Robust Fuzzy Extractors and Helper Data Manipulation Attacks Revisited: Theory vs Practice

Georg T. Becker

Abstract—Fuzzy extractors have been proposed in 2004 by Dodis *et al.* as a secure way to generate cryptographic keys from noisy sources. Originally, biometrics were the main motivation for fuzzy extractors but in recent years their practical relevance stems mainly from their use in secure key generation based on Physical Unclonable Functions (PUFs). Fuzzy extractors are provably secure against passive attackers, i.e., attackers that can observe the helper data. A year later, robust fuzzy extractors were introduced which are also provably secure against an active attacker, i.e., attackers that can manipulate the helper data. Hence, the problem of how to build provably secure robust fuzzy extractors appears to have been solved a long time ago.

However, in this paper we show that from a practical perspective the problem of building a provably secure fuzzy extractor is actually not solved yet. The originally proposed robust fuzzy extractors based on BCH codes either do not have the required error-correction rates for practical applications or violate the parameters in the security proof. Since no helper data manipulation attacks on linear codes are known which work in the robust fuzzy extractor construction, it might be tempting to simply ignore the parameters of the proof. However, we present new helper data manipulation attacks on several decoding strategies for linear codes which *set* a key as opposed to recovering the key. These new attacks show that helper data manipulation attacks are indeed feasible against such constructions if the parameters in the proof are ignored. Robust fuzzy extractors therefore need to be revisited by both engineers and cryptographers to solve the problem of building both provably secure as well as practical robust fuzzy extractors.

Index Terms—Robust Fuzzy Extractor, Physical Unclonable Functions (PUFs), Helper Data Manipulation Attacks

1 Introduction

Fuzzy extractors have been proposed in 2004 by Dodis *et al.* [1] as a provably secure way to generate cryptographic keys from correlated but possibly noisy sources. The main motivation for this work back in 2004 were biometrics: During a generation phase a biometric reading such as a fingerprint or iris picture is used to generate a cryptographic key and some helper data. At a later time, a second, possibly noisy reading of the same biometric source is used in conjunction with the helper data to reproduce the cryptographic key. Fuzzy extractors guarantee that *i)* the derived key is uniform even after revealing the helper data, i.e., the helper data does not reveal information about the key, and *ii)* as long as the distance between the two readings is smaller than or equal to a distance parameter t, the same key is extracted during the recreation step.

While fuzzy extractors are provably secure against passive attackers, this does not say anything about attackers who can alter the helper data. Therefore Boyen *et al.* proposed a construct called *robust fuzzy extractors* in 2005 [2] which is also secure against helper data manipulation attacks. The original robust fuzzy extractor was only provably secure in the random oracle model but a provably secure construction in the general model was proposed a year later in [3] and slightly improved in [4], [5]. It is noteworthy that in theory any fuzzy extractor can be turned into a robust fuzzy extractor. Hence, at least for the Hamming distance metric, how to build provably secure fuzzy extractors and provably secure robust fuzzy extractors seems to be solved.

From a practical perspective, fuzzy extractors have become increasingly important not due to biometrics but due to Physical Unclonable Functions (PUF) based key generation. Storing and generating cryptographic keys in embedded devices can be a challenging task, in particular if they need to be able to withstand physical attacks. Standard non-volatile memory has little protection against unauthorized read-out, especially if the non-volatile memory is not on the same chip. Only dedicated secure non-volatile memory can offer protection against an advanced attacker. PUFs are a promising alternative to such secure non-volatile memory. The idea of PUFs is to use the process variations within each chip to derive a unique "fingerprint" for each chip, which is called the PUF response. A Fuzzy Extractor is then used to derive a cryptographic key and helper data from this PUF response. In the recovery phase this helper data in conjunction with a (possibly noisy) PUF response is used to recover the key again. PUF-based key generation and in particular how to construct fuzzy extractors for PUF-based key storage has been the focus of a lot of research. One important focus of this research is how to handle the significant level of noise that can be present in the PUF responses. Several error-correction codes and decoding strategies have been proposed to be used in fuzzy extractors for PUF-based key generation, e.g. in [6], [7], [8], [9], [10], [11]. It is noteworthy that fuzzy extractors and PUF-based key generation are not purely academic research topics any more. Several

The author is with the Digital Society Institute at the ESMT Berlin, Germany. Most of the work was done while he was with the Horst Görtz Institute for IT Security, Ruhr-Universität Bochum, Germany. E-mail: {firstname.lastname}@rub.de.

The author would like to thank Johannes Tobisch for providing the GMC decoding implementation and for his helpful comments.

high-security products by companies such as Microsemi, Altera and NXP use fuzzy extractors in conjunction with PUF-based key generation[1].

The first helper data manipulation attacks against PUF based key generation were not on fuzzy extractors but on helper data algorithm based on pattern matching [12] and designs specifically for Ring-Oscillators [13]. Another helper data manipulation attack was presented on soft-decision error-correction [14]. While the attack was descried based on the helper data algorithm by Maes *et al.* [9], which is strictly speaking not a fuzzy extractor as no bound on the min-entropy is provided, the attack can also be applied to even number repetition codes and hence fuzzy extractors.

The fact that in practice for most PUF-based key generation solutions robust fuzzy extractors are needed is acknowledged in many papers. However, papers describing actual implementations only used fuzzy extractors and not robust fuzzy extractors.

1.1 Main contribution

In this paper we will show that how to build provably secure as well as practical robust fuzzy extractors is not a solved problem. In particular, we show that:

- It is actually not possible to build a provably secure robust fuzzy extractor based on the BCH construction proposed by Boyen *et al.* [2] for noise levels above 3%.
- The concatenated code constructions used in practical fuzzy extractor implementations that can handle noise levels of 15% or more are actually not well-formed and therefore using them in Boyen *et al.* 's construction violates the security proof.
- We introduce a new helper data manipulation attack strategy on linear codes which we demonstrate based on several decoding strategies for Reed-Muller codes and soft-decision decoding.
- In this new attack strategy the attacker *sets* a key as opposed to recovering a key. Therefore the new attack strategy can be used to attack a robust fuzzy extractor like construction which uses a hash function to check the integrity of the helper data.

Finally, we also propose some promising research directions to solve this problem in the future.

1.2 Outline

The next Sections provides some necessary background information. In particular, robust fuzzy extractors and their related constructions are formally defined and introduced. In Section 3 the robust fuzzy extractor based on BCH codes is examined in greater detail and it is shown that building a robust fuzzy extractor based on BCH codes is not possible for realistic parameters. Section 4 introduces the attack model and strategy of our new helper data manipulation attack and Section 4 presents some concrete examples how the attack works against Reed-Muller codes and soft-decision decoding. Finally, a discussion outlook is provided in Section 6 with promising future research directions before the paper finishes with a short conclusion.

1. NXP announced that the upcoming SmartMX2 security chips will feature a PUF, Altera uses PUFs in their Stratix 10 FPGAs, Microsemi in their SmartFusion FPGAs. All these products use Intrinsic-ID's licenses and PUF solutions.

2 Background

In the following we will provide the formal definitions of robust fuzzy extractors and their building blocks. The corresponding proofs and a more detailed description can be found in the referenced papers. In the following we will use the definitions from [2].

2.1 Notations

In this paper we will use the following notations: A vector is represented with a bold lowercase character, a matrix with a bold uppercase character, a scalar with a lowercase character, a random variable with an uppercase character, and a set with calligraphic character. Table 1 is a summary of the variable names used in this paper to describe the robust fuzzy extractor for PUF based key generation and the helper data manipulation attack. In this context, a variable that is used in the recovery phase before the error-correction is denoted with a tick mark, a variable after the error-correction is denoted with a bar and a value chosen or predicted by the attacker is indicated with an a index. For binary error-correction codes we use to the notion of $[n, k, d]$, with n being the codeword length, k being the dimension and d denoting the minimum distance between codewords.

TABLE 1: Overview of variable definitions

	Original	Before Error Corr.	After Error Corr.	Attacker's choice
PUF response	$\mathbf{w}$	$\mathbf{w}'$	$\bar{\mathbf{w}}$	$\bar{\mathbf{w}}_a$
codeword	$\mathbf{x}$	$\mathbf{x}'$	$\bar{\mathbf{x}}$	$\bar{\mathbf{x}}_a$
helper data	$\mathbf{s}$			$\mathbf{s}_a$
hash value	$\mathbf{h}$		$\bar{\mathbf{h}}$	$\bar{\mathbf{h}}_a$
secret key	$\mathbf{r}$		$\bar{\mathbf{r}}$	$\bar{\mathbf{r}}_a$
error vector	$\mathbf{e}$			
attack vector				$\mathbf{e}_a$

2.2 Definitions of secure sketches and fuzzy extractors

Fuzzy extractors and secure sketches were proposed by Dodis *et al.* in 2004 [1]. A secure sketch is defined as follows: [2]

Definition 1: An (m, m', t)-secure sketch over a metric space $(\mathcal{M}, \mathsf{d})$ comprises a *sketching procedure* $\mathsf{SS} : \mathcal{M} \to \{0,1\}^*$ and a *recovery procedure* Rec, where:

1) **(Security)** For all random variables W over $\mathcal{M}$ such that $\mathsf{H}_\infty(W) \geq m$, we have $\mathsf{H}_\infty(W|\mathsf{SS}(W)) \geq m'$, with H_∞ denoting the min-entropy.
2) **(Error tolerance)** For all pairs of points $\mathbf{w}, \mathbf{w}' \in \mathcal{M}$ with $\mathsf{d}(\mathbf{w}, \mathbf{w}') \leq t$, it holds that $\mathsf{Rec}(\mathbf{w}', \mathsf{SS}(\mathbf{w})) = \mathbf{w}$.

In the context of PUFs, a secure sketch can be used to generate helper data $\mathbf{s}$ for a PUF response $\mathbf{w}$ during the first key generation (sketching procedure). This helper data can then be used to correct up to t errors in a noisy PUF response $\mathbf{w}'$ during the "recovery" phase. The security guarantees state that the remaining min-entropy in the PUF response $\mathbf{w}$ after revealing the helper data $\mathbf{s}$ is at least $m' \leq m$. Hence, $\mathbf{w}$ is not guaranteed to be uniformly distributed and revealing the helper data reduces the guaranteed min-entropy such that it cannot directly be used as a cryptographic key. The main purpose of a Fuzzy Extractor is to extend the secure sketch to generate a string $\mathbf{r}$ that is uniform and has full bit entropy such that it can be be used as a cryptographic key. The formal definition is as follows: [2]

Definition 2: An (m, l, t, δ)-fuzzy extractor over a metric space $(\mathcal{M}, \mathsf{d})$ comprises a (randomized) *extraction algorithm* $\mathsf{Ext} : \mathcal{M} \rightarrow \{0,1\}^l \times \{0,1\}^*$ and a *recovery procedure* Rec such that:

1) **(Security)** For all random variables W over $\mathbf{M}$ that satisfy $\mathsf{H}^{\infty}(W) \geq m$, if $\langle \mathbf{r}, \mathbf{pub} \rangle \leftarrow \mathsf{Ext}(W)$ then $\mathsf{SD}(\langle \mathbf{r}, \mathbf{pub} \rangle, \langle \mathcal{U}_l, \mathbf{pub} \rangle) \leq \delta$, where $SD()$ is the statistical difference as defined in [2] and $\mathcal{U}_l$ the uniform distribution over l-bit strings.
2) **(Error tolerance)** For all pairs of points $\mathbf{w}, \mathbf{w}' \in \mathcal{M}$ with $\mathsf{d}(\mathbf{w}, \mathbf{w}') \leq t$, if $\langle \mathbf{r}, \mathbf{pub} \rangle \leftarrow \mathsf{Ext}(\mathbf{w})$ then it is the case that $\mathsf{Rec}(\mathbf{w}', \mathbf{pub}) = \mathbf{r}$. Where $\mathsf{d}()$ denotes the distance metric for $\mathcal{M}$.

The aforementioned fuzzy extractor is only secure against a passive attacker. I.e., it does not make any guarantees about the security if the attacker can manipulate the helper data **pub**. For this purpose robust fuzzy extractors were proposed by Boyen *et al.* [2] that are also secure against active attackers. These robust fuzzy extractors are based on well-formed secure sketches which are defined as follows: [2]

Definition 3: An (m, m', t)-secure sketch $(\mathsf{SS}, \mathsf{Rec})$ is said to be well-formed if it satisfies the condition of Definition 1 except for the following modifications:

1) Rec may now return either an element in $\mathcal{M}$ or the distinguished symbol $\perp$.
2) For all $\mathbf{w}' \in \mathcal{M}$ and arbitrary $\mathbf{s}'$, if $\mathsf{Rec}(\mathbf{w}', \mathbf{pub}') \neq \perp$ then $\mathsf{d}(\mathbf{w}', \mathsf{Rec}(\mathbf{w}', \mathbf{pub}')) \leq t$.

In other words, a secure sketch only guarantees that if there are t or less errors, the errors will be corrected. It does not say anything about what happens if there are more than t errors. Correcting more than t errors does not have any impact on security. In addition, t does not even need to be a strict lower bound but can be relaxed, which was formalized in [15] as *relaxed notions of correctness*. A well-formed secure sketch on the other hand corrects exactly up to t errors and responds with $\perp$ if there are more than t errors. One cannot simply relax this notion and allow more than t errors to be corrected without violating the assumptions in the proof. It is actually simple to construct a well-formed secure sketch from any secure sketch: For this, one only needs to compute $t' = \mathsf{d}(\mathbf{w}, \mathbf{w}')$. If $t' > t$ return $\perp$ else return $\mathbf{w}$.

A *robust sketch* is resistant against helper data manipulation attacks and is defined as follows: [2]

Definition 4: Given algorithms $(\mathsf{SS}, \mathsf{Rec})$ and random variables $W = \{W_0, W_1, ..., W_n\}$ over the metric space $(\mathcal{M}, \mathsf{d})$, consider the following game between an adversary A and a challenger: Let $\mathbf{w}$ (resp., $\mathbf{w}_i$) be the value assumed by W_0 (resp., W_i). The challenger computes $\mathbf{pub} \leftarrow \mathsf{SS}(\mathbf{w}_0)$ and gives **pub** to A. Next, for $i = 1, ..., n$, the adversary A outputs a "challenge" $\mathbf{pub}_i \neq \mathbf{pub}$ and is given $\mathsf{Rec}(\mathbf{w}_i, \mathbf{pub}_i)$ in return. If there exists an i such that $\mathsf{Rec}(\mathbf{w}_i, \mathbf{pub}_i) \neq \perp$ we say that the adversary succeeds and this event is denoted by *Succ*.

We say that $(\mathsf{SS}, \mathsf{Rec})$ is an (m, m'', n, ϵ, t)-robust sketch over $(\mathcal{M}, \mathsf{d})$ if is is a well-formed (m, m'', t)-secure sketch and:

1) For all t-bounded distortion ensembles W with $\mathsf{H}_{\infty}(W_0) \geq m$ and all adversaries A we have $Pr[Succ] \leq \epsilon$.
2) The average min-entropy of W_0, conditioned on the entire view of A throughout the above game, is at least m''. Which implies that $(\mathsf{SS}, \mathsf{Rec})$ is an (m, m'', t)-secure sketch.

Similar to secure sketches, a robust sketch does not necessarily produce a uniformly distributed string that can be used as a cryptographic key. For this purpose robust fuzzy extractors were defined as follows: [2]

Definition 5: Given algorithms $(\mathsf{Ext}, \mathsf{Rec})$ and random variables $W = \{W_0, W_1, ..., W_n\}$ over a metric space $(\mathcal{M}, \mathsf{d})$, consider the following game between an adversary A and a challenger: Let $\mathbf{w}_0$ (resp., $\mathbf{w}_i$) be the value assumed by W_0 (resp., W_i). The challenger computes $(\mathbf{r}, \mathbf{pub}) \leftarrow \mathsf{Ext}(\mathbf{w}_0)$ and gives **pub** to A. Next, for $i = 1, .., n$, the adversary A outputs $\mathbf{pub}_i \neq \mathbf{pub}$ and is given $\mathsf{Rec}(\mathbf{w}_i, \mathbf{pub}_i)$ in return. If there exists an i such that $\mathsf{Rec}(\mathbf{w}_i, \mathbf{pub}_i) \neq \perp$ we say the adversary succeeds and this event is denoted by *Succ*. We say $(\mathsf{Ext}, \mathsf{Rec})$ is an $(m, l, n, \epsilon, t, \delta)$-robust fuzzy extractor over $(\mathcal{M}, \mathsf{d})$ if the following hold for all t-bounded distortion ensembles W with $\mathsf{H}_{\infty}(W_0) \geq m$:

- **(Robustness)** For all adversaries A, it holds that $Pr[Succ] \leq \epsilon$.
- **(Security)** Let $View$ denote the entire view of A at the conclusion of the above game. Then, $\mathsf{SD}(\langle \mathbf{r}, View \rangle, \langle \mathcal{U}_l, View \rangle) \leq \delta$. $SD()$ is again the statistical difference as defined in [2] and $\mathcal{U}_l$ the uniform distribution over l-bit strings.
- **(Error-tolerance)** For all $\mathbf{w}'$ with $\mathsf{d}(\mathbf{w}_0, \mathbf{w}') \leq t$, we have $\mathsf{Rec}(\mathbf{w}', \mathbf{pub}) = \mathbf{r}$.

2.3 Robust fuzzy extractor constructions

After we have defined the various constructions we will now take a look at how they can be realized. In this paper we only look at constructions for the Hamming distance metric since this is used in PUF-based key generation. The main building block of all constructions is the secure sketch. Two secure sketch constructions for the Hamming

distance metric were proposed by Dodis *et al.* [1], the code-offset and the syndrome construction.

2.3.1 Secure sketches for the Hamming distance metric

The code-offset construction [1] is based on the fuzzy commitment proposed by Juels and Wattenberg [16] and is defined as follows:

Definition 6: Code-offset construction: During sketching, choose a random codeword $\mathbf{x} \in \mathbf{C}$ and compute the helper data $\mathbf{s} = \mathsf{SS}(\mathbf{w}, \mathbf{x}) = \mathbf{w} \oplus \mathbf{x}$. For recovery, compute $\bar{\mathbf{x}} = \mathsf{decode}(\mathbf{w}' \oplus \mathbf{s})$, where $\mathsf{decode}()$ denotes the decoding procedure of the error-correction code $\mathbf{C}$ and $\mathbf{w}'$ the potentially noisy PUF response. Then $\bar{\mathbf{w}} = \mathsf{Rec}(\mathbf{w}', \mathbf{s}) = \mathbf{s} \oplus \bar{\mathbf{x}}$ with $\bar{\mathbf{w}} == \mathbf{w}$ if $\mathsf{d}(\mathbf{w}', \mathbf{w}) \leq t$.

The syndrome construction requires the code $\mathbf{C}$ to be a linear binary error-correction code and works as follows:

Definition 7: Syndrome construction: During sketching, compute $\mathbf{s} = \mathsf{SS}(\mathbf{w}, \mathbf{x}) = \mathbf{w} \cdot \mathbf{H}^{\mathbf{T}}$, where $\mathbf{H}^{\mathbf{T}}$ is the transposed parity check matrix of the used linear error-correction code $\mathbf{C}$. For recovery, compute $\mathbf{s}' = \mathbf{w}' \cdot \mathbf{H}^{\mathbf{T}}$. Determine $\bar{\mathbf{e}} = \mathsf{locate}(\mathbf{s}' \oplus \mathbf{s})$ by using the error-location algorithm $\mathsf{locate}()$ of the code $\mathbf{C}$. Then $\bar{\mathbf{w}} = \mathsf{Rec}(\mathbf{w}', \mathbf{s}) = \mathbf{w}' \oplus \bar{\mathbf{e}}$ with $\bar{\mathbf{w}} = \mathbf{w}$ if $\mathsf{d}(\mathbf{w}', \mathbf{w}) \leq t$.

Both, the code-offset and the syndrome construction are popular for PUF-based key generation.

2.3.2 Hash-based robust fuzzy extractor

Boyen *et al.* showed how robust fuzzy extractors can be built using any well-formed secure sketch in conjunction with a hash function. This construction, which we will denote as *hash-based construction* in the remainder of the paper is provably secure in the random oracle model [2]. This hash-based construction works as follows:

Definition 8: Assume we are given two hash functions $\mathsf{H}_1, \mathsf{H}_2 : \{0,1\}^* \rightarrow \{0,1\}^l$ (in practice a single hash function can be used by prepending a padding) and a well-formed secure sketch $(\mathsf{SS}, \mathsf{Rec})$. The hash-based robust fuzzy extractor $(\mathsf{Ext}, \mathsf{Rec})$ with $(\mathbf{r}, \mathbf{pub}) = \mathsf{Ext}(\mathbf{w})$ and $\bar{\mathbf{r}} = \mathsf{Rec}(\mathbf{w}', \mathbf{pub})$ works as follows:

- $(\mathbf{r}, \mathbf{pub}) = \mathsf{Ext}(\mathbf{w})$: Let $\mathbf{s} = \mathsf{SS}(\mathbf{w})$. Output $\mathbf{pub} = (\mathbf{s}, \mathsf{H}_1(\mathbf{w}, \mathbf{s}))$ and $\mathbf{r} = \mathsf{H}_2(\mathbf{w}, \mathbf{s})$.
- $\bar{\mathbf{r}} = \mathsf{Rec}(\mathbf{w}', \mathbf{pub})$: Parse $\mathbf{pub}$ as $(\mathbf{s}, \mathbf{h})$ and set $\bar{\mathbf{w}} = \mathsf{Rec}(\mathbf{w}', \mathbf{s})$. If $\mathsf{d}(\bar{\mathbf{w}}, \mathbf{w}') \leq t$ and $\bar{\mathbf{h}} == \mathsf{H}_1(\bar{\mathbf{w}}, \mathbf{s})$ then output $\bar{\mathbf{r}} = \mathsf{H}_2(\bar{\mathbf{w}}, \mathbf{s})$ else output $\bar{\mathbf{r}} = \perp$.

How the hash-based robust fuzzy extractor works in conjunction with the code-offset construction is illustrated in Figure 1. One great advantage of this construction is that in this way any fuzzy extractor can be extended to a robust fuzzy extractor. Hence, when discussing fuzzy extractor constructions for PUF-based key generation it is often noted that in case resistance against active attackers is needed, the fuzzy extractor can easily be turned into a robust fuzzy extractor. However, we want to point to a detail that is often overlooked: In a fuzzy extractor $\mathsf{d}(\bar{\mathbf{w}}, \mathbf{w}') \leq t$ is only a correctness requirement, i.e., only guarantees that the errors are corrected. It has nothing to do with security and hence an engineer can simply ignore

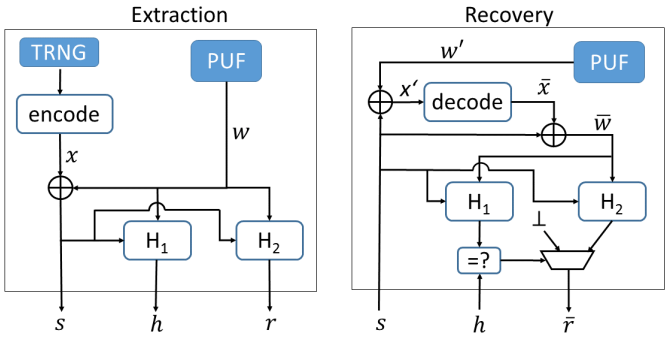


Fig. 1: Illustration of the hash-based fuzzy extractor when it is used in conjunction with the code-offset construct for PUF-based key generation. On the left the extraction phase and on the right the recovery phase is depicted.

this part without jeopardizing security. Indeed, Dodis *et al.* formalized this as relaxed notion of correctness since better error correction rates can be achieved with such relaxed notations.

In a robust fuzzy extractor on the other hand $\mathsf{d}(\bar{\mathbf{w}}, \mathbf{w}') \leq t$ is a security requirement. If this inequality is not satisfied the robust fuzzy extractor needs to return $\perp$. In other words, the robust fuzzy extractor requires a well-formed secure sketch not because of a correctness requirement but because of a security requirement. Hence, this aspect cannot be ignored without security implications.

2.3.3 Fuzzy extractors for PUF-based key generation

In this Subsection we will briefly discuss some of the different fuzzy extractors that have been proposed in conjunction with PUF-based key generation. In earlier works BCH codes have been used [17], [18]. However, the superior error correction capability of concatenated codes were pointed out in the seminal work of Bösch *et al.* from 2008 [6]. In particular, they proposed a simple inner repetition code with an outer Reed-Muller or Golay code. Concatenated codes with BCH codes in conjunction with repetition codes have been used in the literature as well [19]. In 2009 the idea to use soft-decision error correction was first introduced by Maes *et al.* in conjunction with PUF-based error correction [9], [10]. The idea is to collect additional information about the reliability of each PUF response bit and add this as additional helper data during the generation phase. During reconstruction, soft-decision error correction codes can use this information to considerably decrease the probability of a decoding error. For this concatenated codes with soft-decision repetition and Reed-Muller codes were proposed. One disadvantage of this approach is that the soft-decision information needs to be collected first, which might not always be trivial. How to overcome this problem was presented in 2012 by van der Leest *et al.* [7] by using a hard-in soft-out inner code (repetition decoder) in conjunction with a soft-decision outer code (Reed-Muller or Golay code). Compared to hard-decision decoding considerably better error correction rates are achieved without the need to

collect additional information during the generation phase. The same hard-in soft-out decoding was also used in [8]. Similarly, the generalized concatenated code constructions by Puchinger *et al.* [20] also uses hard-in soft-out decoding of Reed-Muller codes.

3 Impossibility results for BCH codes

In order to show the feasibility of fuzzy extractors, Dodis *et al.* showed that BCH codes can be used to construct fuzzy extractors based on the code-offset and syndrome construction [1]. Later work showed how every fuzzy extractor can be turned into a robust fuzzy extractor [2] and hence at least in theory BCH codes can also be used to build robust fuzzy extractors. In this section we look at the problem from a more practical perspective by investigating if the parameters of the security proof can also be met for realistic error-correction rates as they are needed for PUF-based key generation. A typical goal in PUF-based key generation is to achieve a failure rate of less than 10^{-6} or 10^{-9} during key generation with an assumed worst-case PUF reliability of around 85%. This is e.g. assumed in [6], [8], [9], [19], [20]. Note that in all of these papers concatenated code constructions are used that yield much better average error correction rates than non-concatenated code. However, concatenated codes have a significantly worse minimum error-correction capacity t (see Section 3.3) and are therefore impractical for robust fuzzy extractors that follow the security proof from [2]. We therefore want to evaluate in this Section if these results can also be achieved with a single BCH code as it was proposed for robust fuzzy extractors by Boyen *et al.* [2].

3.1 Security bound for Boyen *et al.*'s hash-based robust fuzzy extractor construction

The robust fuzzy extractor proof from [2] provides a formula to compute the success probability ϵ of an active attacker. This equation allows the derivation of a security bound which the binary $[n, k, 2t+1]$ BCH codes need to fulfill in order to be provably secure when used in robust fuzzy extractors.

Theorem 1: Security Bound:
A binary linear $[n, k, 2t+1]$ code used in a robust sketch as defined in [2] that does not fulfill the following bound does not fulfill the corresponding security proof of Boyen *et al.* :

$$k > 1 + log_2(n) + log_2\left(\sum_{i=0}^{t}\binom{n}{i}\right) \tag{1}$$

Proof: From [2] Theorem 1 the success probability ϵ of an attacker is defined with:[2]

$$\epsilon = (4q_H + 2n \cdot Vol_t^M) \cdot 2^{-m'} \tag{2}$$

where q_H denotes the number of times an attacker is allowed to query the robust fuzzy extractor, Vol_t^M denotes the volume of the error correction code and m' the remaining min-entropy after revealing the helper data. For binary linear $[n, k, 2t+1]$ codes Vol_t^M can be expressed as (see, e.g., [4], Sec. III.e.):

$$Vol_t^M = \sum_{i=0}^{t}\binom{n}{i}$$

For a secure sketch based on a $[n, k, 2t+1]$ code the upper bound of the min-entropy m' can be written as $m' \leq n - (n-k) = k$. Note that we are looking for an impossibility result, i.e., we want to show that if the inequality does not hold the construction does not fulfill the security proof from [2]. We do not want to show that if the inequality holds that the constriction is secure. Therefore we derive a lower bound of the attacker's success probability ϵ as follows:

$$\begin{aligned} \epsilon &= (4q_H + 2n \cdot Vol_t^M) \cdot 2^{-m'} \\ \epsilon &\geq 2n \cdot \sum_{i=0}^{t}\binom{n}{i} \cdot 2^{-k} \\ log_2(\epsilon) &\geq log_2\left(2n \cdot \sum_{i=0}^{t}\binom{n}{i} \cdot 2^{-k}\right) \\ log_2(\epsilon) &\geq 1 + log_2(n) + log_2\left(\sum_{i=0}^{t}\binom{n}{i}\right) - k \end{aligned} \tag{3}$$

Since the success probability ϵ needs to be smaller than 1, i.e., $\epsilon < 1$, one can simply define a lower bound as:

$$\begin{aligned} log_2(1) > log_2(\epsilon) &\geq 1 + log_2(n) + log_2\left(\sum_{i=0}^{t}\binom{n}{i}\right) - k \\ 0 &> 1 + log_2(n) + log_2\left(\sum_{i=0}^{t}\binom{n}{i}\right) - k \\ k &> 1 + log_2(n) + log_2\left(\sum_{i=0}^{t}\binom{n}{i}\right) \end{aligned} \tag{4}$$

□

There are parameter choices for BCH codes that fulfill the bound. However, in practice not only the security bound has to hold but the resulting BCH code also needs to achieve the required error correction rate. In particular, usually the probability of a key generation failure P_{fail} in a worst-case scenario should be below a defined threshold. A simple but widely used error-model is that each PUF response bit flips with the same probability $1-p$ (i.e., a homogeneous noise model). Typical parameters found in the literature are $p \approx 0.85$ and P_{fail} around 10^{-6} to 10^{-9}. Table 2 shows the minimum value for t for $[n, k, 2t+1]$ BCH codes and different codeword sizes n and reliability values p to achieve a failure rate of less than $P_{fail} < 10^{-6}$. In a way, the values in the table are a "error-tolerance bound", since only error-correcting codes that can correct at least that many bit errors are reliable enough for a robust fuzzy extractor.

Since we now have a security bound as well as a bound on the minimum required error correction capacity

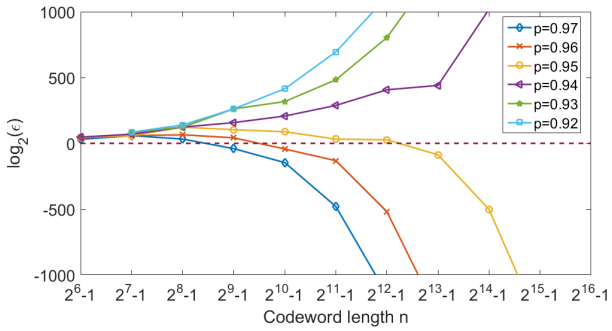
2. We use the simplified lower bound of Theorem 1 [2] for the case that the hash output length was chosen appropriately.

n	63	127	255	511	1023	2047	4095	8191	16383	32767	65535
p =0.92	-	27	45	85	126	231	413	819	1483	2861	5579
p =0.93	-	27	42	85	115	205	367	686	1321	2517	4902
p =0.94	15	23	42	59	102	179	330	597	1170	2340	4681
p =0.95	15	21	42	53	87	153	292	506	955	1829	3545
p =0.96	13	21	29	45	74	127	227	415	793	1483	2863
p =0.97	11	21	25	37	60	101	178	325	599	1133	2184

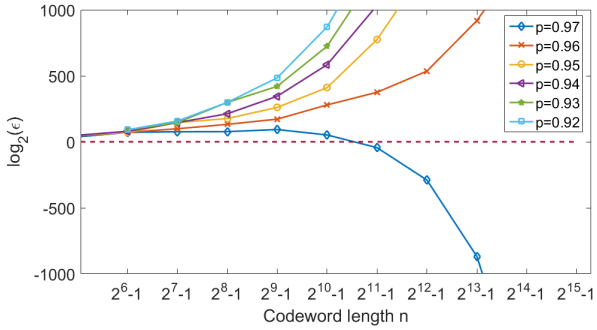
TABLE 2: Lower bound of the required number of correctable errors t for different codeword sizes n and reliabilities p to achieve a failure rate of less than 10^{-6}.

we can verify whether or not building a secure robust fuzzy extractor based on BCH codes is possible. For this purpose we tested the security bound by using the BCH parameters derived in Table 2 and computing the corresponding ϵ value. In particular, we calculated the bounded $log_2(\epsilon)$ according to Equation (3) using Sage for the $[n, k, 2t+1]$ BCH codes that were closest to the values listed in Table 2. The result can be found in Figure 2a. As one can see, only for $p > 0.94$ the value is negative, i.e., fulfills the security bound. For all other values with $p \leq 0.94\%$ the security proof provided in [2] does not hold.

Note that for PUF-based key generation much lower reliabilities than 94% are needed (e.g. $p = 85\%$) and hence it is currently not possible to build a robust fuzzy extractor with BCH codes for PUF-based key generation that is provably secure according to the proof provided by Boyen *et al.* [2].



(a) hash-based construction



(b) standard model construction

Fig. 2: The upper bound of the success probability ϵ of an attacker for a helper data manipulation attack against a) the hash-based construction from [2] and b) the standard-model construction from [3]. For each codeword size n and PUF reliability value p the best BCH parameters were chosen to compute ϵ. Note that a positive value means that for the corresponding reliability and codeword sizes the construction does not offer provable security against helper data manipulation attacks.

3.2 Security bound for Dodis *et al.*'s robust fuzzy extractor construction

The robust fuzzy extractor based on hash functions proposed by Boyen *et al.* [2] is provably secure in the random oracle model but not the general model. Therefore a different construction was proposed by Dodis *et al.* that is also provably secure in the general model [3]. However, this provable security in the general model comes at the cost of a significantly reduced efficiency in terms of security parameters. In particular, it looses half of the min-entropy m'. Therefore the security bound for this construction is reduced to:

$$\frac{k}{2} \geq log\left(\sum_{i=0}^{t}\binom{n}{i}\right) + log\left(2\left\lceil\frac{k}{n-k}+2\right\rceil\right) - \frac{1}{2} \quad (5)$$

Details of how this bound is derived can be found in the Appendix. We again computed the ϵ value for the BCH parameters provided in Table 2. The results can be found in Figure 2b. In this case, even for reliability values of $p = 98\%$ no BCH code exists that fulfills the security proof of Dodis *et al.* [3].

3.3 Impact of concatenated codes on the security bound

Before looking into concatenated codes, let us first consider the case that a message $\mathbf{m}$ is split into l blocks of size n and each block is decoded by a BCH code (or other linear code). Assume a BCH code is used in which the maximum as well as the minimum number of correctable errors within one code word is t. If we concatenate l codeword blocks, the maximum number of correctable errors is $t \cdot l$ while the minimum number of correctable errors is still only t since $t+1$ errors in a single block will result in a decoding failure. Hence, for more than t errors the robust fuzzy extractor needs to output $\perp$, which makes it completely impractical. Otherwise, such a construction would not be in line with the security proofs of Boyen *et al.* [2] and Dodis *et al.* [3].

A very important error correction concept in practice are concatenated codes. Nearly all PUF-based key generation proposals use concatenated codes, which first encode

the message with an inner code and then again with an outer code. In many cases the inner codes is a simple repetition code while the outer code is more complex. The main idea is that while the minimum number of correctable errors is much smaller than using a single large code the average error correction capability is considerably higher. In practice, this average number of correctable errors is what really matters to determine the failure probability P_{fail} for PUF-based key generation. But since t is defined by the *minimum* number of correctable errors and not the average number, this makes it impossible to use concatenated codes in well-formed secure sketches. For example, in [19] a [7,1,7] repetition code concatenated with a [318,174,35] BCH was used. The repetition code can correct 3 errors in a 7-bit codeword and the BCH code can correct up to 17 errors in a 318-bit codeword. The concatenated code has a codeword length of $n = 318 \cdot 7 = 2,226$. The minimum number of bit errors for a decoding error to occur is $18 \cdot 4 = 72$. On the other hand, the maximum number of errors such that a codeword can still be decoded correctly is $17 \cdot 7 + 301 \cdot 3 = 1,022$. When used in a well-formed secure sketch as required for the robust fuzzy extractor constructions from [2] and [3], the number of correctable errors would have to be set to t=72. Hence such a construction would be impractical and considerably worse than non-concatenated codes.

From an engineering perspective it is tempting to ignore the well-formed requirement of the proof and to set t to the average number of correctable errors. While this violates the proof, such a construction might still be secure considering that no attack on such a construction is known. This scenario is discussed in great detail in the next section.

4 Helper data manipulation attack

In this section the general attack strategy for the new helper data manipulation attacks is described before some specific attacks against specific error correction strategies and implementations are introduced in the next Section.

4.1 Attack model

Recently, Delvaux *et al.* [14] introduced a helper data manipulation attack on a soft-decision error-correction strategy, in which modified helper data is sent to the PUF device and the attacker observed if a decoding failure occurred. With each query the attacker learns information about the PUF response and can eventually reconstruct the entire response in a divide-and-conquer like fashion.

In this paper we consider that instead of a fuzzy extractor a RFE-like construct (see Figure 1) is used. The RFE-like construct prevents such helper data manipulation attacks as described above, since the key recovery always fails if the attacker does not transmit a valid hash value $\mathbf{h}$. And without knowing the reconstructed PUF response $\bar{\mathbf{w}}$ the attacker cannot compute a valid hash value $\mathbf{h}_a$ for a modified helper data $\mathbf{s}_a$.

The new attack strategy is therefore not to try to learn the original PUF response $\mathbf{w}$, but instead to try to set the reconstructed PUF response $\bar{\mathbf{w}}$ to a value known by the attacker. This in turn enables the attacker to compute a valid hash value $\mathbf{h}_a$. Note that this also means that the reconstructed key $\bar{\mathbf{r}}$ is known by the attacker but not the same as the original key. Hence, the attacker does not learn the original key $\mathbf{r}$ with such an attack. Whether or not this is a reasonable attack goal depends on the application the key is used for. For example, imaging that the PUF key is used to encrypt an externally stored bitstream. In the helper data manipulation attack from [14], the attacker would have learned the encryption key of the bitstream. This would allow the attacker to both decrypt the original bitstream as well as supply the device with its own bitstream. In our helper data manipulation attack scenario on the other hand the attacker would be able to supply a different bitstream to the device but would not be able to decrypt the original bitstream.

Other scenarios in which setting the key is a legitimate attack goal is if the key is used for access control to the device. In this case it is sufficient for the attacker to set a new key without learning the old key as the main goal is to get access to the device. If on the other hand the goal is to create a software clone of a PUF device, e.g. because it is used as a anti-counterfeiting mechanisms, setting a key is not enough. In this case the original key is needed

However, the presented helper data manipulation attacks can also be extended from only setting a key to additionally recover the original key. When and how this can be achieved is briefly discussed in Section 4.3. But the focus of this paper is only to set the key to defeat the security goal of a robust fuzzy extractor. The attacker's capability can be summarized as follows:

- Attacker's goal:
  - Make the PUF device reconstruct a key $\bar{\mathbf{r}}_\mathbf{a}$ that is known to the attacker (but which is different from the original key $\mathbf{r}$).
- Attacker's capabilities:
  - The attacker has read and write access to the helper data ($\mathbf{s}$ and $\mathbf{h}$)
  - The attacker can verify if the predicted $\bar{\mathbf{r}}_\mathbf{a}$ has been reconstructed by the PUF device

4.2 New attack strategy

The main idea behind the new helper data manipulation attack is to send modified helper data $\mathbf{s}_a$ to the PUF device such that during the recover phase the PUF device will decode to a specific known codeword $\bar{\mathbf{x}}_a = \mathsf{decode}(\mathbf{w}' \oplus \mathbf{s}_a)$ with a very high probability independent of the PUF response $\mathbf{w}'$. This in turn means that the PUF device will recover a PUF response $\bar{\mathbf{w}}_a = \bar{\mathbf{s}}_a \oplus \bar{\mathbf{x}}_a$ which the attacker can predict with a high probability. The attacker can then compute a valid hash value $\mathbf{h}_a = \mathsf{H}_1(\mathbf{s}_a, \bar{\mathbf{w}}_a)$. If the attack succeeded the PUF device and the attacker then share a common key $\bar{\mathbf{r}}_a = \mathsf{H}_2(\mathbf{s}_a, \bar{\mathbf{w}}_a)$.

TABLE 3: General description of the new helper data manipulation attack on an RFE-like construct.

Attacker		**PUF device**
Given: helper data $\mathbf{s}$, with $\mathbf{s} = \mathbf{x} \oplus \mathbf{w}$		Given: noisy PUF function, $\mathsf{PUF}() = \mathbf{w} \oplus \mathbf{e}$
Choose attack vector $\mathbf{e}_a$ such that $\bar{\mathbf{x}}_a = \mathsf{decode}(\mathbf{c}_i \oplus \mathbf{e}_a), \forall \mathbf{c}_i \in \mathbf{C_a}$ with $\mathbf{C_a} \subseteq \mathbf{C}$ and $\lvert\mathbf{C_a}\rvert$ maximized $\mathbf{s}_a = \mathbf{s} \oplus \mathbf{e}_a$ $\bar{\mathbf{w}}_a = \mathbf{s}_a \oplus \bar{\mathbf{x}}_a$ $\mathbf{h}_a = \mathsf{H}_1(\mathbf{s}_a, \bar{\mathbf{w}}_a)$ $\mathbf{r}_a = \mathsf{H}_2(\mathbf{s}_a, \bar{\mathbf{w}}_a)$		
	$\xrightarrow{\mathbf{s}_a, \mathbf{h}_a}$	
		$\mathbf{w}' = \mathsf{PUF}() = \mathbf{w} \oplus \mathbf{e}$ $\mathbf{x}' = \mathbf{w}' \oplus \mathbf{s}_a$ $= (\mathbf{w} \oplus \mathbf{e}) \oplus (\mathbf{w} \oplus \mathbf{x} \oplus \mathbf{e}_a) = \mathbf{x} \oplus \mathbf{e} \oplus \mathbf{e}_a$ $\bar{\mathbf{x}} = \mathsf{decode}(\mathbf{x}') = \mathsf{decode}(\mathbf{x} \oplus \mathbf{e} \oplus \mathbf{e}_a)$ $\bar{\mathbf{w}} = \mathbf{s}_a \oplus \bar{\mathbf{x}}$ $\bar{\mathbf{h}} = \mathsf{H}_1(\mathbf{s}_a, \bar{\mathbf{w}})$ IF $\bar{\mathbf{h}} == \mathbf{h}_a$ set $\bar{\mathbf{r}} = \mathsf{H}_2(\mathbf{s}_a, \bar{\mathbf{w}})$ ElSE set $\bar{\mathbf{r}} = \perp$
	$\xleftarrow{\bar{\mathbf{r}}}$	
IF $\bar{\mathbf{r}} == \perp$ repeat attack ELSE success and $\mathbf{r}_a == \bar{\mathbf{r}}$ and $\bar{\mathbf{w}}_a == \bar{\mathbf{w}}$		

The entire attack is depicted in Table 3. The crucial point in the attack is whether the attacker is able to modify the helper data $\mathbf{s}$ in a way that during the error-correction phase the PUF device will decode towards the attacker's prediction $\bar{\mathbf{x}}_\mathbf{a}$ with a high probability. Whether or not this is possible strongly depends on *i)* the used error-correction code and *ii)* the implementation of the error-correction decoding. In the next chapter we will show some concrete attacks on implementations of different error-correction codes that have been proposed in the context of PUF based key storage.

4.3 Extending attack to extract original key

Depending on the applications and used error-correction codes it might be possible to not only set the key but to also recover the key. In practice often not a single long code is used for the codeword $\mathbf{x}$ but instead the the codeword $\mathbf{x}$ actually consists of l smaller codewords that are concatenated together. This is actually true for all the codes we consider in our attack section. In this case an attacker can perform a helper data manipulation attack that tries to set the codeword $\bar{\mathbf{x}}$ for $l-1$ codeword blocks but leaves one untouched. For the helper data manipulation attack to succeed the attacker then guesses the unmodified codeword and tests the guess using the appropriate $\bar{\mathbf{w}}_a$ to compute the hash $\mathbf{h}_a$. If the unmodified code word has k bit of entropy, the attacker has in average to guess $k/2$ times till he succeeds and can proceed to the next codeword block. This way the attacker could learn the entire original codeword $\mathbf{x}$ and hence also the original PUF response $\mathbf{w} = \mathbf{x} \oplus \mathbf{s}$ and key $\mathbf{r}$ at the costs of an increased attack complexity of $l \cdot k/2$.

Note that this strategy also works if only a fuzzy extractor (as opposed of a RFE-like fuzzy extractor) as long as the attack can verify if the recovered key is the one he assumed it is. This greatly increases the practical relevance of the attacks discussed in the next section.

5 Example attacks

In this Section several attacks on popular error-correction codes for PUF based key generation are shown. In particular, several decoding strategies for Reed-Muller codes are attacked as well as soft-decision and even-numbered repetition decoding.

One of the most popular error-correction codes for PUF-based key generation are Reed-Muller codes, which are for example used in [6], [7], [9], [10], [20]. In the following we will show that different implementation strategies for Reed-Muller codes exists that can be attacked using the new helper data manipulation attack. For a successful attack we need to find an error pattern $\mathbf{e}_a$ such that the decoding algorithm decodes the codeword $\mathbf{x}' \oplus \mathbf{e}_a$ to the same codeword $\bar{\mathbf{x}}_a$ for most codewords $\mathbf{x}'$. There are several possible decoding strategies for Reed-Muller codes. We consider three popular decoding strategies: soft- and hard-decision maximum-likelihood decoding (SDML, ML), soft-decision Generalized Multiple Concatenated codes (GMC) decoding, and classic Reed decoding based on majority logic vote.

5.1 Attacking SDML decoding

The idea of SDML decoding is very simple. During decoding all possible codewords are generated and the Hamming distance between the received codeword and all codewords is computed. The codeword with the smallest Hamming distance is then chosen as the decoded codeword. The key observation for our attack on SDML decoding is that for some error vectors $\mathbf{e}_a$ several codewords have the same minimal Hamming distance. In this case always

the first or last tested codeword will be chosen in most implementations[3].

5.1.1 The noise free case

To provide a concrete example, let us look at a [16,5,8] Reed-Muller code which contain $2^k = 2^5 = 32$ different 16-bit codewords $\mathbf{x}_i$. Let us denote the codeword $\mathbf{x}_0$ as the codeword consisting of all-zeros $\mathbf{x}_0 = [0, .., 0]$, and $\mathbf{x}_1 = [1, .., 1]$ the codeword consisting of all-ones. If we add the following error vector $\mathbf{e}_a = [0110101011000000]$ to $\mathbf{x}_i$ then:

$$\mathsf{HD}(\mathbf{x}_0, \mathbf{x}_i \oplus \mathbf{e}_a) = 6 \text{ or } \mathsf{HD}(\mathbf{x}_1, \mathbf{x}_i \oplus \mathbf{e}_a) = 6$$

$$\text{and } \mathsf{HD}(\mathbf{x}_j, \mathbf{x}_i \oplus \mathbf{e}_a) = \{6, 10\} \; \forall \; i, j \geq 2$$

In this case, depending on the codeword $\mathbf{x_i}$, the maximum-likelihood decoding always decodes to either $\mathbf{x}_0$ or $\mathbf{x}_1$ for attack vector $\mathbf{e}_a$. In particular, there are 16 codewords $\mathbf{x_i}$ such $\mathbf{x}_0 = \mathsf{decode}(\mathbf{x}_i \oplus \mathbf{e}_a)$ and 16 codewords $\mathbf{x}_i$ such that $\mathbf{x}_1 = \mathsf{decode}(\mathbf{x}_i \oplus \mathbf{e}_a)$. Hence, by manipulating the helper data with error vector $\mathbf{e}_a$, only one bit of entropy remains from the original 5 bits of entropy in the noise-free case.

For their PUF design, van der Leest *et al.* proposed to use 35 blocks of a concatenated code construction consisting of a hard-in soft-out [7,1,7] repetition code as an inner code and a [16,5,8] Reed-Muller code with SDML decoding as an outer code to derive a 175-bit secret. Let us first consider the noise free case. In order to defeat the RFE-like construction, the attacker has to correctly guess 35 blocks, i.e. predict 35 times the decoded codeword $\bar{\mathbf{x}}_\mathbf{a}$ correctly to be able to compute the correct hash $\bar{\mathbf{h}}_a$ and key $\bar{\mathbf{r}}_a$. After the helper data manipulation one bit of entropy is left per block and a resulting entropy and min-entropy of 35 all codeword blocks. One way to interpret the min-entropy is that the success probability for the most likely decoded codeword $\bar{\mathbf{x}}_a$ is 2^{-35}. But it should be noted that testing a key requires the attacker to send a manipulated helper data to the PUF device. Even for an entropy of 35 the attack would be quite hard to execute in practice since the PUF would be needed to be challenge around 2^{35}. But please also note that a full bit-entropy of the PUF is assumed which in practice might be reduced e.g. due to bias.

5.1.2 Impact of noise on the attack

In practice the PUF response $\mathbf{w}'$ will be noisy and hence a legitimate question is how well the attack works in the presence of noise. To evaluate this question we performed following analysis. In a first step 100,000 random binary response strings are generated in MATLAB and the corresponding helper data for the code-offset construction and the error correction code are computed. Then each response is flipped with probability $1 - p$ to simulate noise (i.e. we use the simple homogeneous noise model in which each response bit has the same failure probability). The helper data is manipulated according to the attack strategy. The noisy responses and the modified helper data are passed to the Rec algorithm and the decoded responses are stored. Based on this analysis the probability that a decoding to a certain response $\bar{\mathbf{w}}_i$ occurs is computed which in turn allows us to compute the min-entropy and entropy of the recovered response $\bar{\mathbf{w}}$ in the presence of the attack. One thing to consider is that to estimate the required error-correction capacity of a code the worst-case PUF reliability under environmental conditions is used. But during an attack the environmental conditions can be kept constant to reduce the noise to a minimum. Therefore the reliability during an attack is considerably higher than the worst-case reliability used to select the code. For SRAM PUFs for example, reliability values around 96-98% are not uncommon when the environmental conditions are kept constant while their worst case reliably can be easily 10% worse [21].

3. It is also possible to choose a random codeword instead, in which case the attack would not work. However, this is typically more difficult to implement especially for hardware implementation.

reliability	min-entropy (per codeword)	entropy (per codeword)	min-entropy (per 175 bits)
soft-decision			
100%	1.0	1.0	35.0
99%	2.0	3.6	69.6
98%	2.8	4.5	98.2
97%	3.4	4.8	118.5
96%	3.8	4.9	130.8
95%	4.0	4.9	138.8
hard-decision			
100%	1.0	1.0	35.0
95%	1.0	1.0	35.0
90%	1.0	1.2	36.4
85%	1.2	1.8	41.1
80%	1.5	2.7	51.8
Without attack			
–	5	5	175

TABLE 4: Top: Overview of helper data manipulation attacks on a hard-in soft-out [7,1,7] repetition code in conjunction with a [16,5,8] Reed-Muller code using soft-decision SDML decoding. Bottom: hard-decision ML decoding of a concatenated [7,1,7] repetition code with a [16,5,8] Reed-Muller code

The top of Table 4 shows the results for the attack on the SDML construction from [7] while the bottom shows the same attack for hard-decision maximum likelihood decoding as proposed in [6]. The attack on soft-decision decoding quickly becomes very difficult to perform in the presence of noise. Even with a reliability of 99% the min-entropy within a 175-bit block is only reduced to 69.6. However, when hard-decision maximum likelihood decoding is used, the attack is very resistant to noise. This is due to the fact that the inner repetition code corrects most of the noise. In this case the min-entropy of a 175-bit block is only slightly increased from 35 to 36.4 for a reduction of the PUF reliability to 90%. This shows that helper data manipulation attacks are not restricted to soft-decision decoding and hard-decision decoding can actually be easier to attack in some cases.

reliability	min-entropy (per codeword)	entropy (per codeword)	min-entropy (per 175 bits)
100%	0	0	0
99%	0.4	2.2	14.4
98%	0.9	3.7	31.3
97%	1.4	4.5	49.2
96%	1.9	5.0	66.6
95%	2.4	5.2	82.7
Without attack			
–	5	5	175

TABLE 5: Overview of helper data manipulation attacks on a hard-in soft-out [7,1,7] repetition code in conjunction with a [16,5,8] Reed-Muller code using soft-decision GMC decoding.

5.2 Attacking GMC decoding

Another popular decoding algorithm for Reed-Muller codes is the Generalized Multiple Concatenated code (GMC) decoding [22]. It is e.g. used in [10] and [20] in conjunction with PUF-based key generation. The main idea of this soft-decision decoding algorithm is to treat the Reed-Muller code as a concatenation of multiple smaller codes and recursively decode these codes. The smallest code that is decoded is simply a repetition code, which has a very low decoding complexity. The algorithm can be used as a soft-decision decoder and with some slight modifications also supports erasures. We implemented the GMC decoding in MATLAB as a target implementation. Again, the [16,5,8] Reed-Muller code was used and an error pattern for a helper data manipulation attack was found by simply testing all 2^{16} possible error pattern. It turned out that the helper data manipulation attack on our GMC decoding implementation is actually more powerful than the one on SDML decoding.

For the noise-free case, our GMC implementation always decodes to the same codeword when supplied with following error vector $\mathbf{e}_a = [0000001011001010]$, i.e.:

$$\mathbf{x}_0 = \mathsf{decode}(\mathbf{x}_i \oplus \mathbf{e}_a) \ \forall \mathbf{x}_i \in \mathbf{C}$$

Hence, the resulting entropy is zero, since always $\mathbf{x}_0$ is decoded. The attack complexity for a [7,1,7] hard-in soft-out repetition code as an inner code in conjunction with the tested soft-decision GMC $[16, 5, 8]$ Reed-Muller implementation for different error rates is summarized in Table 5. For a noise level of 98% the attack complexity is quite reasonable and hence the attack can be considered practical. However, for larger noise levels the attack complexity increases significantly.

5.3 Attacking Reed decoding

Another decoding algorithm considered in this paper is the classic hard-decision Reed decoding based on majority logic. For this purpose we took a publicly available MATLAB implementation of a Reed-Muller majority logic decoding and again searched for an error vector suitable for a helper data manipulation attack. The attack vector we found was actually better than for the SDML decoding but slightly worse than GMC decoding. For error vector

reliability	min-entropy (per codeword)	entropy (per codeword)	min-entropy (per 175 bits)
100%	0.2	0.5	6.8
95%	0.2	0.6	6.8
90%	0.2	0.7	7.4
85%	0.3	1.3	11.0
Without attack			
–	5	5	175

TABLE 6: Results of helper data manipulation attacks on a hard-decision $[7, 1, 7]$ repetition code in conjunction with a $[16, 5, 8]$ Reed-Muller code using classic majority decoding.

$\mathbf{e}_a = [1001011000000000]$ the implementation we used decoded to the all-zero codeword $\mathbf{x}_0$ for 28 of the 32 codewords $\mathbf{x}_i$. The four codewords that did not decode to $\mathbf{x}_0$ decoded to the bit vector consisting of all zeros besides the least significant bit which was a one. Hence, in the noise-free case a min-entropy of $28/32 = 0.2$ is achieved per block. However, when considering noise the helper data manipulation attack on classic hard-decision Reed decoding actually exhibits the best performance amongst all considered concatenated codes. This is once again due to the fact that the inner repetition code corrects most of the noise. If we use a [7,1,7] repetition code in conjunction with a $[16, 5, 8]$ Reed-Muller code even with a PUF reliability of only 85% the attack complexity is in the order of 2^{11}. The results of our attack on the concatenated construction can be found in Table 6.

For comparison, Figure 3 shows the development of the min-entropy of this attack when only a single Reed-Muller code is being used. Without the inner repetition coding, the attack would be considerably more difficult in a noisy environment.

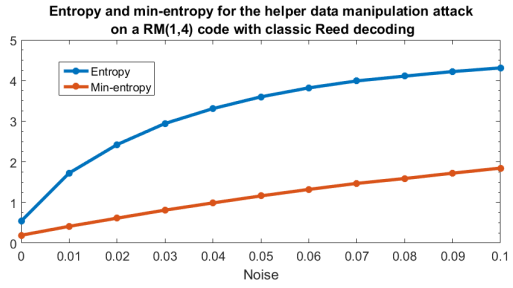


Fig. 3: The resulting entropy and min-entropy for a helper data manipulation attack on a [16,5,8] Reed-Muller code with classic Reed decoding based on majority logic. Note that the information content of a $[16, 5, 8]$ Reed-Muller code is 5 bits, i.e., without the attack the entropy and min-entropy would be 5.

5.4 Attacks on soft-decision and even-numbered repetition codes

Maes *et al.* were the first to propose soft-decision decoding for PUFs in 2009 [9]. They proposed to derive a reliability value for each PUF bit by querying the PUF multiple times during the setup phase. This reliability vector $\mathbf{p}$ is then

stored and provided as additional helper data to a soft-decision error correction code. Hence, in their scheme a second helper data string $\mathbf{p}$ is provided to the PUF device in addition to the helper data $\mathbf{s}$.

Such a soft-decision error-correction code was used by Delvaux *et al.* [14] to demonstrate their helper data manipulation attack on a fuzzy extractor like construct(the soft-decision helper data algorithm [9] is not a fuzzy extractor as no bound on the min-entropy loss is provided). The attack is based on the fact that if the probability $\mathbf{p}$ is set to exactly 0.5, then the corresponding bit will essentially be ignored during the soft-decision decoding. The authors showed a divide-and-conquer manipulation attack using SDML decoding of a simple $[7, 4, 3]$ code as an example. The idea is to set $p_i = 0.5$ for some bits and observe if a decoding failure occurs. With each error pattern the attacker recovers information about the codeword until only one possible codeword remains. This attack idea is illustrated in Figure 4 [14].

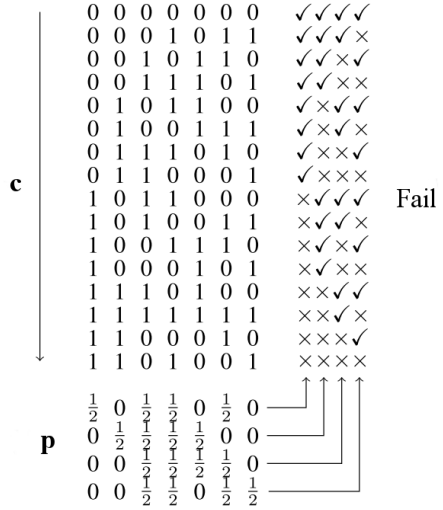


Fig. 4: Soft-decision manipulation attack for a [7,4,3] code with SDML decoding, taken from [14]. It is assumed that the first codeword is selected in case of a likelihood tie.

While the divide-and-conquer approach is very efficient for fuzzy extractors, the RFE-like construction prevents this type of helper data manipulation attack as it will always return a decoding failure unless a valid hash $\bar{\mathbf{h}}_a$ is supplied. However, applying the new attack strategy is straight forward for such a soft-decision decoding algorithm. As also observed in [14], by setting all values to $\mathbf{p_a} = (0.5, .., 0.5)$ in repetition codes, the codeword corresponding to 1 is equally likely than the codeword corresponding to 0. In this case the decoder will decode always to either 1 or 0 depending on the implementation[4]. Hence, by setting $\mathbf{p_a} = (0.5, .., 0.5)$ for all codeword bits, the response bits are all decoded to either 0 or 1.

4. Note that just like the previous attacks this attack strongly depends on the implementation.

reliability	min-entropy (per codeword)	entropy (per codeword)	min-entropy (per 132 bits)
100%	0	0	0
99%	1.2	3.8	12.9
98%	2.0	6.1	22.4
97%	2.8	7.6	30.5
96%	3.4	8.7	37.9
95%	4.1	9.4	44.9
Without attack			
–	12	12	132

TABLE 7: Result of a helper data manipulation attack on a [8,1,8] hard-in soft-out repetition code as an outer code and a [24,12,8] Golay code with a soft-decision Hackett decoder for different noise levels.

Of great practical relevance is also that the same attack works on even-numbered repetition codes which have been proposed as a hard-in soft out outer code e.g. in [7]. The principle of a hard-in soft-out repetition code is fairly simple. The output is the Hamming weight of the codeword divided by the length of the codeword. The resulting value is basically the probability that the corresponding message bit is a 1. Three different constructions have been proposed in [7] based on repetition codes in conjunction with either Reed-Muller or Golay codes. Two of the proposals use an even numbered repetition code including a $[8, 1, 8]$ hard-in soft-out repetition code with a $[24, 12, 8]$ soft-decision Golay code. If we flip half of the bits in an [8,1,8] repetition code, then a 0 and a 1 are equally likely and we basically get $p_i = 0.5$ and the corresponding bit is decoded to either always 0 or always 1 in most implementation for the noise free case. Hence, by flipping half of the bits of the helper data the attacker forces the decoded codeword $\bar{\mathbf{x}}_a$ to be the all zero codeword. Note that for an uneven numbered repetition code, such a helper data manipulation attack is not possible.

5.4.1 Impact of noise

Of course, in the presence of noise the decoder might decode to a different codeword depending on which bits are noisy. To get a feeling how well such an attack would work in practice, we simulated the attack for several error rates and the [8,1,8] repetition code in conjunction with a [24,12,8] Golay code and a soft-decision Hackett decoder. In [7] 11 blocks of the concatenated code construction are used to generate a key with an entropy of 132 bit. Table 7 shows the result of the noise analysis based on the same experimental setup as discussed before.

As one can see, if the PUF reliability is very high, e.g. 99%, the attack is very practical with a min-entropy per codeword of 1.2 and a min-entropy for 132 bits of 12.9. One way to interpret the min-entropy for 132 bits is that the success probability of a helper data manipulation attack is $2^{-12.9}$ when the attacker chooses the most likely codeword to compute h_1. Hence, the min-entropy can be viewed as the attack complexity at least in case that the noise is identically and independently distributed (which is not necessarily true in practice, see [14]). For smaller reliability values such as 95% the attacks become quite difficult to

perform in practice with a min-entropy of roughly 44.9.

6 Discussion

So far we have mainly presented negative results, i.e., showed that it is currently not possible to build a robust fuzzy extractor that fulfills the security proof and that several practical error correction constructions are attackable. However, by no means does that mean that building a provably secure fuzzy extractor is in general impossible. What we need are new proofs and constructs.

6.1 Outlook: Secure decoding strategies

Looking forward, what is needed is a provably secure robust fuzzy extractor that works with efficient concatenated code constructions to achieve the required error correction rates. As a starting point the hash-based construction seems to be very promising since it has considerably better performance than the construction for the general model proposed in [3]. The helper data manipulation attacks discussed in Section 4 show that these attacks strongly depend on the used error correction code as well as the decoding strategy an its implementation. But by no means do our results show that any error correction code or any implementation could be attacked using helper data manipulation attacks! In particular, some error correction codes have a very interesting property that makes them secure against the helper data manipulation attacks from Section 4. The most common decoding strategy of a BCH code is *syndrome decoding* which works as follows: Assume that a noisy codeword $\mathbf{x}' = \mathbf{x} \oplus \mathbf{e}$ with $\mathbf{x} \in \mathbf{C}$ and some noise vector $\mathbf{e}$ has been received. In the first step a syndrome $\mathbf{s}'$ is computed with $\mathbf{s}' = \mathbf{x}' \cdot \mathbf{H}^T$. Then from this syndrome an error location polynomial $\bar{\mathbf{e}}$ is computed $\bar{\mathbf{e}} = \mathsf{locate}(\mathbf{s}')$. If $\mathbf{e} \leq t$ then $\mathbf{e} = \bar{\mathbf{e}}$ and $\mathbf{x} = \mathbf{x}' \oplus \bar{\mathbf{e}}$. What is important for us is that there are decoding strategies for BCH codes based on syndrome decoding for which the following equation holds for all codewords $\mathbf{x}_i$ of the $[n, k, 2t+1]$ code $\mathbf{C}$:

$$\begin{gathered} \bar{\mathbf{e}}_j = \mathsf{locate}\big((\mathbf{x}_i \oplus \mathbf{e}_j) \cdot \mathbf{H}^{\mathbf{T}}\big) \\ \forall\ \mathbf{x}_i \in \mathbf{C},\ \forall\ \mathbf{e}_j \in \{0,1\}^n \text{ and } \bar{\mathbf{e}}_j \in \{0,1\}^n \end{gathered} \tag{6}$$

In other words, the error polynomial $\bar{\mathbf{e}}_j$ is independent of the codeword $\mathbf{x}_i$ and only depends on the error polynomial $\mathbf{e}_j$. While it is possible to flip specific bits of the decoded codeword $\bar{\mathbf{x}}$ with a helper data manipulation attack, it is not possible to *set* specific bits. The attacker can no longer predict $\bar{\mathbf{x}}$ with an increased probability and can therefore also not compute a hash value $\bar{\mathbf{h}}_a$ which the PUF device will accept as valid. Hence, the helper data manipulation attacks presented in this paper do not work any longer if a concatenated code construction is used in the RFE-like construction for which Equation (6) holds.

It therefore seems that it should be possible to build secure robust fuzzy extractors if the used error correction codes fulfills Equation (6). However, note that we only presented a security argument. Proving this in a more formal setting would be very interesting since this would enable us to use a relaxed notion of correctness for robust fuzzy extractors similar as it has been proposed for fuzzy extractors in [15]. This would allow the use of efficient concatenated code constructions and possibly even soft-decision decoding. But it is important to note that this way not a specific error-correction code construct can be shown to be provably secure. Instead only specific error correction strategies could be proven to be secure. In other words, using a BCH code does not guarantee that the implemented decoding strategy indeed fulfills Equation (6). To give an example, consider an $[n, k, 2t+1]$ BCH code with a small k. The typical decoding strategy for a BCH code is syndrome decoding. But for codes with a small k, maximum-likelihood decoding can be used as well and might be very efficient, especially in hardware implementations. However, for maximum-likelihood decoding Equation (6) does not hold and helper data manipulation attacks are possible (see the attack in Section 5.1).

Another important but difficult aspect is the remaining min-entropy for concatenated codes if the PUF response $\mathbf{w}$ does not have full bit entropy. While there is a clear higher bound of the entropy loss in fuzzy extractors [1], using this bound makes building fuzzy extractors very challenging [23]. In [8], [24] a more in-depth discussion regarding this aspect is presented, including a considering tighter bound than the one assumed in [23]. In how far the presented attacks can be improved by incorporating such reductions in the PUF response entropy is an interesting research question.

6.2 Random oracle model vs general model

From a practical perspective, the first robust fuzzy extractor proposal by Boyen *et al.* [2] based on hash functions is very compelling since it only requires a hash function which needs to be implemented for the fuzzy extractor anyway. Furthermore, it is very straightforward and easy to implement and to understand. The fact that it is "only" secure in the random oracle model is not seen as a big problem from a practical perspective. To put it bluntly, hardware security engineers have much bigger problems than the assumptions in the random oracle model. For example, in practice it is impossible to determine the exact entropy within a PUF or a biometric reading. The best we can do is perform measurements and simulations and approximate the entropy based on some assumptions. However, this will never be hundred percent accurate and proofing the assumptions made about the entropy is impossible. Therefore sacrificing nearly half of the entropy so that the system is also provably secure in the standard model does not really make sense from a practical perspective. As a result the construction by Dodis *et al.* [3] that is provably secure in the general model has basically been ignored by the PUF community. It should also be noted that the problem of robust fuzzy extractors in general has been largely neglected by the PUF community. Usually, the need of security against an active attacker is acknowledged but this is usually only

followed by the remark that in this case hash-based robust fuzzy extractor construction should be used. However, it appears that the details of the proofs and definitions of the robust fuzzy extractor have not been really considered. Therefore, the fact that it is actually not possible to extend the popular fuzzy extractor constructions to robust fuzzy extractors has not been discussed.

From a theoretical perspective, the construction that is secure in the general model is more compelling since it has less assumptions. The construction for the standard model therefore has gained much more attention in theory oriented papers [5], [4]. However, the fact that the small error correction rate of robust fuzzy extractors is actually the most limiting factor for a provable as well as practical robust fuzzy extractor has not been identified. Coming up with a provably secure robust fuzzy extractor with a relaxed notion of correctness (i.e., based on a non well-formed secure sketch) would have a big practical impact, even if it is proven in a weaker security model than the random oracle model or general model.

7 Conclusion

This work shows that *i)* currently no robust fuzzy extractor construction exists that fulfills the security proof while also achieving the required error correction rates for PUF based key generation and *ii)* many implementations of decoding algorithms for error correction codes used in PUF-based key generation schemes are susceptible to helper data manipulation attacks even when used in a hash-based construction. Hence, our results show that the problem of building robust fuzzy extractors is actually not solved yet. Our attack on the widely used Reed-Muller decoding shows that there is a great need to build robust fuzzy extractors that are secure against helper data manipulation attacks. While this paper mainly presented negative results and attacks, this by no means mean that building secure robust fuzzy extractors is a lost cause: By considering specific decoding strategies it seems that it should be possible to build both (provably) secure and practical robust fuzzy extractor. However, for this a combined effort of both practitioners as well as theorist is needed.

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Appendix

Security bound for the general construction from Dodis *et al.*

Theorem 2: Security Bound for the general construct from Dodis *et al.* [4] :
A binary linear $[n, k, 2t+1]$ code used in a robust sketch as defined in [4] that does not fulfill the following bound is not provably secure according to the proof provided in [4]:

$$\frac{k}{2} \geq log\Big(\sum_{i=1}^{t}\binom{n}{i}\Big)+log\Big(2\left\lceil\frac{k}{n-k}+2\right\rceil\Big)-1 \quad (7)$$

Proof: Our Theorem is based on Theorem 3 from [4]. In [4] slightly different notions are used than in this paper and the paper from Boyen [2]. In particular the new notion of pre- and post-application robustness is introduced. We will not discuss these definitions and refer the interested reader to [4]. A few variables used in Theorem 3 [4] can be confusing since we use them differently in this paper and therefore we marked variables from Theorem 3 [4] that have a different meaning in this paper with a tilde. Furthermore, Vol_t^M is denoted in [4] with B. In Theorem 3 from [4] it is stated that for any ϵ, δ satisfying

$$l \leq 2m-n-\tilde{k}-2\mathsf{max}\left\{log(Vol_t^M)+log\Big(2\left\lceil\frac{n-\tilde{k}}{\tilde{k}}+2\right\rceil\Big)+log\Big(\frac{1}{\tilde{\delta}}\Big), 2log\Big(\frac{1}{\tilde{\epsilon}}\Big)\right\} \quad (8)$$

(Gen, Rec) is an $(m, l, t, \tilde{\epsilon})$- fuzzy extractor for M with pre-application robustness $\tilde{\delta}$. The pre-application robustness is basically the chance that an active attacker can perform a helper data manipulation attack while $\tilde{\epsilon}$ is the success probability of a passive attacker. Hence, $\epsilon = \tilde{\delta}$ as we denoted the attack probability of an active attacker with ϵ in this paper. For a linear $[n, k, 2t-1]$ code $\tilde{k} = n-k$ and the maximum possible entropy m is $m = n$. The variable l is the length of the resulting key in bits. We are again interested in an impossibility result that shows that codes not fulfilling the bound cannot be secure (but again fulfilling the bound does not necessarily mean the robust fuzzy extractor is secure). One can bound the attackers success probability ϵ with:

$$\begin{aligned} l \leq 2m-n-\tilde{k}-2(log(Vol_t^M)+log\Big(&(2\left\lceil\frac{n-\tilde{k}}{\tilde{k}}\Big))+2\right\rceil+log\Big)(\frac{1}{\epsilon}\Big)) \\ \frac{l}{2} &\leq \frac{k}{2}-log\Big(\sum_{i=1}^{t}\binom{n}{i}\Big)-log\Big(2\left\lceil\frac{k}{n-k}+2\right\rceil\Big)+log(\epsilon) \\ log(\epsilon) &\geq log\Big(\sum_{i=1}^{t}\binom{n}{i}\Big)+log\Big(2\left\lceil\frac{k}{n-k}+2\right\rceil\Big)-\frac{k}{2}-\frac{l}{2} \end{aligned} \quad (9)$$

Since $l \geq 1$ and $\epsilon \leq 1$ we can define the following bound that needs to be fulfilled:

$$\begin{aligned} log(1) \geq log(\epsilon) &\geq log\Big(\sum_{i=1}^{t}\binom{n}{i}\Big)+log\Big(2\left\lceil\frac{k}{n-k}+2\right\rceil\Big)-\frac{k}{2}-\frac{l}{2} \\ \frac{k}{2} &\geq log\Big(\sum_{i=1}^{t}\binom{n}{i}\Big)+log\Big(2\left\lceil\frac{k}{n-k}+2\right\rceil\Big)-\frac{1}{2} \end{aligned} \quad (10)$$

□