

§ 1.3 克莱姆法则

讨论 n 个未知数 n 个方程的线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \cdots \cdots \cdots \cdots \cdots \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases} \quad (*)$$

的求解问题.

行列式 $D = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$ 称为方程组(*)的系数行列式.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases} \quad ---(*), \quad D = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}.$$

❖ 克莱姆法则

如果线性方程组(*)的系数行列式D不等于零，则方程组(*)有唯一解

$$x_1 = \frac{D_1}{D}, \quad x_2 = \frac{D_2}{D}, \quad \dots, \quad x_n = \frac{D_n}{D},$$

其中 $D_j (j=1, 2, \dots, n)$ 是把系数行列式D中第j列的元素 $a_{1j}, a_{2j}, \dots, a_{nj}$ 对应地换为方程组的常数项 b_1, b_2, \dots, b_n 后所得到的n阶行列式.

例9 解线性方程组

$$\begin{cases} 2x_1 + x_2 - 5x_3 + x_4 = 8 \\ x_1 - 3x_2 - 6x_4 = 9 \\ 2x_2 - x_3 + 2x_4 = -5 \\ x_1 + 4x_2 - 7x_3 + 6x_4 = 0 \end{cases}$$

解：系数行列式为：

$$D = \begin{vmatrix} 2 & 1 & -5 & 1 \\ 1 & -3 & 0 & -6 \\ 0 & 2 & -1 & 2 \\ 1 & 4 & -7 & 6 \end{vmatrix} = \begin{vmatrix} 0 & 7 & -5 & 13 \\ 1 & -3 & 0 & -6 \\ 0 & 2 & -1 & 2 \\ 0 & 7 & -7 & 12 \end{vmatrix} = - \begin{vmatrix} 7 & -5 & 13 \\ 2 & -1 & 2 \\ 7 & -7 & 12 \end{vmatrix} = 27$$

其中：

$$D_1 = \begin{vmatrix} 8 & 1 & -5 & 1 \\ 9 & -3 & 0 & -6 \\ -5 & 2 & -1 & 2 \\ 0 & 4 & -7 & 6 \end{vmatrix} = 81 \quad D_2 = \begin{vmatrix} 2 & 8 & -5 & 1 \\ 1 & 9 & 0 & -6 \\ 0 & -5 & -1 & 2 \\ 1 & 0 & -7 & 6 \end{vmatrix} = -108$$

$$D_3 = \begin{vmatrix} 2 & 1 & 8 & 1 \\ 1 & -3 & 9 & -6 \\ 0 & 2 & -5 & 2 \\ 1 & 4 & 0 & 6 \end{vmatrix} = -27 \quad D_4 = \begin{vmatrix} 2 & 1 & -5 & 8 \\ 1 & -3 & 0 & 9 \\ 0 & 2 & -1 & -5 \\ 1 & 4 & -7 & 0 \end{vmatrix} = 27$$

因此可得： $x_1 = 3, x_2 = -4, x_3 = -1, x_4 = 1$