{x'^2 + y'^2}{2z} - \frac{xx' + yy'}{z}$$

$$\tilde{U}(x', y') = \frac{a}{r} e^{ikr}$$

平面波 $\tilde{U}(x', y') = \frac{a}{z} e^{ikz}$

$$\begin{cases} \tilde{U}(x',y') = \dfrac{a}{r} e^{ikr} \\ \tilde{U}(x',y') = \dfrac{a}{z} e^{ikz} \\ r \approx z + \dfrac{x^2+y^2}{2z} + \dfrac{x'^2+y'^2}{2z} - \dfrac{xx'+yy'}{z} \end{cases}$$



(a)傍轴条件： $r = z$ $\begin{cases} z^2 >> x^2, y^2 & \text{物点} \\ z^2 >> x'^2, y'^2 & \text{场点} \end{cases}$

(b)物点远场条件： $z >> \dfrac{x^2}{l}, \dfrac{y^2}{l}$

(c)场点远场条件： $z >> \dfrac{x'^2}{l}, \dfrac{y'^2}{l}$

在物点和场点满足傍轴，场点满足远场条件下：

$$z^2 >> x^2, y^2 \qquad z^2 >> x'^2, y'^2 \qquad z >> \frac{x'^2}{l}, \frac{y'^2}{l}$$

$$\tilde{U}(x', y') = \frac{a}{z} e^{ik\left(z + \frac{x^2+y^2}{2z} + \frac{x'^2+y'^2}{2z} - \frac{xx'+yy'}{z}\right)}$$

$$= \frac{a}{z} e^{ik\left(z + \frac{x^2+y^2}{2z}\right)} e^{ik\left(\frac{x'^2+y'^2}{2z}\right)} e^{-ik\left(\frac{xx'+yy'}{z}\right)}$$

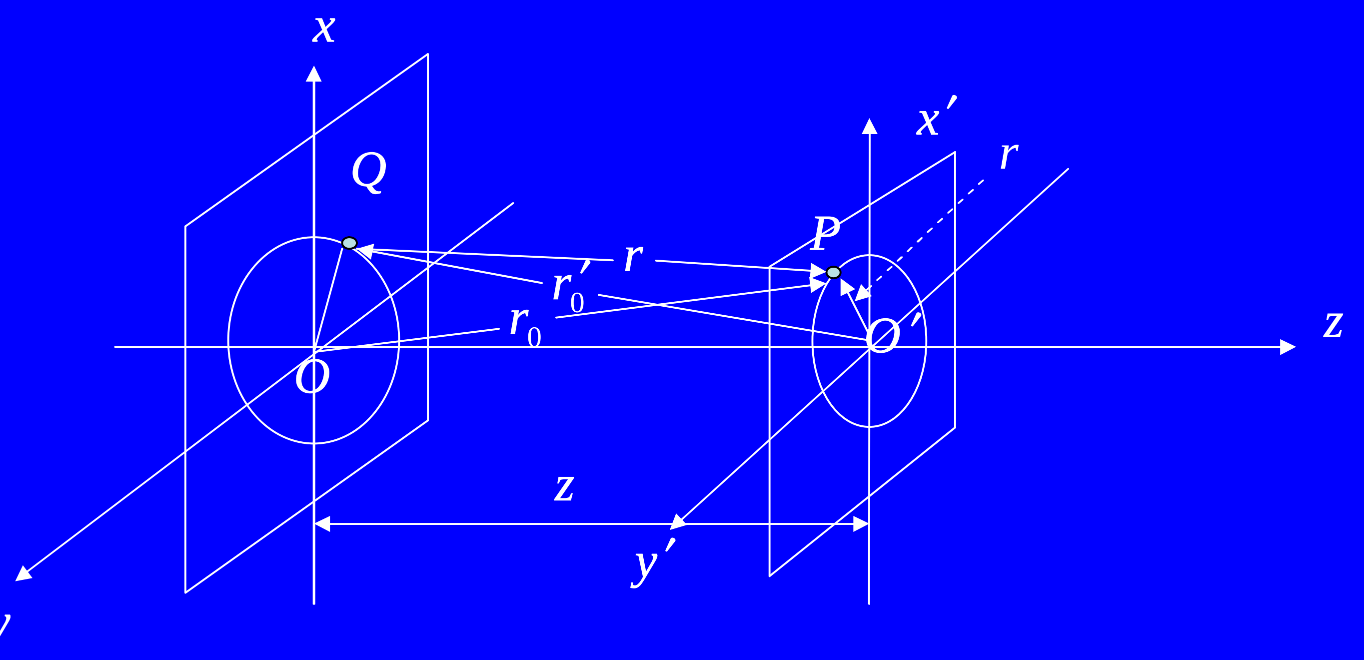
$$e^{ik\left(\frac{x'^2+y'^2}{2z}\right)} \to 0$$

$$\approx \frac{a}{z} e^{ikr_0} e^{-ik\left(\frac{xx'+yy'}{z}\right)} = \frac{a}{z} e^{ikr_0} e^{i\left[kx'\left(-\frac{x}{z}\right) + ky'\left(-\frac{y}{z}\right)\right]}$$

$$= \frac{a}{z} e^{ikr_0} e^{i[kx'\cos a' + ky'\cos b']}$$

平面波的方向是：$\tilde{U}(x',y') = \frac{a}{z} e^{ikr_0} e^{i[kx'\cos a' + ky'\cos b']}$

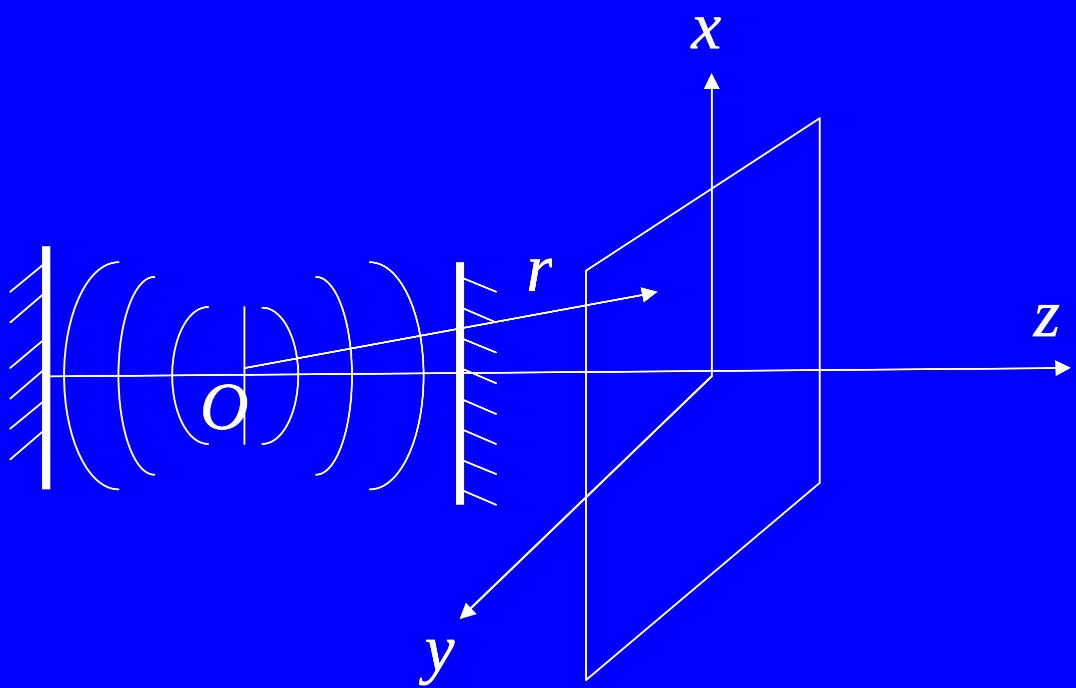
$$\cos a' = -\frac{x}{z} \qquad \cos b' = -\frac{y}{z}$$ ---即QO'连线的方向



$$\cos g' = \frac{z}{\sqrt{x^2 + y^2 + z^2}} \approx 1 - \frac{x^2 + y^2}{2z^2} \approx 1$$

2.4 高斯光束

激光谐振腔发出的光束

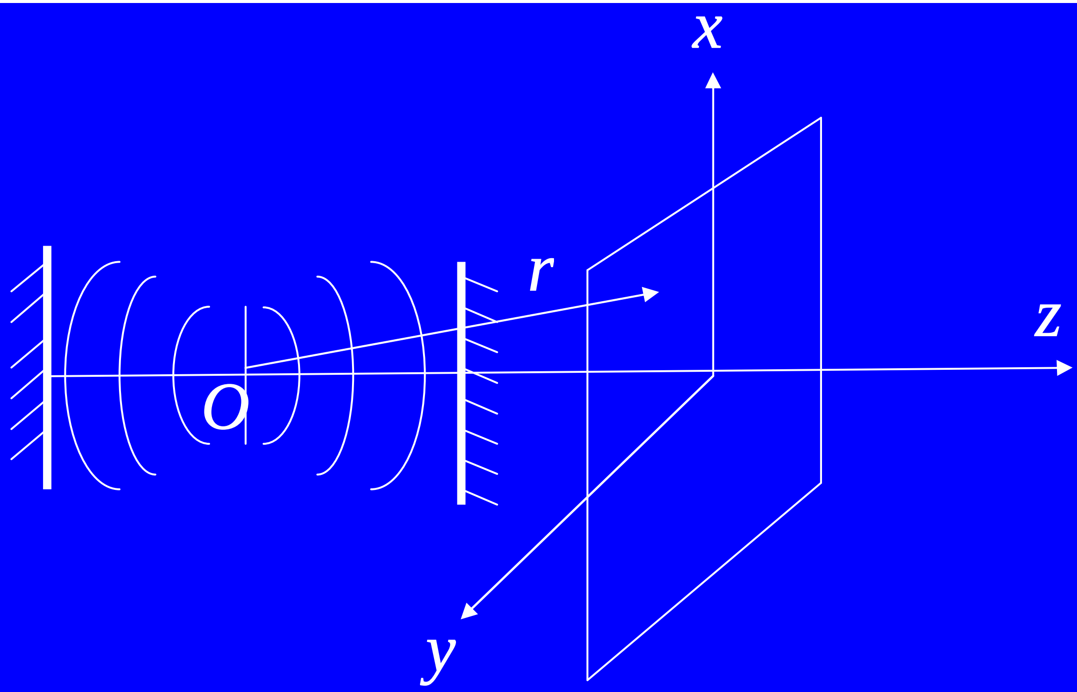


复振幅

$$\tilde{U}(x,y,z)=\frac{A}{w(z)}e^{-\frac{x^2+y^2}{w^2(z)}}e^{-ik(\frac{x^2+y^2}{2r(z)}+z)}$$

$w(z)$ ---光束在Z处的有效半径

$r(z)$ ---轴上Z点处等相面的曲率半径



$$w(z)=w_0(1+\frac{l^2z^2}{pw_0^4})^{1/2}$$

---光束有效半径

$$r(z)=z(1+\frac{p^2w_0^4}{l^2z^2})$$

---等相面的曲率半径

作业：

习题1，2