Evolution of electron phase-space holes in 3D

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Abstract. Electron phase-space holes are regions of depleted electron density commonly generated during the nonlinear stage of the two-stream instability. Recently, bipolar electric field structures — a signature of electron holes — have been identified in the acceleration region of the auroral ionosphere. This paper compares the evolution of electron holes in 2-D and 3-D using massively-parallel PIC simulations. In 2-D, the holes decay after hundreds of plasma periods while emitting electrostatic whistler waves. In the 3-D simulations, electron holes also go unstable and generate whistlers but, due to physical processes not present in 2-D, energy flows out of the whistlers and into highly perpendicular lower hybrid modes. As a result of this difference, 3-D holes do not decay as far as 2-D holes. The differences between 2-D and 3-D evolution may have important implications for hole longevity and wave generation in the auroral ionosphere.

1. Introduction and Background

Electric field probes aboard the FAST and POLAR satellites frequently measure solitary, bipolar electric field pulses with amplitudes sometimes exceeding 1V/m in the auroral ionosphere, making these among the most energetic wave phenomena in space physics [*Carlson et al.*, 1998; *Ergun et al.*, 1998; *Franz et al.*, 1998]. Recent research has established that these pulses result from electron phase-space holes similar to those generated in the nonlinear stage of the two-stream plasma instability [*Matsumoto et al.*, 1994; *Muschietti et al.*, 1999]. Simulations have contributed to our understanding of the complex, nonlinear dynamics associated with electron phase-space holes in 1-D and 2-D [*Omura et al.*, 1996; *Miyake et al.*, 1998; *Goldman et al.*, 1999; *Oppenheim et al.*, 1999a]. In this paper we extend this understanding through the use of the first large-scale, long duration, and 3-D simulations [*Oppenheim et al.*, 1999b]. These simulations reveal a new physical process which does not appear in 1-D or 2-D.

This paper is organized as follows. First, we briefly review the history and literature of bipolar electric field pulses and electron phase-space holes. Second, we discuss the techniques in our 3-D simulations of electron holes and the results, focusing particularly on the new features not seen in earlier simulations. Finally, we analyze these new results and summarize.

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In 1994, *Matsumoto, et al.* [1994] first identified holes as the origin of bipolar electric fields measured by the GEOTAIL satellite in the Earth's magnetotail. In 1998, FAST satellite instruments measured bipolar electric field pulses in the downward current regions of the auroral ionosphere at $\sim$ 2000 km altitude [*Ergun et al.*, 1998]. The FAST observation was quickly followed by reports that the POLAR antennae had measured similar waveforms at altitudes between 2 and 8.5 Earth radii and by WIND in the Earth's foreshock region [*Franz et al.*, 1998; *Bounds et al.*, 1999; *Bale et al.*, 1998].

The theoretical study of electron holes extends back to the earliest days of kinetic simulations of plasmas [*Morse and Nielson*, 1969]. In 1-D, coincident counter-streaming electrons beams generate the two-stream instability which eventually evolves into long-lived structures of depleted electron density. These structures appear in phase-space as rotating vortices of trapped particles around "holes." The nonlinear evolution of these holes has been the subject of substantial theoretical and computational investigations, principally in 1-D [*Dupree*, 1982; *Turikov*, 1984; *Lynov et al.*, 1985]. The phase space structure of these holes has been modeled using stationary BGK modes of the Vlasov equation [*Bernstein et al.*, 1957; *Krasovsky et al.*, 1997]. In higher dimensions, electron phase-space hole physics has received recent attention [*Miyake et al.*, 1998; *Goldman et al.*, 1999; *Oppenheim et al.*, 1999a] in the interpretation of spacecraft data.

The earliest multidimensional simulations of the two-stream instability showed that phase-space holes, which remain stable in 1-D, quickly dissipate in an unmagnetized 2-D or 3-D plasma [*Morse and Nielson*, 1969]. However, a magnetic field enables holes to persist in higher dimensions for hundreds of plasma periods[*Miyake et al.*, 1998]. *Singh, et al.*[2000] shows results from 3-D simulations of magnetized electron holes and observes whistler wave development similar to that described by *Oppenheim, et al.* [1999] but does not make any comparisons with 2-D simulations nor do they resolve the evolution of the modes from whistler waves to lower hybrid waves as shown in this letter.

2. Simulation Methods and Results

Our kinetic simulations use a massively-parallel, electrostatic, particle-in-cell (PIC) algorithm capable of modeling either a finite or an infinite magnetic field in 1, 2, or 3-D [*Birdsall and Langdon*, 1985]. This code applies periodic boundary conditions to an initial-value problem and uses "quiet-start" algorithms to minimize particle noise. Running the code on super-computers enables us to employ over

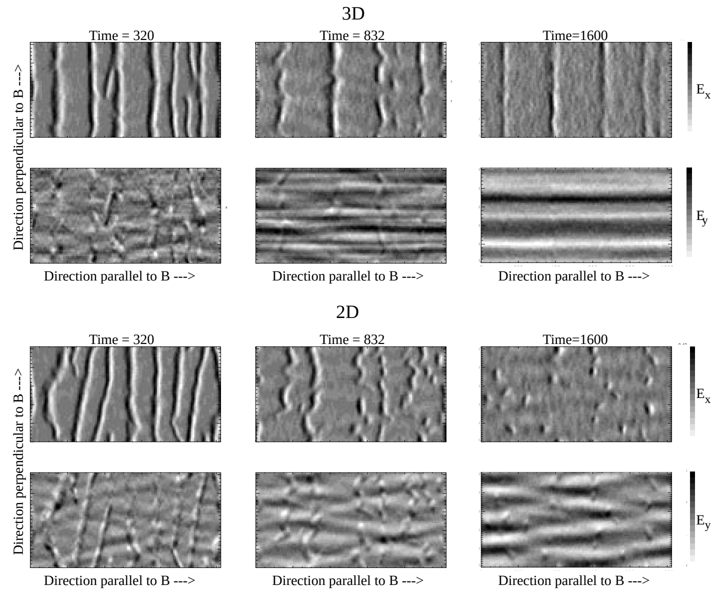

Figure 1. Electric field amplitudes, E_x and E_y for nearly identical 2-D and 3-D baseline simulations at 3 times. E_x emphasizes structures with fields pointing parallel to $\mathbf{B}_0$, principally the kinked electron holes, and E_y emphasizes structures with fields pointing perpendicular to $\mathbf{B}_0$, principally the electrostatic whistler waves. Darkly shaded regions show positive E_x and E_y while light shades show negative Ex and E_y. Both simulations span roughly 400 Debye lengths parallel to $\mathbf{B}_0$ and 250 perpendicular. The 3-D images show-cross section of the full 3-D data set.

4×10^8 particles on meshes resolving up to $512 \times 64 \times 64$ cells. Our PIC simulation results have been validated, in the case of an infinite magnetic field, through comparisons with a simulator that solves the Vlasov equation numerically and, in the case of finite magnetic fields, through comparison with linear theory.

We initiate both 2-D and 3-D simulations with counter-streaming beams of equal density and temperature where each beam begins with a velocity ± 2.5 times its initial thermal velocity. A magnetic field, $\mathbf{B}_0$, parallels the $\hat{x}$ axis with an amplitude such that the ratio of the electron cyclotron, Ω_e, to the electron plasma frequency, ω_e, is 2.5. The time step of $\Delta t = 0.25\omega_e^{-1}$ just resolves Ω_e, where ω_e results from the combined density of the beams. The grid resolves $256 \times 128 \times 64$ cells and spans $1024 \times 256 \times 128$ initial Debye lengths, λ_{D0}. Since the two stream instability leads to substantial heating parallel to $\mathbf{B}_0$, the parallel cells span only $1.5\lambda_D$ after a few tens of plasma periods. The simulated electron holes range in size from $x =\sim 12\lambda_D$ to $x =\sim 24\lambda_D$ which means the simulator resolves them well. Further, halving the grid spacing and timestep so that a cell spans $0.75\lambda_D$ does not change hole size or evolution substantially.

Ion populations measured in the downward current region have a range of masses, temperatures and mean velocities. To illustrate the simplest physical processes, we initially describe a baseline simulation containing a uniform immobile (infinitely massive) ion population. We then compare this simulation with runs containing populations of ions with different temperatures, mean velocities, and masses to illustrate the effects ions have on the evolution of electron holes.

In the earliest stages, 2-D and 3-D simulations follow the same evolutionary path described in *Goldman et al.* [1999] while in later stages striking differences appear. Initially, waves from the two-stream instability evolve into electron phase-space holes extending $\sim 20\lambda_D$ parallel to $\mathbf{B}_0$ and $\sim 200\lambda_D$ perpendicular to $\mathbf{B}_0$. Each hole has a distinct electrical field amplitude, size, and velocity similar to those measured in the auroral ionosphere. When two holes collide, they usually merge into a single, larger amplitude, hole. Hence, after $\sim 300\omega_e^{-1}$, relatively few holes persist, all traveling at almost the same velocity, as seen in Fig. 1 at $t = 320\omega_e^{-1}$. Additionally, these holes have electron distributions where the parallel temperature substantially exceeds the perpendicular temperature as measured by FAST instruments.

Electron holes in 2-D and 3-D are subject to the slow instability described in *Oppenheim, et al.* [1999]. Over hundreds of plasma periods, the holes develop kinks and generate electrostatic whistler waves (magnetized Langmuir waves propagating obliquely to the geomagnetic field, $\mathbf{B}_0$). This phenomenon appears in both the 2-D and 3-D simulations as illustrated by Fig. 1 at $t = 320\omega_e^{-1}$ and $t = 832\omega_e^{-1}$.

The power spectra shown in Fig. 2 identify the modes containing the most electrostatic field energy. The modes having substantial $k_\parallel$ but small $|\mathbf{k}_\perp|$ result from the energy stored in the electron holes. The modes lying close to perpendicular ($k_\parallel = 0$) result from the electrostatic whistlers. Both 2-D and 3-D spectra show these oblique modes at both $t = 320\omega_e^{-1}$ and $t = 832\omega_e^{-1}$.

By $t = 832\omega_e^{-1}$, the 2-D and 3-D simulations are evolving along distinctly separate paths. In 2-D, progressively more energy shifts from the electron holes to the electrostatic whistler waves until almost all the energy resides in the whistlers. In 3-D, a third mode along $k_\parallel = 0$ develops only after substantial energy accumulates in the oblique whistler modes. It appears that energy from the oblique whistlers waves flows into $k_\parallel = 0$ modes. These modes are also electrostatic whistler waves but with a very small frequency.

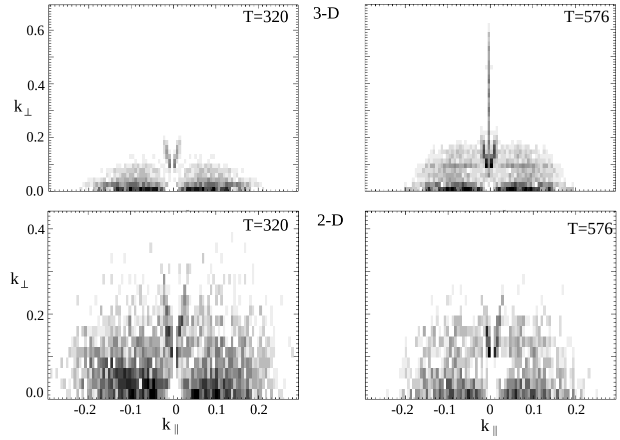

Figure 2. Power spectra showing $E^2(k_x, k_y)$ from 2-D and 3-D baseline simulations at 2 times. The energy amplitude ranges from $|E|^2_{max}$ in each frame, shown in black, to energies less than or equal to $|E|^2_{max}/1000$, shown in white, on a logarithmic scale. The 3-D spectra are averaged for all $k_\perp$.

Ultimately, after $t = 1600\omega_e^{-1}$, these $k_\parallel = 0$ modes end up containing most of the simulation's wave energy. This energy conversion does not occur in 2-D, nor does it occur in 3-D when we suppress perpendicular electron dynamics ($\mathbf{B}_0 \to \infty$). This implies that the crucial difference between 2-D and 3-D results from plasma drifts such as $\mathbf{E} \times \mathbf{B}$ or gradient drifts. Animations of $\mathbf{E}$ and potential show these dynamical processes more clearly.
[1]

Ion dynamics modifies the rate at which energy flows into perpendicular modes and, also, modifies the particular modes energized. Fig. 3 shows power spectra from a simulation similar to the baseline case except it contains H^+ ions starting with $T_i = T_e$ where the mean velocity of the ions matches one of the beams. Further, the direction parallel to $\mathbf{B}_0$ is twice as long as in the baseline case, spanning 512 grid cells, while the perpendicular directions both span 64 grid cells. At $t = 832\omega_e^{-1}$ little energy appears in the perpendicular modes, a clear distinction from the baseline case with immobile ions. However, by $t = 1600\omega_e^{-1}$ much of the energy now appears in shorter wavelength, mostly perpendicular, lower hybrid modes. Also the strictly perpendicular modes contain far less energy than in the baseline case, implying that these modes fail to be energized or that they are damped. Additionally, substantial perpendicular heating of the faster moving ions appears in both 2-D and 3-D simulations (ie., tail heating).

3. Discussion and Conclusions

In both 2-D and 3-D, electron holes go unstable, kink, and generate electrostatic whistler waves but in 3-D the holes do not decay as far as in 2-D. Instead, the kinking halts and the holes appear to straighten into elongated, irregular structures. The striking difference between 2-D and 3-D hole structure appears clearly in the E_x images of Fig 1 at $t = 1600\omega_e^{-1}$. *Franz et al.* [2000] observed that holes appear elongated, perpendicular to $\mathbf{B}$, when $\Omega_e < \omega_e$ and become more spherical with increasing magnetic field. Electron holes generated by simulations in 2-D or 3-D with infinite $\mathbf{B}_0$ become spherical – a fact which agrees with the observations. Simulations with weaker fields in 3-D, such as $\Omega_e/\omega_e = 1.25 - 2.5$, generate holes which remain more elongated than the *Franz et al.* observations predict. This difference may result from one of the following effects. First, the substantial variations of $\mathbf{E}$ along the length of the hole perpendicular to $\mathbf{B}$ might make the hole appear to have a finite extent. Second, the simulation excludes highly oblique modes which may play an essential role in hole evolutions as discussed in the following paragraph.

Why do the reduced amplitude holes seen at the end of the 3-D simulations remain stable? By comparing hole evolution in simulations with three different sizes, we show that simulation size affects hole stability and wave growth. Two simulations, having lengths parallel to $\mathbf{B}$ of $1024\lambda_{D0}$ and $512\lambda_{D0}$ follow evolutionary paths quite similar to the baseline case describe above. However, a simulation spanning only $256\lambda_D$ deviates substantially. This run develops electron holes which, because of the short box length along $\mathbf{B}_0$, merge into a single hole by $t = 450\omega_e^{-1}$. This final hole spans the simulation perpendicular to $\mathbf{B}_0$ and extends over roughly 1/4 of the direction parallel to $\mathbf{B}_0$. During the final merging of the holes, some kinking and oblique whistler waves appear as well as $k_\parallel = 0$ modes, but the final hole maintains its amplitude and extent. A 2-D simulation with the same length shows similar evolution to 3-D except no energy flows into $k_\parallel = 0$ modes. We conclude that the short box length excludes long parallel wavelengths, artificially preventing the holes from giving up a large fraction of their energy to whistler waves and, subsequently, $k_\parallel = 0$ modes.

Linear theory enables us to understand many of the characteristics of the mostly perpendicular modes and the role ions play in these simulations. The cold fluid dispersion relation governing the combined behavior of electrostatic whistler waves and lower hybrid waves in the regime where $\omega \gg \Omega_i$ is

$$0 = 1 + \frac{\omega_i^2}{\omega^2}\left(\frac{\Omega_i^2}{\omega^2}\cos^2\theta - 1\right) + \omega_e^2\left[\frac{\Omega_e^2/\omega^2\cos^2\theta - 1}{\omega^2 - \Omega_e^2}\right] \quad (1)$$

where θ is the angle between the magnetic field and the wave vector; ω, ω_i, and Ω_i are the wave, ion plasma, and ion cyclotron frequencies respectively. In the regime where $\Omega_e^2 \gg \omega_e^2$ and $\Omega_e^2 \gg \omega_e^2$ then eqn. (1) simplifies to $\omega^2 = \omega_e^2\cos^2\theta + O(m_e/m_i)$. Ions only become important in eqn. (1) when $w^2 \sim \omega_i^2$ which occurs when $\theta \sim \pi/2 - \omega_i/\omega_e$. For our simulation parameters, $\theta = \pi/2 - \sqrt{m_e/m_i} = 88.7°$

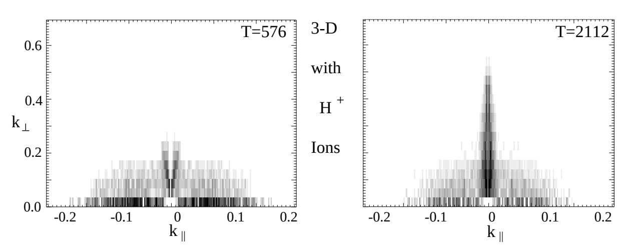

Figure 3. Power spectra showing $E^2(k_x, k_y)$ from simulations containing H^+ ions comparable to those in Fig. 2.

[1]Available via Web browser or FTP from ftp://kosmos .agu.org (username="anonymous", password="guest"), directory "append"; subdirectories arranged by paper number. Information available at http://www.agu.org/pubs/e-supp_about.html.

making the ions inconsequential for all but nearly perpendicular, lower hybrid, modes.

The generation of perpendicular waves appears, from the spectra, to result from mode coupling between the whistler waves and the perpendicular waves. It is easy to satisfy the wavevector and frequency matching conditions for three-wave coupling. The mechanism which generates such a wide range of wavevectors perpendicular to $\mathbf{B}_0$ remains unclear. Since the 2-D and infinitely magnetized simulations do not show this mode coupling it appears likely that the mode coupling is mediated by the components of the equations responsible for plasma drifts.

Dynamic H^+ ions change the oscillation frequency of the perpendicular modes from close to zero to the lower hybrid frequency. This difference modifies the frequency and wavenumber matching conditions and may explain the reduced growth rates and amplitudes of the perpendicular modes when comparing the cases with and without ions. Additionally, ion Landau damping may play a role in eliminating strictly perpendicular modes since waves with wavelengths shorter than twice the ion Larmor radius may damp.

Ions can play additional roles in electron hole dynamics. If a large fraction of the ion population travels at electron hole velocities then parallel ion-acoustic modes will couple energy out of the electron holes and cause the eventual destruction of the electron holes [*Saeki and Genma*, 1998]. However, the observed auroral electron holes generally travel much faster than the bulk of the ion population. That fact originally made it clear that bipolar electric fields were an electron phenomenon. Using artificially light ions, as frequently done in plasma simulations, will modify the critical angle at which ions play a role and will change the frequencies of the oblique modes. Hence these ions can substantially modify the resonant interaction between the holes and the whistler waves.

These simulations show that many aspects of electron hole evolution in the auroral ionosphere are governed by fully 3-D dynamics while some processes are well represented in 2-D. Both 2-D and 3-D simulations show electron holes evolving along similar paths initially; developing from streaming instabilities, merging, and generating oblique electrostatic whistler waves while decaying. In 3-D, energy flows from the whistlers into lower hybrid waves, probably as a result of mode coupling which is suppressed in 2-D and in infinitely magnetized systems. This implies that one might expect to measure more energy in highly perpendicular lower hybrid modes coincident with electron holes than in the less perpendicular electrostatic whistler modes. The ratio of energy in these modes may allow us to learn something about the stability, lifetime and origin of the holes. Finally, 3-D simulations predict longer electron hole longevity and greater hole elongation perpendicular to $\mathbf{B}$ than do 2-D simulations.

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