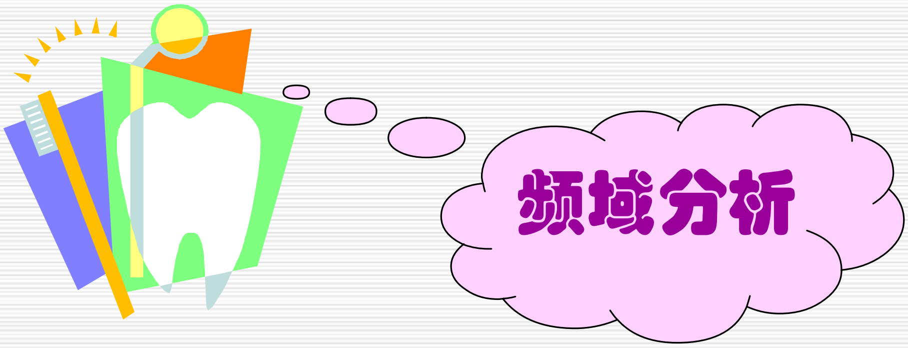
第三章 周期信号的傅里叶级数表示

Fourier Series Representation of Periodic Signals



复习

指数形式的傅里叶级数

$$\mathbf{x}(t) = a_0 + 2\sum_{n=1}^{\infty}\left(an\cos n\omega_1 t + bn\sin n\omega_1 t\right)$$

$$\mathbf{x}(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

综合公式

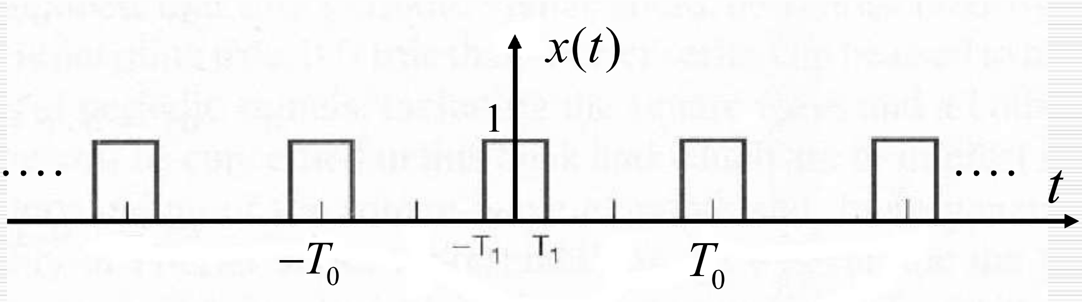
$$a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt$$

分析公式

2. 周期信号的傅里叶级数计算

例题1：求周期方波的傅里叶级数系数

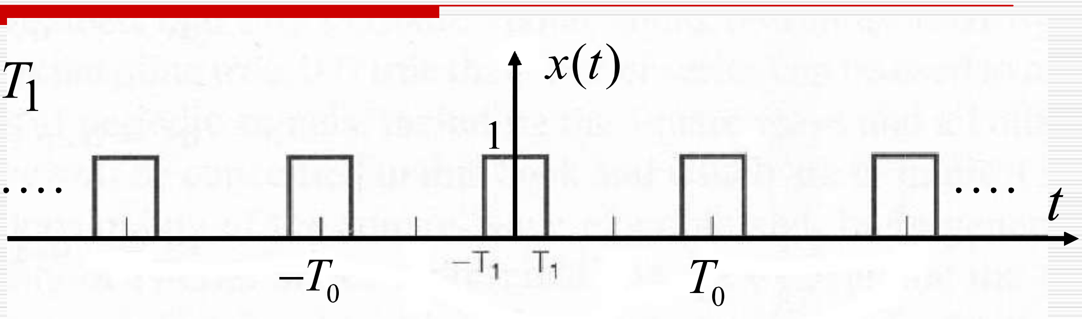
$$x(t)=\begin{cases}1, & |t|<T_1 \\ 0, & \mathbf{T_1}<|t|<\dfrac{T_0}{2}\end{cases}$$



$$a_k=\frac{\sin(k\omega_0 T_1)}{k\pi}=\frac{2T_1}{T_0}Sa(k\omega_0 T_1)$$

2. 周期信号的傅里叶级数计算

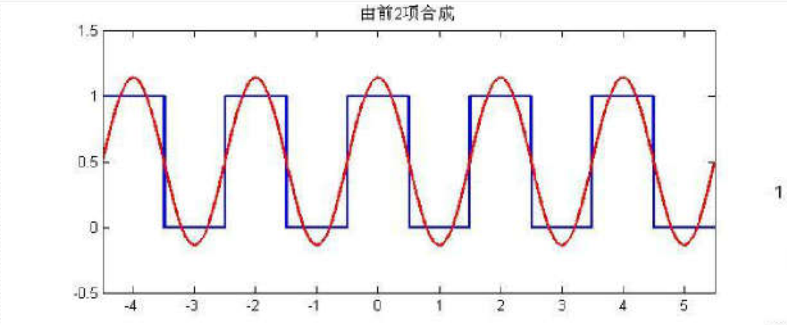
讨论1：当 $T_0 = 4T_1$

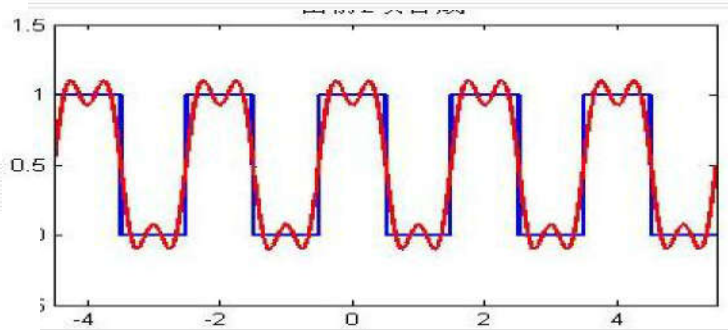


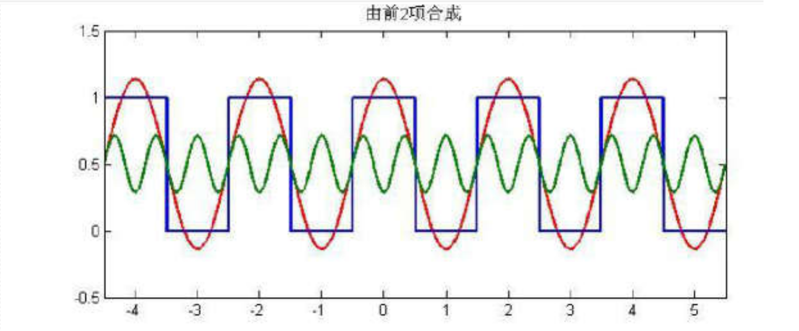
$$x(t) = \frac{1}{2} + \frac{1}{\pi}\cos(\pi t) + \frac{1}{3\pi}\cos(3\pi t) + \frac{1}{5\pi}\cos(5\pi t) + \frac{1}{7\pi}\cos(7\pi t) + \ldots$$

2. 周期信号的傅里叶级数计算

$$x(t)=\frac{1}{2}+\frac{1}{\pi}\cos(\pi t)+\frac{1}{3\pi}\cos(3\pi t)+\frac{1}{5\pi}\cos(5\pi t)+\frac{1}{7\pi}\cos(7\pi t)+\ldots$$

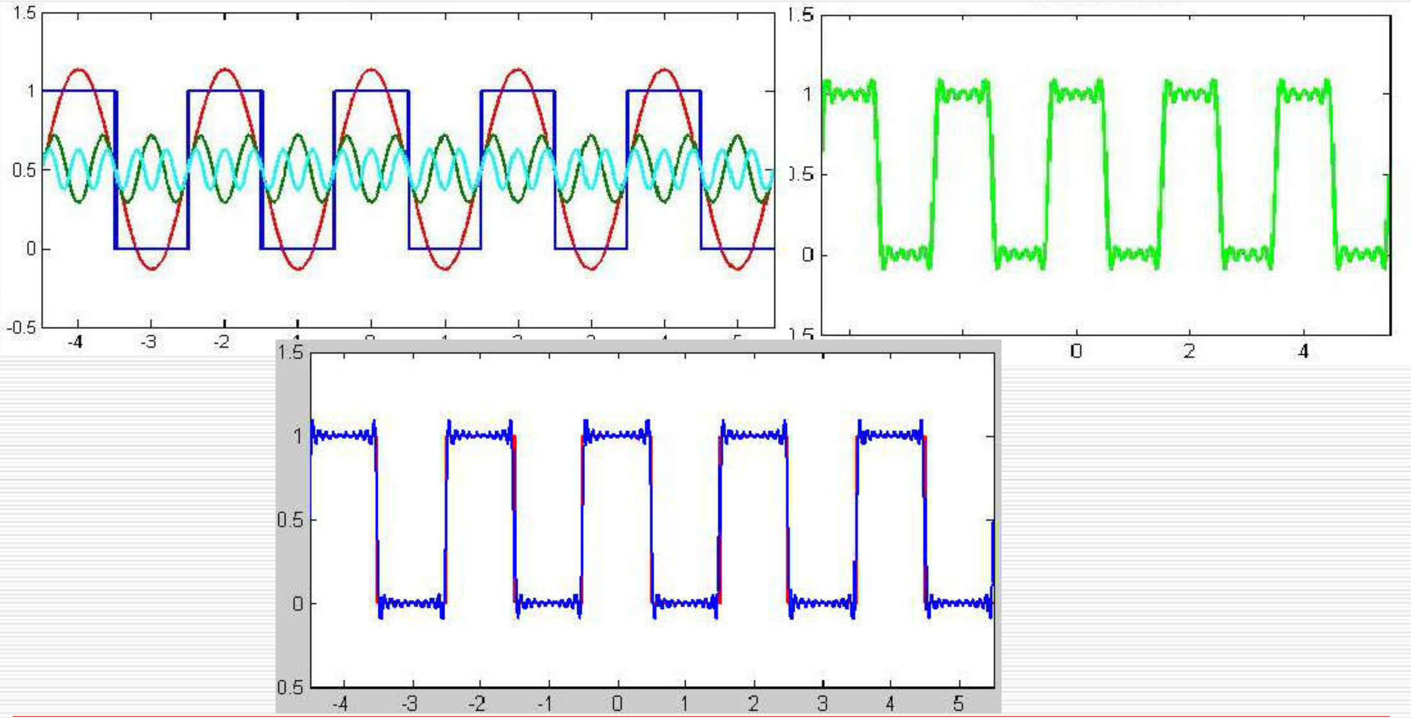






2. 周期信号的傅里叶级数计算

$$x(t)=\frac{1}{2}+\frac{1}{\pi}\cos(\pi t)+\frac{1}{3\pi}\cos(3\pi t)+\frac{1}{5\pi}\cos(5\pi t)+\frac{1}{7\pi}\cos(7\pi t)+\ldots$$

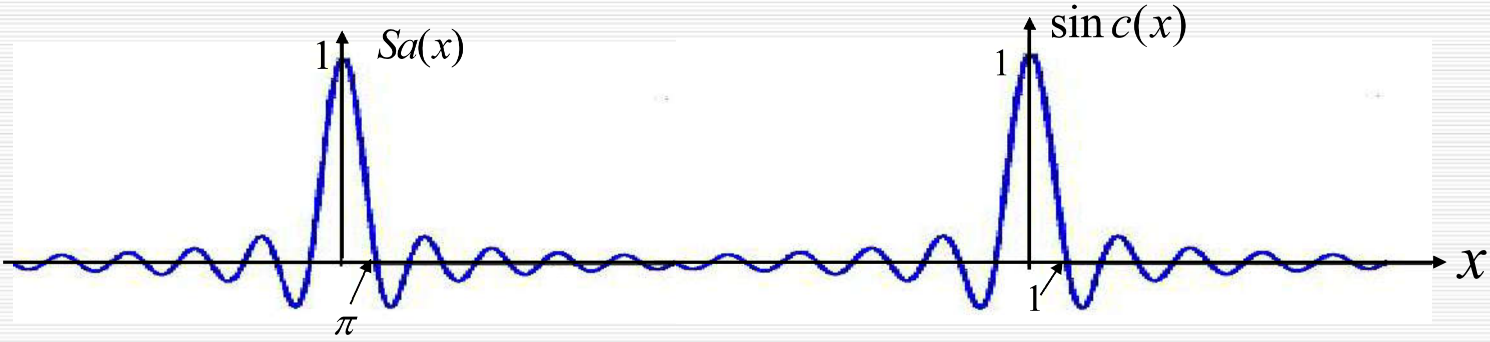


3. 周期信号的频谱

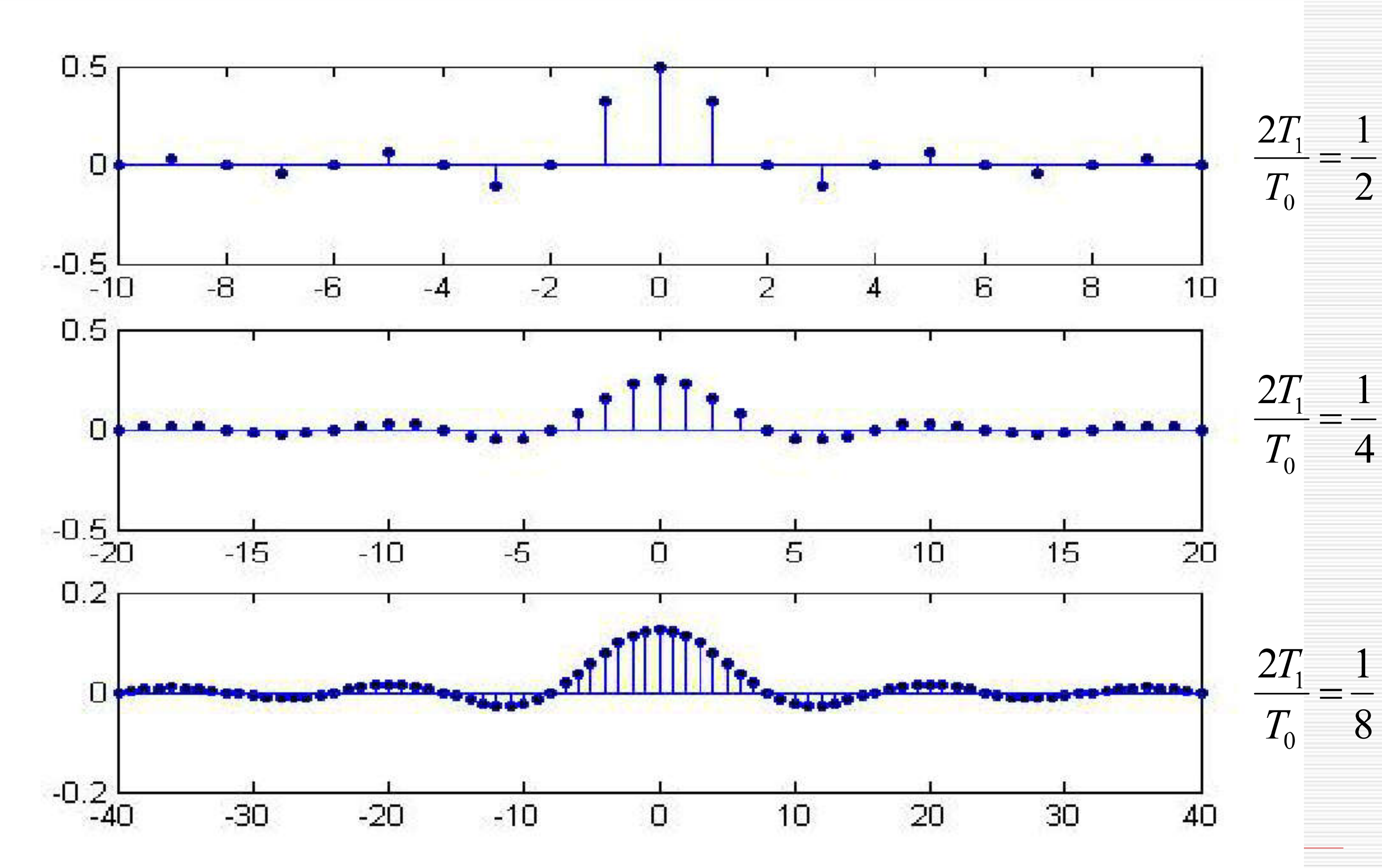
讨论2：方波的频谱

$$a_k = \frac{\sin(k\omega_0 T_1)}{k\pi} = \frac{2T_1}{T_0} Sa(k\omega_0 T_1)$$

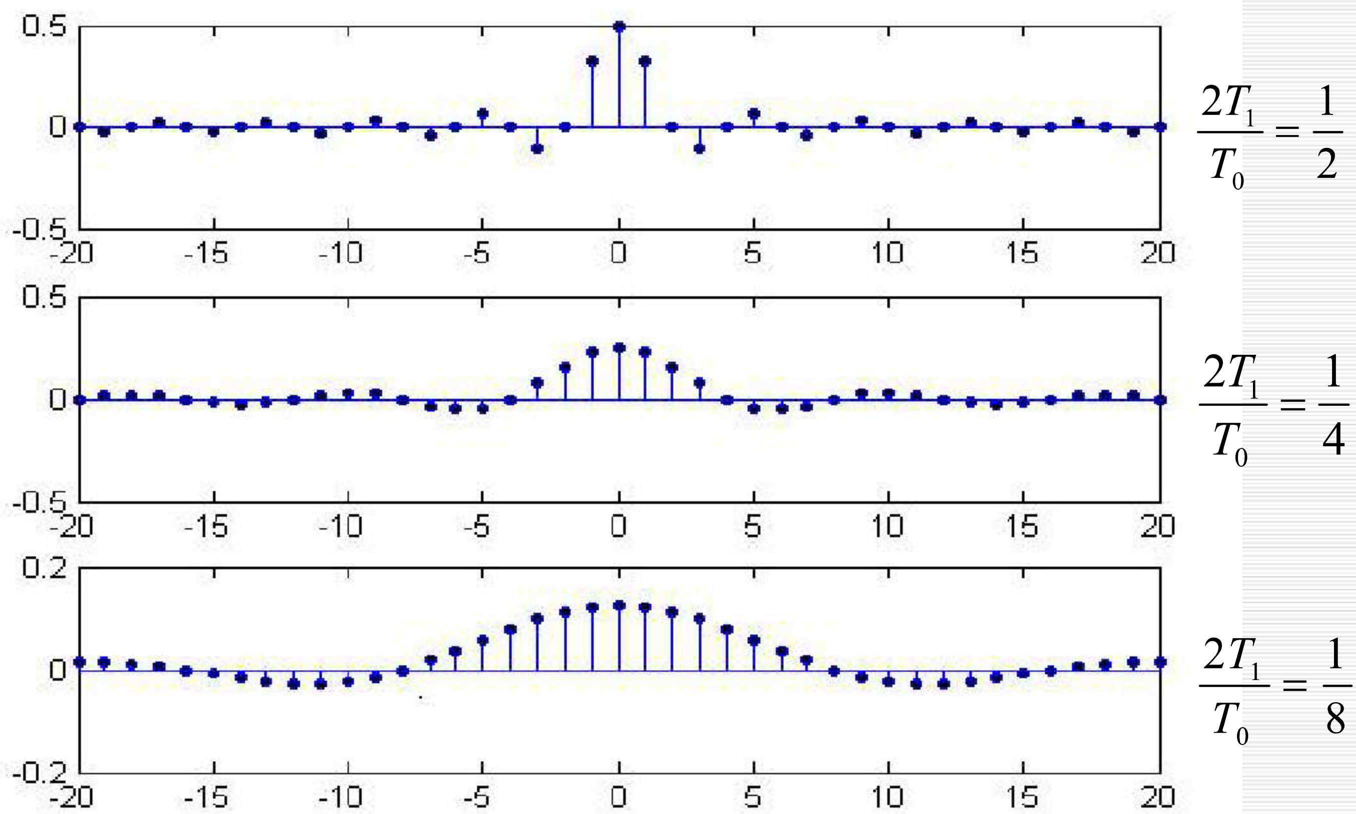
其中 $\mathrm{Sa}(x) = \dfrac{\sin x}{x}$ $\quad \mathrm{sinc}(x) = \dfrac{\sin \pi x}{\pi x}$



T_1 不变 $T_0\uparrow$ 时



T_0不变 $T_1\downarrow$时



3. 周期信号的频谱

周期信号频谱特点

- 离散性
- 谐波性
- 收敛性

例题2：求周期冲激串的频谱 $x(t)=\sum_{k=-\infty}^{\infty}\delta(t-kT_0)$