Post-Quantum Key Exchange on FPGAs

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Abstract. The National Institute of Standards and Technology (NIST) announces the post-quantum crypto project, aiming to select cryptographic standard in the post-quantum era. The key establishment algorithm is one of the most important primitives. At Usenix Security 2016, Alkim, Ducas, Pöpplemann, and Schwabe proposed a post-quantum key exchange scheme called NewHope, based on the ring-learning-with-error (RLWE) problem [ADPS16]. In this work, we propose the first hardware implementation of NewHope. Our implementation requires 12,707 FFs, 19,781 LUTs, 13,025 slice registers, 32 DSPs and 13 BRAMs on Xilinx Zynq-7000 equipped with 28mm Artix-7 7020 FPGA. For NewHope key exchange, the three phase of key exchange costs 75.4, 99.1, and 24.6 μs, respectively.

Keywords. Post-quantum cryptography, lattice-based cryptography, LWE, RLWE, key exchange, FPGA implementation.

1 Introduction

In the last decade, post-quantum cryptography has drawn widespread interest. Not only will post-quantum cryptography potentially save us from the threat from large quantum computers, but also provide provable security in some cases. Lattice-based cryptography is a candidate for post-quantum cryptography that provides strong theoretical security gaurantees such as worst-case to average-case reduction. It also provides the initial constructions of many new cryptographic functionalities, e.g., fully-homomorphic encryption [Gen09]. Furthermore, such cryptosystems are usually very efficient. For example, the computation of some public-key encryption based on (Ring-)LWE is faster than RSA/ECC, even though the key size is usually larger [GFS$^+$12,FY14].

Recently, National Institute of Standards and Technology (NIST) announces a post-quantum crypto project, aiming to select new standard cryptographic primitives for the post-quantum era [oSN16]. The key establishment algorithm is one of the most important primitives in this project. At Usenix 2016, Alkim, Ducas, Pöpplemann, and Schwabe proposed the NewHope post-quantum key agreement scheme, based on the ring-learning-with-error (RLWE) problem [ADPS16]. Google conducted a set of experiments using NewHope on internet through the Google Chrome Canary Browser starting from July, 2016. The results show that NewHope is computationally inexpensive, with only a slight increase in latency for some slow internet connections.

Since the computation of NewHope is more complicated than ECDH/ECDHE, the server load will become heavier [ADPS16,Cho15]. A way to offset this load is to use hardware accelerators, where the computation is off-loaded from CPU. However, application-specific integrated circuits (ASICs) are very expensive, so a more cost-effective way to deploy hardware accelerators is to use Field-Programmable Gate Arrays (FPGAs).

As a result, FPGAs are one of the most popular ways today to deploy hardware accelerators. An FPGA contains an array of programmable logic components and a hierarchy of reconfigurable interconnects. The popularity of FPGAs can be seen by the fact that soon it will be possible to rent FPGAs from Amazon Web Service (AWS), who has announced their EC2 F1 instance, a kind of computing instances with FPGAs. Therefore, many foresee the use of cloud services with on-demand FPGAs to increase computation resource when load is high.

Why Key Agreement? A key establishment protocol may be one of two types: key agreement or a key transport protocol. However, key agreement is a common way to implement forward secrecy since both side can influence a shared key. This is why public key encryption can not supplant key agreement protocol. Moreover, compare to the key agreement protocol whose shared key is decided by both parties, the shared key in key transport protocol is decided by a single party. This opens the door to problems in implementation. For example, if the randomness source is compromised (e.g. a backdoor is in the random number generator) in the key transport protocol, the key is directly stolen by the adversary. On the other hard, if the randomness of one party in NewHope is compromised, an adversary still has to solve an RLWE problem to get a shared key.

2 Background

Notation. Let χ be a probability distribution over a set S. We use $x \xleftarrow{\$} \chi(S)$ to denote x sampled from S according to χ, and $x \xleftarrow{\$} S$ to denote x uniformly sampled from S. We define ring $\mathcal{R} = \mathbb{Z}[X]/(X^n + 1)$ as the ring of integer polynomials modulo $X^n + 1$ and $\mathcal{R}_q = \mathcal{R}/q\mathcal{R}$ as the ring of $\mathcal{R}$, where each coefficient is reduced modulo q.

LWE and RLWE. The LWE problem is first introduced by Regev [Reg09] and can be quantum-reduced to certain worst-case lattice problems. Moreover, Peikert [Pei09] and Brakerski *et al.* [BLP$^+$13] further improve the situation by providing classical reductions to lattice problems. LWE based public key cryptosystem is proposed in various variant schemes [ABB10b,ABB10a,Pei09,BV11b,BV11a,LP11]. One important variant of LWE is Ring-LWE, which introdues ring structure into play [LPR10]. The RLWE problem is defined as following: let $s \in \mathcal{R}_q$ be the secret, generate $a \xleftarrow{\$} \mathcal{R}_q$ and $e \xleftarrow{\$} \chi(\mathcal{R}_q)$, compute $b = a * s + e$, and the search version of RLWE is to find s given a list of (a, b). For the setting of most cryptosystems, only one pair of (a, b) is given.

Post-Quantum Key Exchange. To the best of our knowledge, there are two ways to construct a post-quantum key exchange: lattice-based and isogeny-based. Supersingular isogeny Diffie-Hellman key exchange is the key exchange scheme based on isogeny [CLN16]. At this point, it is far from being practical, as the running time is typically 100 times larger than similar schemes based on (R-)LWE. Thus, RLWE may be the most practical post-quantum key exchange scheme so far.

The first LWE-based key exchange is proposed by Ding [Din12], subsequently modified by Peikert [Pei14]. At 2015, Bos *et al.* implemented Peikert's version of RLWE key exchange with a parameter set of their choice [BCNS15]. They also integrated their implementation into the TLS protocol into OpenSSL.

NewHope is the key exchange scheme proposed by Alkim *et al.* [ADPS16], which further improves the performance from [BCNS15] by choosing a different set of parameters. Their analysis shows that the new scheme still remains secure while using a smaller modulus, efficient noise sampling, and fast reconciliation. The details of NewHope will be introduced in next section.

Frodo is the key exchange scheme based on LWE problem instead of RLWE problem, proposed by Bos *et al.* after NewHope [BCD+16]. Without the additional assumption of ring-structure, they selected the parameter with a smaller security margin. Because it based on LWE rather than RLWE, Frodo is still less efficient than NewHope. More precisely, the computation cost of Frodo is around ten times larger, and the communication size is around six times larger than NewHope. However, Frodo is an alternative choice for post-quantum key exchange since RLWE-based cryptosystem might be potentially insecure [ELOS15,CIV16] due to the ring structure.

2.1 NewHope Protocol

As mentioned earlier, NewHope is a variant of Ding's and Peikert's protocols [Din12,Pei14]. The protocol is described in Protocol 1. All the variables except for $\boldsymbol{r} \in \mathbb{R}^4$ are in the ring $R_q = \mathbb{Z}[X]/(X^n + 1)$, where $n = 1024$ and $q = 12289$. This parameter setting is suitable for a number-theoretic transform (NTT) since $q \equiv 1 \mod 2n$.

The key idea of the protocol is: Use the property of $\boldsymbol{ass'} + \boldsymbol{es'} = \boldsymbol{bs'} \approx \boldsymbol{us} = \boldsymbol{ass'} + \boldsymbol{e's}$, where Alice can compute the left-hand side part and Bob can compute the right-hand side part. A problem arises in this situation: The codeword is decided by ass', so the rounding technique usually used in a LWE-based cryptosystem does not work. More precisely, the value of ass' may be near the boundary between where a point rounds to 0 and where it rounds to 1. Then Alice and Bob will add different noise vectors, which may lead to different rounding results. The technique to solve this problem is called *reconciliation*. The main idea is that one party (in NewHope, Bob) sends a hint to the other party (in NewHope, Alice), and the two parties can use the hint to decode the message into the same shared secret. The algorithm to generate hint is shown in Algorithm 1, and the reconciliation algorithm is shown in Algorithm 2.

Finally, to transmit a 256-bits key with 1024 coefficients, NewHope encodes 1 bit of codeword into 4 coefficients in order to increase the error resilience and (hopefully) better security.

Protocol 1: NewHope Key Exchange Scheme

Parameters: $q = 12289 < 2^{14}, n = 1024$		
Error Distribution: ψ_{16}		
Alice (server)		**Bob (client)**
$seed \xleftarrow{\$} \{0,1\}^{256}$		
$\boldsymbol{a} \leftarrow \text{Parse}(\text{SHAKE-128}(seed))$		
$\boldsymbol{s}, \boldsymbol{e} \xleftarrow{\$} \psi_{16}$		$\boldsymbol{s}', \boldsymbol{e}', \boldsymbol{e}'' \xleftarrow{\$} \psi_{16}$
$\boldsymbol{b} \leftarrow \boldsymbol{as} + \boldsymbol{e}$	$\xrightarrow{(\boldsymbol{b}, seed)}$	$\boldsymbol{a} \leftarrow \text{Parse}(\text{SHAKE-128}(seed))$
		$\boldsymbol{u} \leftarrow \boldsymbol{as}' + \boldsymbol{e}'$
		$\boldsymbol{v} \leftarrow \boldsymbol{bs}' + \boldsymbol{e}''$
$\boldsymbol{v}' \leftarrow \boldsymbol{us}$	$\xleftarrow{(\boldsymbol{u}, \boldsymbol{r})}$	$\boldsymbol{r} \xleftarrow{\$} \text{HelpRec}(v)$
$\boldsymbol{v} \leftarrow \text{Rec}(\boldsymbol{v}', \boldsymbol{r})$		$\boldsymbol{v} \leftarrow \text{Rec}(\boldsymbol{v}, \boldsymbol{r})$
$\boldsymbol{\mu} \leftarrow \text{SHA3-256}(\boldsymbol{v})$		$\boldsymbol{\mu} \leftarrow \text{SHA3-256}(v)$

2.2 Algorithms

Reconciliation We follow [ADPS16] in implementing the reconciliation function. The main idea of the recovery mechanism is to encode and decode over the lattice $\tilde{D}_4$, which is the densest lattice sphere packing in dimension 4 so that it provides the lowest failure rate. $\tilde{D}_4$ consists the two

Algorithm 1: HelpRec

Parameter : r-bits reconciliation information
Input : $w \in \mathbb{Z}_q^4$
Output : 4 dimension r-bits reconciliation information $\{0, 1, ..., 2^r - 1\}^4$

$b \xleftarrow{\$} \{0, 1\}$
$\boldsymbol{x} \leftarrow (\frac{2^r}{q}(\boldsymbol{w} + b \cdot (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})^t))$
$\boldsymbol{v}_0 \leftarrow \lfloor \boldsymbol{x} \rceil$
$\boldsymbol{v}_1 \leftarrow \lfloor \boldsymbol{x} - (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})^t \rceil$
$k \leftarrow (\| \boldsymbol{x} - \boldsymbol{v}_0 \|_1 < 1) ? 0 : 1$
$(v_0, v_1, v_2, v_3)^t \leftarrow \boldsymbol{v}_k$
return $(v_0, v_1, v_2, k)^t + v_3 \cdot (-1, -1, -1, 2)^t \mod 2^r$

Algorithm 2: Rec

Parameter : r-bits reconciliation information
Input : $w \in \mathbb{Z}_q^4, \boldsymbol{r} = (r_0, r_1, r_2, r_3)$
Output : 1-bit shared information

$\boldsymbol{x} \leftarrow (\frac{1}{q}\boldsymbol{w} - \frac{1}{2^r} \cdot (r_0, r_1, r_2, \frac{r_0 + r_1 + r_2 + r_3}{2})^t))$
$\boldsymbol{v} = \boldsymbol{x} - \lfloor \boldsymbol{x} \rceil$
return 0 if $\| \boldsymbol{v} \|_1 \leq 1$, and 1 otherwise

shifted copies $\mathbb{Z}^4$ with the shift vector $\boldsymbol{g} = (1/2, 1/2, 1/2, 1/2)^t$. The basis of $\tilde{D}_4$ is $(\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3, \boldsymbol{g})$.

$$\tilde{D}_4 = \mathbb{Z}^4 \cup (\mathbb{Z}^4 + \boldsymbol{g})$$

The encoding method is to equally split the 4-dimensional space by the 1-norm distance to $\boldsymbol{g}$, that is, the regular 24-cells icositetrachoron shape. The r-bit assisted reconciliation algorithm, the algorithm to generate hints, is shown in Algorithm 1, the reconciliation algorithm is shown in Algorithm 2, and NewHope selects the parameter $r = 2$.

Number-Theoretic Transform. Direct multiplication (using school-book algorithm) between two elements in polynomial ring costs n^2 multiplications and roughly as many additions or subtractions. The best way to accelerate the computation is to use fast Fourier transform. The number theoretic transform (NTT) is a discrete version of fast Fourier transform defined over a finite ring $\mathbb{Z}_p$. The NTT algorithm is shown in Algorithm 3, and the inverse number theoretic transform, INTT is very similar to NTT except for an additional final multiplication by n^{-1} for each coefficient of the polynomial.

Negative Wrapped Convolution. The NewHope uses the anti-cyclic ideal $\mathbb{Z}_q[X]/(X^n + 1)$, which does not lead to a classical cyclic convolution when we multiply two ring elements. We use what is called "negative wrapped convolution" to solve the problem. Negative wrapped convolution is first introduced in [LMPR08], and Chen *et al.* implemented the algorithm on FPGA [CMV+15]. Let $\boldsymbol{c} = (c_0, c_1, ..., c_{n-1})$ be the negative wrapped convolution of $\boldsymbol{a} = (a_0, a_1, ..., a_{n-1})$

Algorithm 3: Number-Theoretic Transform, NTT

Parameter : ω is a primitive n-th root of unity in $\mathbb{Z}_q[X]$, n and q
Input : $\boldsymbol{a} \in \mathbb{Z}_q[X]/(X^n+1)$
Output : $\boldsymbol{A} = NTT_\omega^n(\boldsymbol{a})$

$\boldsymbol{a} \leftarrow Order\_reverse(\boldsymbol{a})$
for $i = 0$ *to* $\log_2 n - 1$ **do**
  for $j = 0$ *to* $n/2 - 1$ **do**
    $P_{ij} \leftarrow \lfloor \frac{j}{2^{\log_2 n - 1 - i}} \rfloor \times 2^{\log_2 n - 1 - i}$
    $A_j \leftarrow a_{2j} + a_{2j+1}\omega^{P_{ij}} \mod q$
    $A_{j+n/2} \leftarrow a_{2j} - a_{2j+1}\omega^{P_{ij}} \mod q$
  if $i \neq \log_2 n - 1$ **then**
    $\boldsymbol{a} \leftarrow \boldsymbol{A}$
return A

Algorithm 4: Polynomial Multiplication using NTT over $\mathbb{Z}_q[X]/(X^n+1)$

Parameter : ω is a primitive n-th root of unity in $\mathbb{Z}_q[X]$, $\phi^2 = \omega, n$, and q
Input : $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{Z}_q[X]/(X^n+1)$
Output : $\boldsymbol{c} = \boldsymbol{a} * \boldsymbol{b} \in \mathbb{Z}_q[X]/(X^n+1)$

Precompute: $\omega^i, \omega^{-i}, \phi^i, \phi^{-1}$, where $i = 0, 1, ..., n-1$
for $i = 0$ *to* $n - 1$ **do**
  $\overline{a}_i \leftarrow a_i \phi^i \mod q$
  $\overline{b}_i \leftarrow b_i \phi^i \mod q$
$\overline{\boldsymbol{A}} \leftarrow NTT_\omega^n(\overline{\boldsymbol{a}})$
$\overline{\boldsymbol{B}} \leftarrow NTT_\omega^n(\overline{\boldsymbol{b}})$
for $i = 0$ *to* $n - 1$ **do**
  $\overline{C}_i \leftarrow \overline{A}_i \overline{B}_i \mod q$
$\overline{\boldsymbol{c}} \leftarrow INTT_\omega^n(\overline{\boldsymbol{C}})$
for $i = 0$ *to* $n - 1$ **do**
  $c_i \leftarrow \overline{c}_i \phi^{-i} \mod q$
return c

and $\boldsymbol{b} = (b_0, b_1, ..., b_{n-1})$, it is defined by

$$c_i = \sum_{j=0}^{i} a_j b_{i-j} - \sum_{j=i+1}^{n-1} a_j b_{n+i-j}.$$

This is exactly the polynomial multiplication over $\mathbb{Z}_q[X]/(X^n+1)$. Using the NTT multiplication with negative wrapped convolution, the complexity of multiplication over the polynomial ring $\mathbb{Z}_q[X]/(X^n+1)$ becomes $O(n \log n)$. The pseudo-code of negative wrapped convolution is shown in Algorithm 4.

Noise Sampling. The Knuth-Yao algorithm [KY76] is a common way to sample high-precision discrete Gaussian distribution, which is implemented in [RVV13]. However, such near optimality may result in non-constant execution time, which might lead to side-channel attack. Thus, we do not use the algorithm in this work. NewHope samples the noise from the binomial distribution instead of discrete Gaussian distribution, which needs high precision and much more computation resources. Moreover, sampling from the centered binomial distribution ψ_{16} is cheap in both hardware and software. One can use the property that the centered binomial distribution follows $\sum_{i=0}^{15} b_i - b_i'$, where the b_i, b_i' are random bits. Thus, the sampling algorithm needs 32 random bits to generate one coefficient.

2.3 FPGA

The basic building block of FPGAs is the look-up tables (LUTs). In Xilinx 7 series FPGA, each LUT can be programmed either as a 6-input 1-output function or two 5-input 1-output functions. To implement sequential circuits, each LUT can be connected to two flip-flops. Certain number of LUTs are then grouped into a slice, and a few slices are grouped into a configurable logic block (CLB). Building around CLBs, FPGAs have other circuitries for, e.g., multiplexing input and output, carry-propagation chains for accelerating arithmetic computation, as well as routing fabrics for connecting LUTs. Furthermore, FPGAs also have fixed multipliers in so-called "DSP slices" that can carry out (fixed-point) arithmetic operations, as well as block RAM as the fast on-die working memory. We use Xilinx Zynq-7000 all programmable SoC (AP SoC), which is equipped with a dual-core ARM Cortex-A9 processors running at 667 MHz and integrated with 28nm Artix-7 Z-7020 FPGA. This FPGA has 46,200 look-up tables and 220 DSP slices.

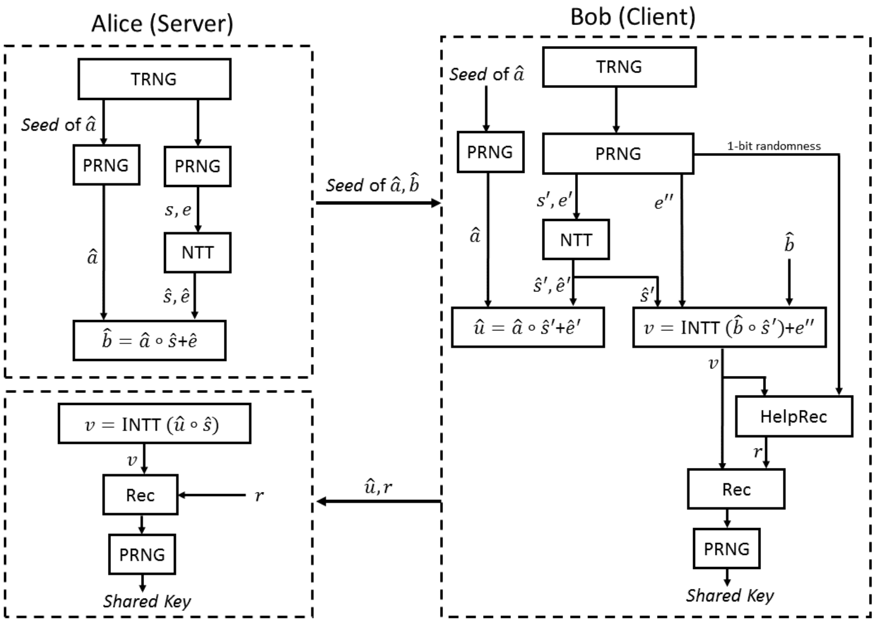
3 Implementation

The block diagram is in Figure 1. There are three main blocks in the diagram representing the flowchart of our hardware implementation of NewHope. First, Alice (Server) uses the TRNG and PRNG to generate the seed of $\widehat{a}$, and computes $b = as + e$ in NTT domain. Bob (Client) receives the seed of $\widehat{a}$ and $\widehat{b}$ (b in NTT domain), computes $u = as' + e'$ in NTT domain and the his shared secrete $v = bs' + e''$, and compute the reconciliation information and the shared key. In the last step, Alice (Server) receives $\widehat{u}$ (u in NTT domain) and reconciliation information r, compute their shared secret $v = us$, and derive the shared key though the reconciliation function with r. We explain the techniques in our implementation.

3.1 Random Number Generator

There are two phases in generating the randomness: TRNG (true random number generator) and PRNG (pseudorandom number generator). In the TRNG phase, we use a credible way from

Fig. 1: Flowchart of our implementation



Wold and Tan's work to generate the randomness by oscillator rings, which has passed NIST and DIEHARD statistical tests [WT09]. The throughput of the implementation from Wold and Tan is 100Mbps with less than 100 logic elements in an Altera Cyclone II FPGA. In our implementation, we use 32 oscillators rings to generate the randomness, and their experiment showed if the number of oscillator rings exceeds 25, the result can pass the statistical tests. In the PRNG phase, NewHope uses SHAKE128 as the PRNG, which is the Extendable Output Functions (XOF's) of SHA-3 family. NewHope uses the extendable property to generate 1024 uniform coefficients in $\mathbb{Z}_p$ with 256 bits true randomness since the randomness is sufficient resist either classic brute-force attack or quantum attack (Grover's algorithm). We extract the SHAKE128 portion from open-source code [Ope12], which usually provides only standard SHA-3 on FPGA.

3.2 Number-Theoretical Transform

We use the design of optimized NTT hardware implementation in [CMV$^+$15,RVM$^+$14]. The main differences are that we use 4 butterfly units, and the modulus is different.

Figure 2 is the high level design of our NTT implementation, it combined both NTT and INTT. For NTT, it processes multiplication on ϕ^i, order reverse, and butterfly units in order. In contrast, INTT processes order reverse, butterfly units and multiplication on ϕ^i in order.

Butterfly Units. In [CMV$^+$15], they use 8 and 2 butterfly units and compare the performance. In [RVM$^+$14]'s implementation, they use a single butterfly unit to compute the NTT function in order to optimize the area usage. We use 4 butterfly units to compute the NTT since our implementation aims to be more speed-optimized. Also, we follow the idea of [CMV$^+$15] we use the architecture shown in Figure 3 that places the data into the memory in the correct positive in order to achieve higher efficiency.

Fig. 2: Overview of our NTT implementation, which consists three components of circuit: multiplication on ϕ^i, order reverse, and butterfly units.

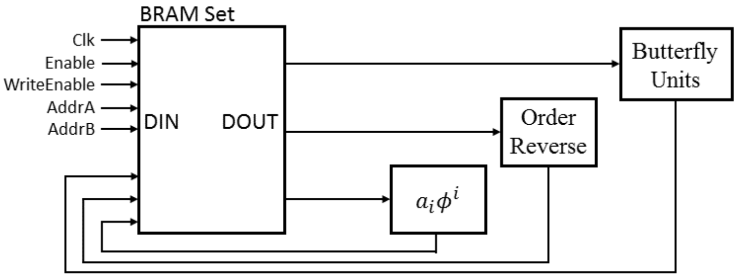
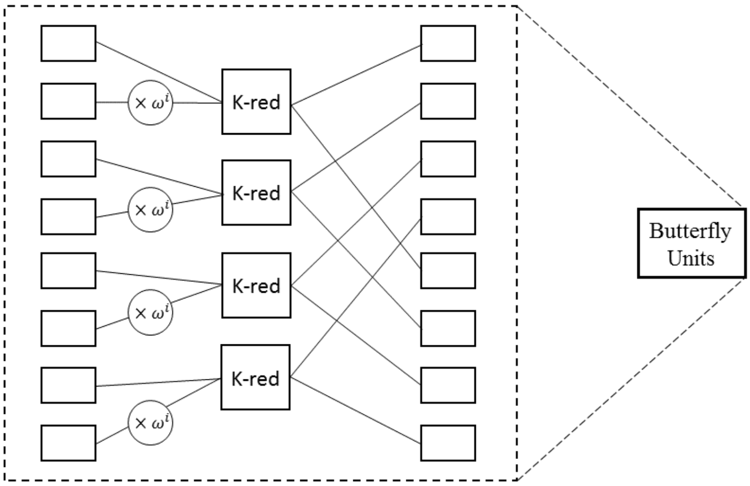


Fig. 3: Illustration of the design of the butterfly unit



Modular Reduction. A common way to do modular reduction is Barrett reduction.

$$c \mod p = c - \lfloor (c \cdot \tfrac{1 \ll 32}{12289}) \gg 32 \rfloor \cdot 12289$$

In this viewpoint, we can use DSP to multiply the reciprocal of 12289 without computing the floating number. Since the algorithm chops rather than rounds the result, the result is possibly slightly large than p. Thus, the algorithm subtracts p if it is larger than p in the final step. We can further improve the computation since $12289 = (1 \ll 13) + (1 \ll 12) + 1$ by following equation, where $\bar{a}$ is the complement of $\lfloor (c \cdot \frac{1 \ll 32}{12289}) \gg 32 \rfloor$

$$c \mod p = (c + (\bar{a} \ll 13)) + ((\bar{a} \ll 12) + \bar{a})$$

So a Barrett modular reduction with $q = 12289$ is around 5 cycles. But there is a multiplication between 32 bit- and 19-bit numbers leading to a long critical path and limiting the frequency.

Therefore we opt for the efficient reduction method from [LN16] for modular reduction. The method is a variant of Montgomery reduction with the auxiliary modulus k, which is defined by $q = k \cdot 2^m + 1$. For $q = 12289$, we have $m = 12$ and $k = 3$.

This algorithm is suitable for hardware implementation, since the operations in the function `K-RED` and `K-RED2x` are bit selections plus a final step which is equal to $(C_0 \ll 1) + C_0 - C_1$ and

function `K-RED(C)`
  $C_0 \leftarrow C \mod 2^m$
  $C_1 \leftarrow C/2^m$
  return $kC_0 - C_1$
end function

function `K-RED-2x(C)`
  $C_0 \leftarrow C \mod 2^m$
  $C_1 \leftarrow C/2^m \mod 2^m$
  $C_2 \leftarrow C/2^{2m}$
  return $k^2C_0 - kC_1 + C_2$
end function

$(C_0 \ll 3) + C_0 - (C_1 \ll 1) - C_1 + C_2$, respectively. Using this technique, we replace Line 5&6 in Algorithm 3 and get Algorithm 5.

Algorithm 5: Number-Theoretic Transform with `K-RED`

Parameter : ω is a primitive n-th root of unity in $\mathbb{Z}_q[X]$, n and q
Input : $\boldsymbol{a} \in \mathbb{Z}_q[X]/(X^n + 1)$
Output : $\boldsymbol{A} = NTT_\omega^n(\boldsymbol{a})$

1 $\boldsymbol{a} \leftarrow Order\_reverse(\boldsymbol{a})$
2 **for** $i = 0$ *to* $\log_2 n - 1$ **do**
3   **for** $j = 0$ *to* $n/2 - 1$ **do**
4     $P_{ij} \leftarrow \lfloor \frac{j}{2^{\log_2 n-1-i}} \rfloor \times 2^{\log_2 n-1-i}$
5     $U \leftarrow$ **K-RED**(a_{2j})
6     $V \leftarrow$ **K-RED2x**$(a_{2j+1}\omega^{P_{ij}})$
7     $A_j \leftarrow U + V$
8     $A_{j+n/2} \leftarrow U - V$
9   **if** $i \neq \log_2 n - 1$ **then**
10     $\boldsymbol{a} \leftarrow \boldsymbol{A}$
11 return A

We replace Line 3, 4, 8, 9 and 11 in Algorithm 4 to get Algorithm 6.

Note that `K-RED` function does not compute the exact value $C \mod q$ but $kC \mod q$. . Correspondingly `K-RED2x` function computes $k^2C \mod q$, and we eliminate the extra factor of k by storing $\omega_{ij}^P k^{-1}$ instead of ω_{ij}^P. Thus, after multiplication of $\omega_{ij}^P k^{-1}$ and `K-RED2x` function, the result kC has the correct value. Since $n = 1024 = 2^{10}$, there are ten stages in NTT function, the output vector from NTT with `K-RED` is $k^{10}v$, where v is the correct output vector of NTT. It is easy to transform the output vector to correct one, but we wait until the last step of INTT, which now becomes a final multiplication by the pre-computable $n^{-1}k^{-14}$.

One trick in the modified algorithm is to pre-compute $\phi^i k^{-(2+\log n)}$ instead of ϕ^i. This ensures that the output of our modified algorithm is exactly the same as that from the original NTT. We also replace $INTT_\omega^n$ by $NTT_{-\omega}^n$, and multiply instead by $n^{-1}\phi^{-i}$ (which can also be pre-computed and stored in the block RAM) in Line 11,. This way we only need 1024 multiplications.

Note that the output of both functions are bounded by not a fixed value but by $q + |C|/2^m$ which is related the input value C. Applying results of [LN16] to our algorithm, the input size of function `K-RED` and `K-RED2x` are 16 bits and 30 bits, respectively. One technique to maintain a plus sign for the output of these two functions (in order to multiply using DSP slices in the next stage) is to add multiples of $q = 12289$. It can be verified that $U + V$ and $U - V$ are larger than $-2q$ and $-4q$, respectively. But directly adding $2q$ and $4q$ to $U + V$ and $U - V$ causes a

Algorithm 6: Polynomial Multiplication using NTT with **K-RED** over $\mathbb{Z}_q[X]/(X^n+1)$

Parameter : ω is a primitive n-th root of unity in $\mathbb{Z}_q[X]$, $\phi^2 = \omega, n$, and q
Input : $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{Z}_q[X]/(X^n+1)$
Output : $\boldsymbol{c} = \boldsymbol{a} * \boldsymbol{b} \in \mathbb{Z}_q[X]/(X^n+1)$

Precompute: $\omega^i, \omega^{-i}, \phi^i, \phi^{-1}$, where $i = 0, 1, ..., n-1$
for $i = 0$ *to* $n-1$ **do**
  $\overline{a}_i \leftarrow$ **K-RED2x**$(a_i(\phi^i k^{-(2+\log n)}))$
  $\overline{b}_i \leftarrow$ **K-RED2x**$(b_i(\phi^i k^{-(2+\log n)}))$
$\overline{\boldsymbol{A}} \leftarrow NTT_\omega^n(\overline{\boldsymbol{a}})$
$\overline{\boldsymbol{B}} \leftarrow NTT_\omega^n(\overline{\boldsymbol{b}})$
for $i = 0$ *to* $n-1$ **do**
  $\overline{C}_i \leftarrow$ **K-RED2x**$(\overline{A}_i \overline{B}_i)$
$\overline{\boldsymbol{c}} \leftarrow NTT_{-\omega}^n(\overline{\boldsymbol{C}})$
for $i = 0$ *to* $n-1$ **do**
  $c_i \leftarrow$ **K-RED2x**$(\overline{c}_i(\phi^{-i} k^{-(4+\log n)} n^{-1}))$
return c

new problem: it may exceeds 16 bits. BRAM reads 64 bit at a time, so 17 bits as the input of `K-RED` slows each BRAM read to 3 data points.

Thus, we propose the method to solve the problem:
Let s be bit 11 (corresponding to 2048) of $a_{2j+1}\omega^{P_{ij}}$ in Line 6 in Algorithm 5.

If $s = 0$, $A_j \leftarrow U + V + 2q$ and $A_{j+n/2} \leftarrow U - V + 2q$.
If $s = 1$, $A_j \leftarrow U + V$ and $A_{j+n/2} \leftarrow U - V + 4q$.

Note that both sets of values are computed and then selected using s to avoid side-channels. This modification makes sure that the results of that step are positive. This method is a consequence of the properties of the `K-RED` and `K-RED2x` functions, and we give the proof in Appendix A. Note that the outputs of function `K-RED` and `K-RED2x` are signed 14 bits and signed 16 bits, respectively. Combined all the techniques describe above, the design of `K-RED` in the butterfly unit is shown in Figure 4.

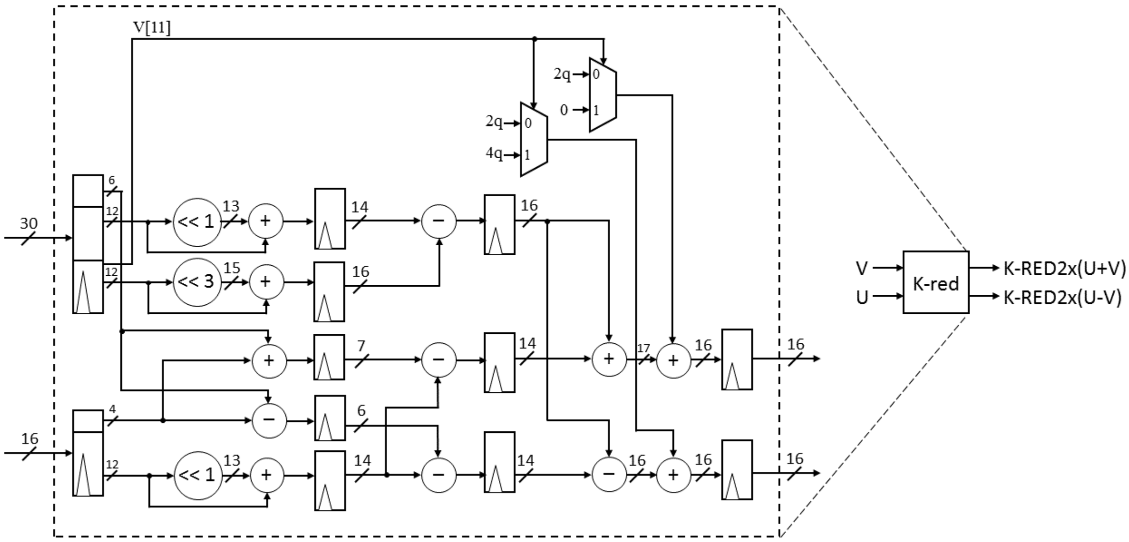
3.3 Reconciliation

A naive way to implement the `HelpRec` and `Rec` function on FPGA is to pre-compute $1/q$ and to use DSPs to compute the multiplication in runtime. This way is inefficient and wastes many logic elements. In our implementation of reconciliation, instead of trying to determine $\sum_{n=0}^{3} x_i/q < 1$ or not, we determine where $\sum_{n=0}^{3} x_i$ is less than q or not, in order to avoid floating-point number computation. Other divisors do not need this trick because they are all powers of 2.

4 Results

The three phases of key exchange cost 90.1, 162.4, and 80.8 μs, respectively. The resource consumption of each component is shown in Table 2. The area of PRNG (SHAKE from SHA-3) is quite large among the components. However, it is not the focus of this work. In theory we could have taken any FPGA SHA-3 implementation, such as the area-optimized one from [KDV+11] which only uses one tenth of the area. Alternatively, one can use a lightweight PRNG to generate

Fig. 4: illustration of the design of `K-RED`



the randomness for ψ_{16}.
The area of NTT component is reasonable since it is around 4 times that of [RVM+14]. Note that we use 4 butterfly units in each NTT component, and they use only one. Obviously, our implementation is the fastest post-quantum key exchange, which is 74% smaller and 62.7 times faster than that of SIDH [BK16].
In Fig 3, we also show the best record of hardware implementation of lattice-based PKE. At first glance, our results is worse than the hardware implementation of PKE. But the computation of NewHope is about 3.3 times larger than the computation of RLWE with $(p, q, \sigma) = (512, 12289, 4.92)$.

Runtime. The computation of NTTs dominates both schemes (in fact, NewHope has higher load because it has to expand a and compute `Rec` and `HelpRec`) Totally, NewHope has 6 NTT parts (include INTT) and RLWE has 4 NTT parts (include INTT). And considering that the size of the NTT is $n \log n$, the overall computation ratio is at least 3.3. The total time of our implementation is 333.3 μs, and the total time of RLWE(512,12289,4.92) is 58.9μs. Taking into account the relative sizes of computations, our implementation did 1.71 times slower than theirs. One reason is the maximum frequency is bounded by the critical path in PRNG component, other components all can go higher. Another is that the two primitives are different. For a public-key encryption scheme to provide forward secrecy, a one-time public key needs to be generated and transmitted every time before being used. That would probably make up much of the difference.

Area-time product. Ignoring the PRNG, we did about 6.1× worse in area-time product than RLWE$(512, 12289, 4.92)$, considering the computation effort. We emphasise that our implementation is not optimized for area-time product, and again, the structures of the algorithms are also much different.

Table 2: The resource consumption of each component

Component	Area					Clock Count
	#FFs	#LUTs	#Slices Registers	#DSPs	#BRAMs	
TRNG	130	310	258	0	0	1
PRNG (SHA-3)	2,958	4,848	2,963	0	0	24
pipelined NTT	2,934	2,832	1,317	12	10	2486
-multiply ϕ^i	-	-	-	-	-	132
-Order Reverse	-	-	-	-	-	1024
-Butterfly Units	-	-	-	-	-	1330
HelpRec	0	308	0	0	0	1
Rec	0	422	0	0	0	1

Table 3: Hardware comparison of post-quantum key exchange and some post-quantum public key encryption

Work	Scheme	Parameters	Security Parameter	Area				Time		
				#FFs	#LUTs	#DSPs	#BRAMs	Freq (MHz)	Latency ($\times 10^3$)	Total time (μs)
[KAKJ17]	SIDH	prime: 511 bits	128 bits	30,031	24,499	192	27	177	5,967	33,700
[BK16]	SIDH	prime: 503 bits	125 bits	26,659	19,882	192	40	181.4	3,800	20,900
This Work	NewHope	$n = 1024, p = 12289$, noise dist. ψ_{16}	128 bits	6,098	12,340	29	14	114	8.6/11.3 /2.8	75.4/99.1 /24.6
[RVM+14]	RLWE (PKE)	$n = 256, q = 7681$, $\sigma = 4.516$	80 bits	860	1,349	1	2	313	6.3/2.8	20.1/9.1
[RVM+14]	RLWE (PKE)	$n = 512, q = 12289$, $\sigma = 4.92$	128 bits	953	1,536	1	3	278	13.3/5.8	47.9/21
[PG13]	RLWE (PKE)	$n = 256, q = 7681$, $\sigma = 12.18$	80 bits	3,624	4,549	1	12	262	6.86/4.40	26.19/16.8
[HMO+16]	LWE (PKE)	$n = 256, q = 4096$, $\sigma = 3.39$	128 bits	4,804	6,152	1	73	125	98.3/32.8	786/262
[LW16]	NTRU $ees761ep1$	$n = 761, q = 2048$, $p = 3$	128 bits	#logic elements: 42,642, #registers: 16,746				75.36	0.44	5.89

However, as we mentioned in the introduction, the functionality of key transport is not the same as key agreement. Therefore, there is a need for a post-quantum key exchange scheme as well as its hardware implementation.

5 Results and Discussion

In this work, we proposed the first hardware implementation of lattice-based key exchange, which is also the fastest hardware implementation of post-quantum key exchange so far. Compare to the most optimized RLWE hardware implementation, our implementation did $1.71\times$ worse due to differences in the algorithm structures. To achieve perfect forward secrecy using PKE scheme, it needs to generate an ephemeral key each time. Thus to compare the key agreement speed fairly, key generation time needs to be accounted for as well. This code will be open-sourced after we clean it up.

5.1 Future Work

We will consider a pipelined butterfly unit to improve resource usage. Second, a countermeasures for side channel attacks (SCA) is an urgent priority. For example, we may use a method such as the masked RLWE decryption implementation resistant to first-order SCA is proposed in [RRVV15]. and apply it in our implementation. It is also interesting to optimize the SCA countermeasures for post-quantum key exchange scheme.

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A Proof of the input size and output size of `K-RED` butterfly

Here we consider the case of $q = 12289$, $k = 3$. The input U, V satisfies the following condition.

$$
\begin{aligned}
V &= V_0 + V_1 \cdot 2^{12} + V_2 \cdot 2^{24} \\
U &= U_0 + U_1 \cdot 2^{12}
\end{aligned}
$$

where

$$
\begin{aligned}
0 &\leq V_0 < 2^{12} \\
0 &\leq V_1 < 2^{12} \\
0 &\leq V_2 < 2^{6} \\
0 &\leq U_0 < 2^{12} \\
0 &\leq U_1 < 2^{4}
\end{aligned}
$$

$$
\begin{aligned}
A &= \texttt{K-RED}(U) = 3U_0 - U_1 \\
B &= \texttt{K-RED2x}(V) = 9V_0 - 3V_1 + V_2
\end{aligned}
$$

Thus, the output of the butterfly unit is:

$$
\begin{aligned}
A + B &= 9V_0 + 3U_0 + V_2 - (3V_1 + U_1) \\
A - B &= 3(U_0 + V_1) - (9V_0 + V_2 + U_1)
\end{aligned}
$$

Define $s = (V_0 \gg 11)\%2$; let's first consider $s = 0$, $V_0 < 2^{11}$.

$$
\begin{aligned}
9V_0 + 3U_0 + V_2 &\geq A + B \geq -(3V_1 + U_1) \\
9V_0 + 3U_0 + V_2 &\geq A + B \geq -(3V_1 + U_1) \\
9 \cdot 2^{11} + 3 \cdot 2^{12} + 2^{12} &\geq A + B \geq -(3 \cdot 2^{12} + 2^{12}) \\
2^{16} - 2q &\geq A + B > -2q \\
2^{16} &> A + B + 2q > 0
\end{aligned}
$$

$$
\begin{aligned}
A - B &\geq -(9 \cdot 2^{11} + 2^6 + 2^4) > -2q \\
A - B &\leq 3(U_0 + V_1) < 6 \cdot 2^{12} < 2^{16} - 2q \\
2^{16} &> B - A + 2q \geq 0
\end{aligned}
$$

We prove that when $s = 0$, adding $2q$ always make the output between 0 and 2^{16}

When $s = 1$, $V_0 \leq 2^{11}$

$$
2^{16} > A + B \geq 9 \cdot 2^{11} - (3V_1 + U_1) > 9 \cdot 2^{11} - 2^{14} > 0
$$

$$
\begin{aligned}
A - B &\geq -(9v_0 + V_2 + U_1) \geq -(9 \cdot 2^{12} + 2^{12} + 2^{12}) > -4q \\
A - B &\leq 3(U_0 + V_1) - 9 \cdot 2^{11} < 3(2^{12} + 2^{12}) - 9 \cdot 2^{11} < 2^{16} - 4q \\
2^{16} &> A - B + 4q \geq 0
\end{aligned}
$$

We prove that adding $4q$ to $B - A$ makes the output between 0 and 2^{16}.