5.3 线性2DOF汽车模型对前轮角输入的响应

Bicycle model or single track model

1 线性2DOF汽车模型的运动微分方程

☆忽略转向系的影响，以前轮转角作为输入；

☆忽略悬架作用，只在地面上做平面运动；

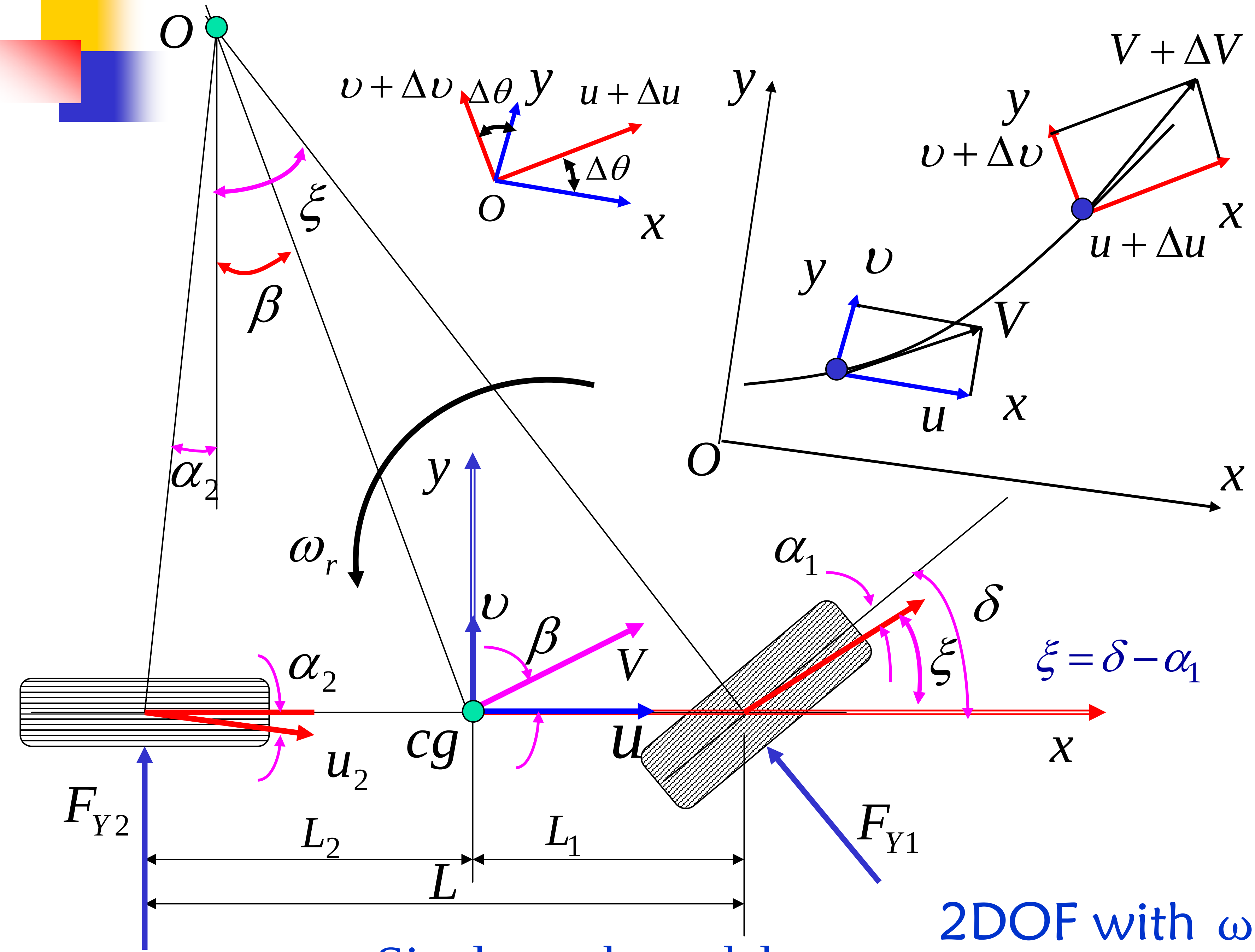
☆忽略因载荷变化引起左、右轮胎特性的变化；

☆驱动力不大，对侧偏特性无影响；

☆忽略回正力矩的变化；

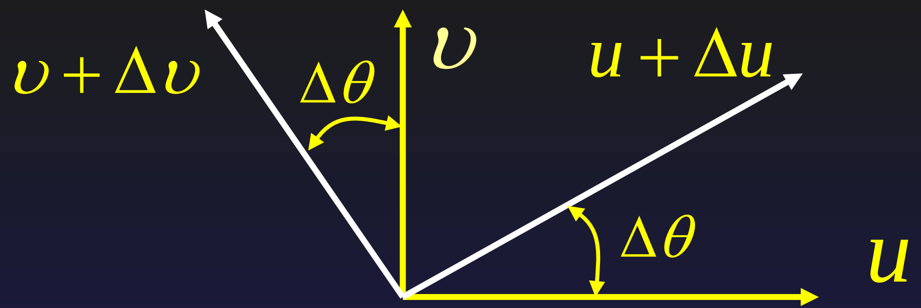
☆忽略空气阻力作用；

☆前进(纵轴)速度u_a不变，只有沿y轴的侧向速度 v 和绕z轴的横摆运动 ω_r，且a_y<0.4g.



2DOF with ω, υ

Single track model



$$u_0 = u, \upsilon_0 = \upsilon$$

$$[(u+\Delta u)\cos\Delta\theta - (\upsilon+\Delta\upsilon)\sin\Delta\theta] - u$$

$$\approx u + \Delta u - \upsilon\Delta\theta - \Delta\upsilon\Delta\theta - u \approx \underline{\Delta u - \upsilon\Delta\theta}$$

$$\underline{(\cos\Delta\theta \approx 1, \sin\Delta\theta \approx \Delta\theta)} \qquad \underline{\Delta\upsilon\Delta\theta \approx 0}$$

$$[(u+\Delta u)\sin\Delta\theta + (\upsilon+\Delta\upsilon)\cos\Delta\theta] - \upsilon$$

$$\approx u\Delta\theta + \Delta u\Delta\theta + \upsilon + \Delta\upsilon - \upsilon \approx \underline{u\Delta\theta + \Delta\upsilon}$$

$$\Delta u\Delta\theta \approx 0$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta u - \upsilon \Delta\theta}{\Delta t} \approx \dot{u} - \upsilon \omega_r$$

$$a_y = \lim_{\Delta t \to 0} \frac{u\Delta\theta + \Delta\upsilon}{\Delta t} = u\omega_r + \dot{\upsilon}$$

Y向力平衡

$$F_{Y1}\cos\delta + F_{Y2} = m(u\omega_r + \dot{\upsilon})$$

$$\Rightarrow F_{Y1} + F_{Y2} \approx m(u\omega_r + \dot{\upsilon})$$

对质心取矩

$$L_1 F_{Y1}\cos\delta - L_2 F_{Y2} = I_z \dot{\omega}_r$$

$$\Rightarrow L_1 F_{Y1} - L_2 F_{Y2} \approx I_z \dot{\omega}_r$$

$$F_{Y1}\cos\delta + F_{Y2} = m(u\omega_r + \dot{\upsilon})$$

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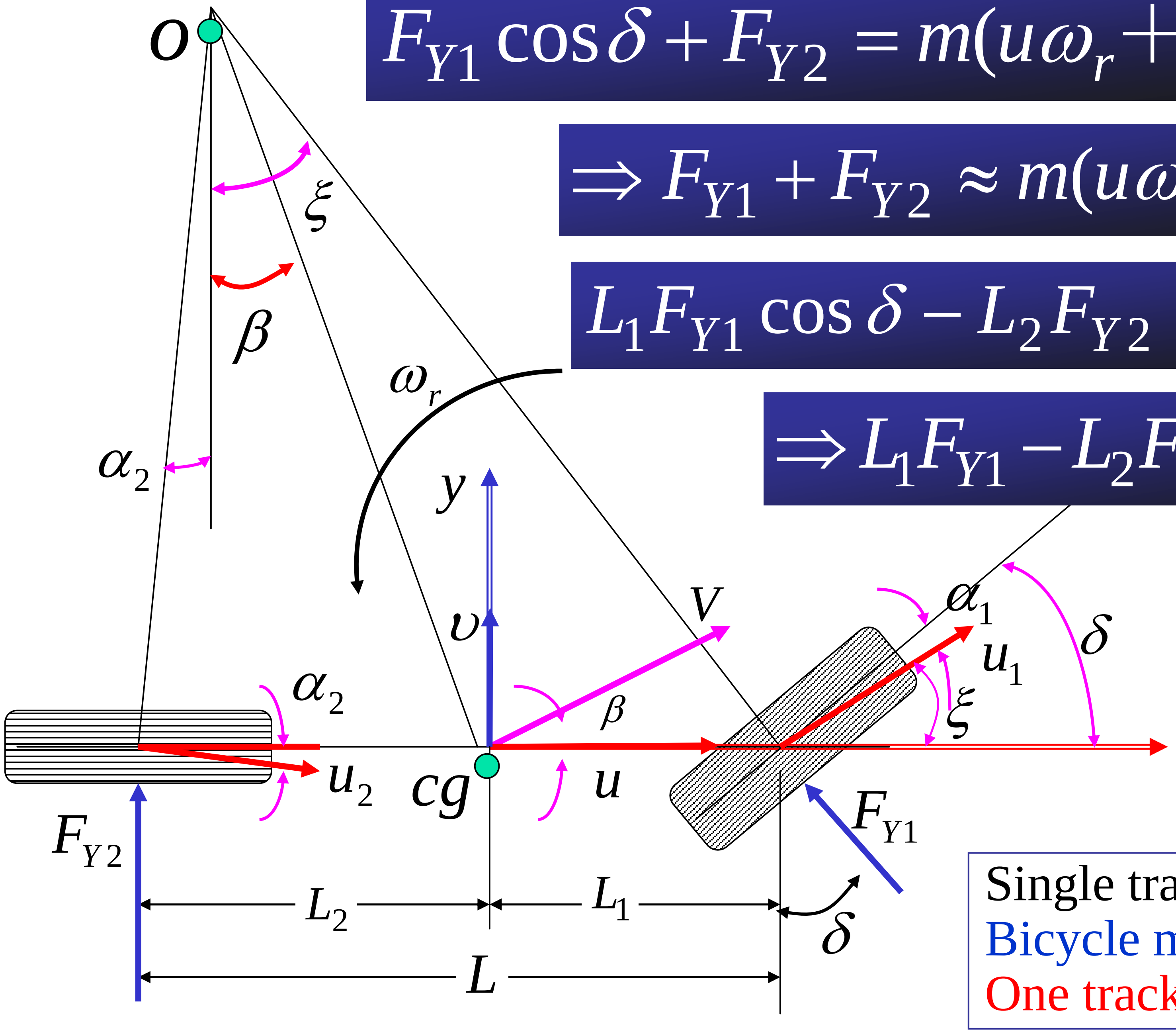
$$\Rightarrow L_1 F_{Y1} - L_2 F_{Y2} \approx I_z\dot{\omega}_r$$



$$\xi = \delta - \alpha_1$$

Single track model
Bicycle model
One track model

2DOF model

$$\tan\beta \approx \beta$$

$$\tan\xi \approx \xi$$

$$\tan\alpha \approx \alpha$$

$$\beta = \frac{\upsilon}{u}, \xi = \frac{\upsilon + L_1\omega_r}{u}$$

$$\alpha_1 = -(\delta - \xi) = \beta + \frac{L_1\omega_r}{u} - \delta$$

$$\alpha_2 = \frac{\upsilon - L_2\omega_r}{u} = \beta - \frac{L_2\omega_r}{u}$$

$$F_{Y1} = k_1\alpha_1$$

$$F_{Y2} = k_2\alpha_2$$

$$F_{Y1} + F_{Y2} \approx m(u\omega_r + \dot{\upsilon})$$

$$F_Y = ma_y$$

$$L_1F_{Y1} - L_2F_{Y2} \approx I_z\dot{\omega}_r$$

$$M_Z = I\dot{\omega}_r$$

$$\begin{cases} k_1\alpha_1 + k_2\alpha_2 = m(u\omega_r + \dot{\upsilon}) \\ L_1k_1\alpha_1 - L_2k_2\alpha_2 = I_z\dot{\omega}_r \end{cases}$$

$$\begin{cases} \alpha_1 = -(\delta - \xi) = \beta + \dfrac{L_1\omega_r}{u} - \delta \\ \alpha_2 = \dfrac{\upsilon - L_2\omega_r}{u} = \beta - \dfrac{L_2\omega_r}{u} \end{cases}$$

$$
\begin{cases}
(k_1 + k_2)\beta + (L_1 k_1 - L_2 k_2)\dfrac{\omega_r}{u} - k_1\delta = m(\dot{\upsilon} + u\omega_r) \\
(L_1 k_1 - L_2 k_2)\beta + (L_1^2 k_1 - L_2^2 k_2)\dfrac{\omega_r}{u} - L_1 k_1 \delta = I_z \dot{\omega}_r
\end{cases}
$$

角阶跃进入稳态转向

$$
\begin{cases}
\omega_r = \text{const.} \\
\dot{\upsilon} = 0 \\
\dot{\omega}_r = 0
\end{cases}
$$

$$
\beta = \frac{\upsilon}{u}
$$

Angle step

Steady response

Steady yaw velocity

Angle velocity gain

$$
\begin{cases}
(k_1+k_2)\dfrac{\upsilon}{u}+\dfrac{1}{u}(L_1k_1-L_2k_2)\omega_r-k_1\delta=mu\omega_r \\
(L_1k_1-L_2k_2)\dfrac{\upsilon}{u}+\dfrac{1}{u}(L_1^2k_1-L_2^2k_2)\omega_r-L_1k_1\delta=0
\end{cases}
$$

消除方程中 υ,则

$$
\left.\frac{\omega_r}{\delta}\right)_s=\frac{u/L}{1+\dfrac{m}{L^2}\left(\dfrac{L_1}{k_2}-\dfrac{L_2}{k_1}\right)u^2}=\frac{u/L}{1+Ku^2}
$$

稳定性因数

Stability factor

$$
\boxed{K=\frac{m}{L^2}\left(\frac{L_1}{k_2}-\frac{L_2}{k_1}\right)}
$$

稳定性因素K

$$K = \frac{m}{L^2}\left(\frac{L_1}{k_2} - \frac{L_2}{k_1}\right)$$

$$\begin{cases} m_1 = \dfrac{mL_2}{L} \\ m_2 = \dfrac{mL_1}{L} \end{cases} \longrightarrow \quad K = \frac{1}{L}\left(\frac{m_2}{k_2} - \frac{m_1}{k_1}\right)$$

$$K = \frac{1}{gL}\left(\frac{F_{z2}}{k_2} - \frac{F_{z1}}{k_1}\right)$$

2 稳态响应的三种类型

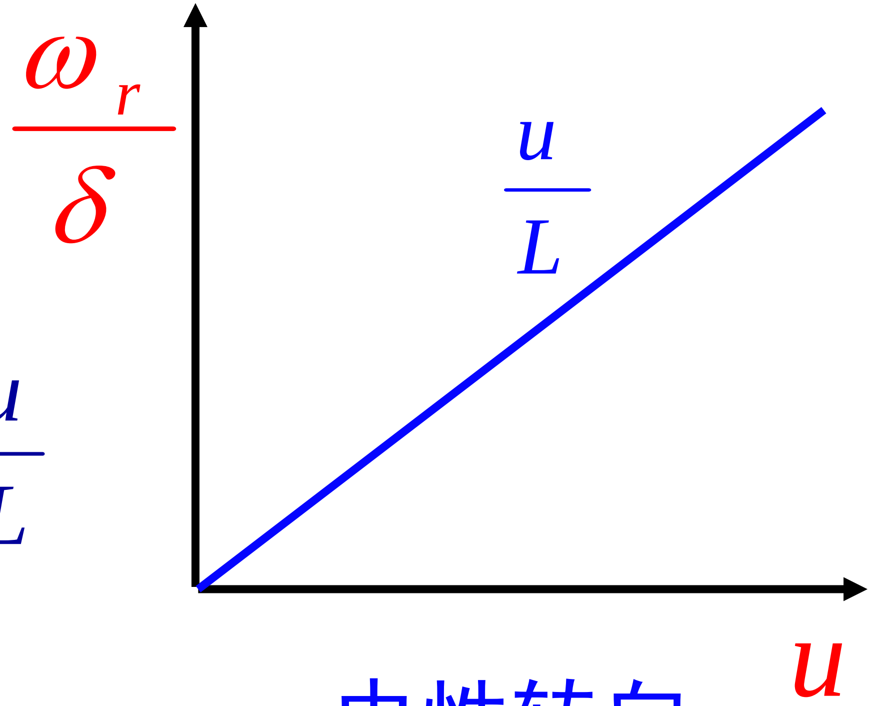
不足转向 Under- Steering	中性转向 Neutral-Steering	过度转向 Over-Steering

$$\left.\frac{\omega_r}{\delta}\right)_s = \frac{u/L}{1+Ku^2}$$

$$K = 0 \Rightarrow \left.\frac{\omega_r}{\delta}\right) = \frac{u}{L}$$

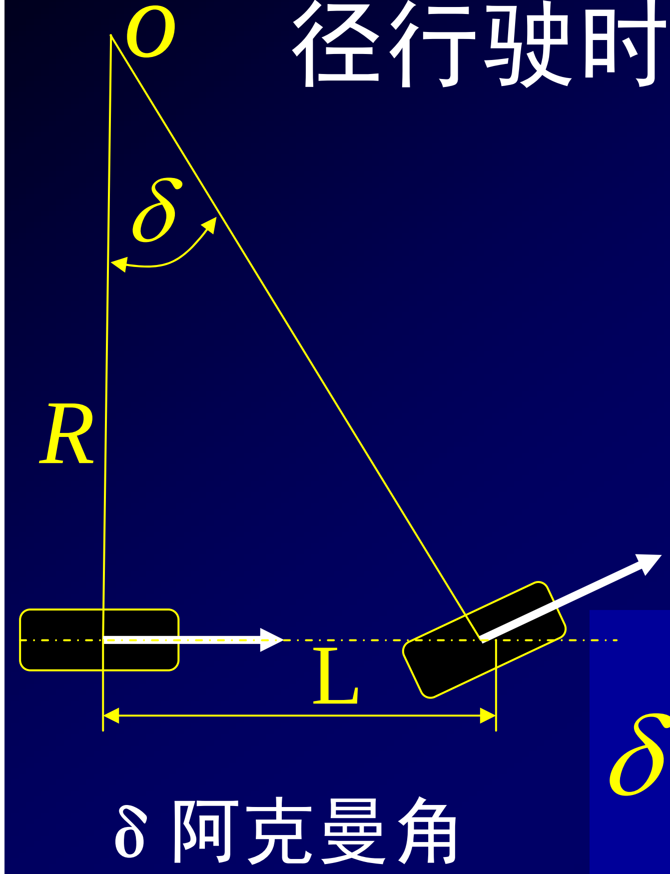
$$u \uparrow \Rightarrow \frac{\omega_r}{\delta} \uparrow$$

在实践中



中性转向

当汽车以很低的速度和/或很大转向半径行驶时，侧偏角很小，有



$$\alpha_1 = \alpha_2 \approx 0, \xi \approx \delta$$

i.e. $\tan\delta \approx \delta$

$$\delta \approx \frac{L}{R}, R \approx \frac{L}{\delta}, \omega_r = \frac{u}{R} = \frac{u}{L}\delta$$

阿克曼几何关系 Ackermann’s Geometry

特征车速

不足转向时， $K > 0$,

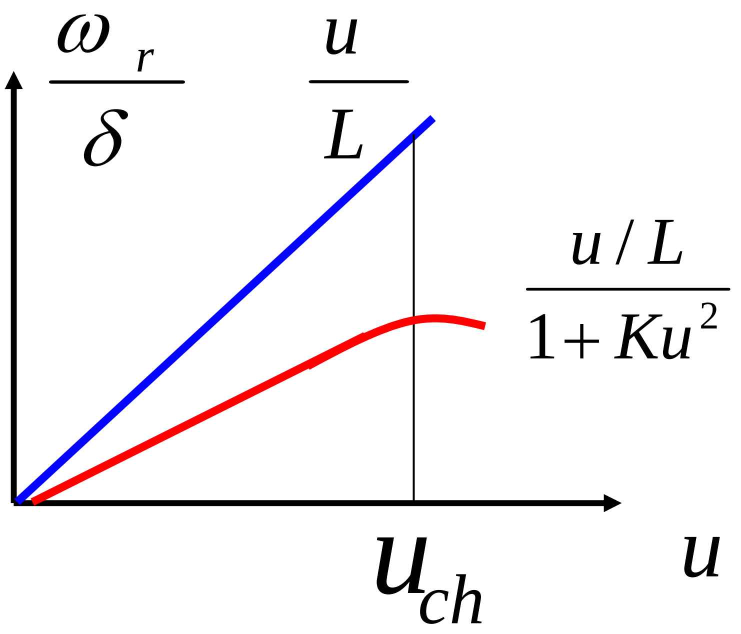
$\left.\frac{\omega}{\delta}\right)$是随$u_a$向下弯曲的曲线，

首先$u\uparrow \Rightarrow \left.\frac{\omega_r}{\delta}\right)\uparrow$，然后$\left.\frac{\omega_r}{\delta}\right)\downarrow$

当$u_{ch} = \sqrt{\frac{1}{K}}$时(特征车速)，

此时$\left.\frac{\omega_r}{\delta}\right)_{K>0} = \frac{1}{2}\left.\frac{\omega_r}{\delta}\right)_{K=0}$

$$\left.\frac{\omega_r}{\delta}\right)_s = \frac{u/L}{1+Ku^2} < \frac{u}{L}$$



Characteristic velocity

临界车速

过度转向时，$K<0$，

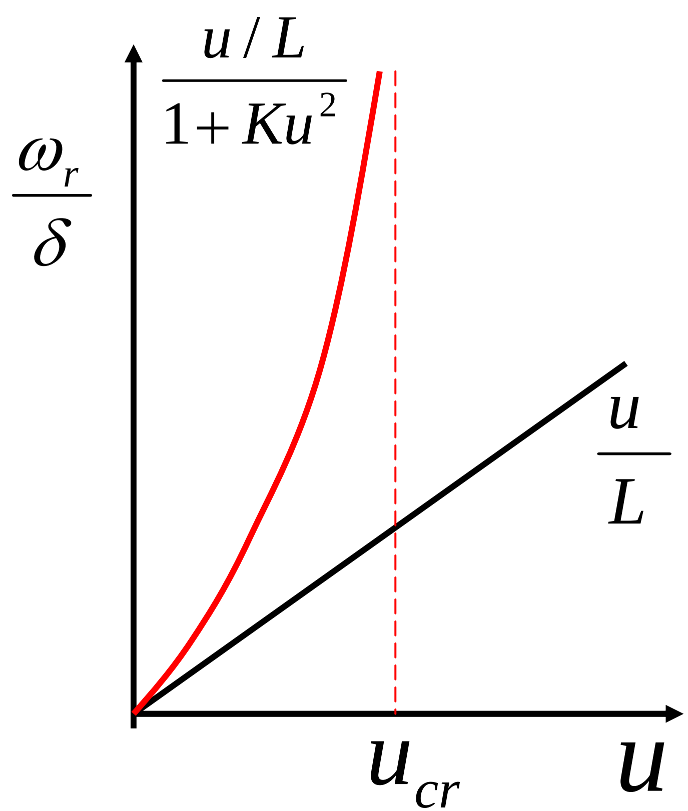
$\left.\frac{\omega}{\delta}\right)$是随$u_a$向上弯曲的曲线，

$|K|\uparrow \Rightarrow \left.\frac{\omega_r}{\delta}\right)\uparrow$，

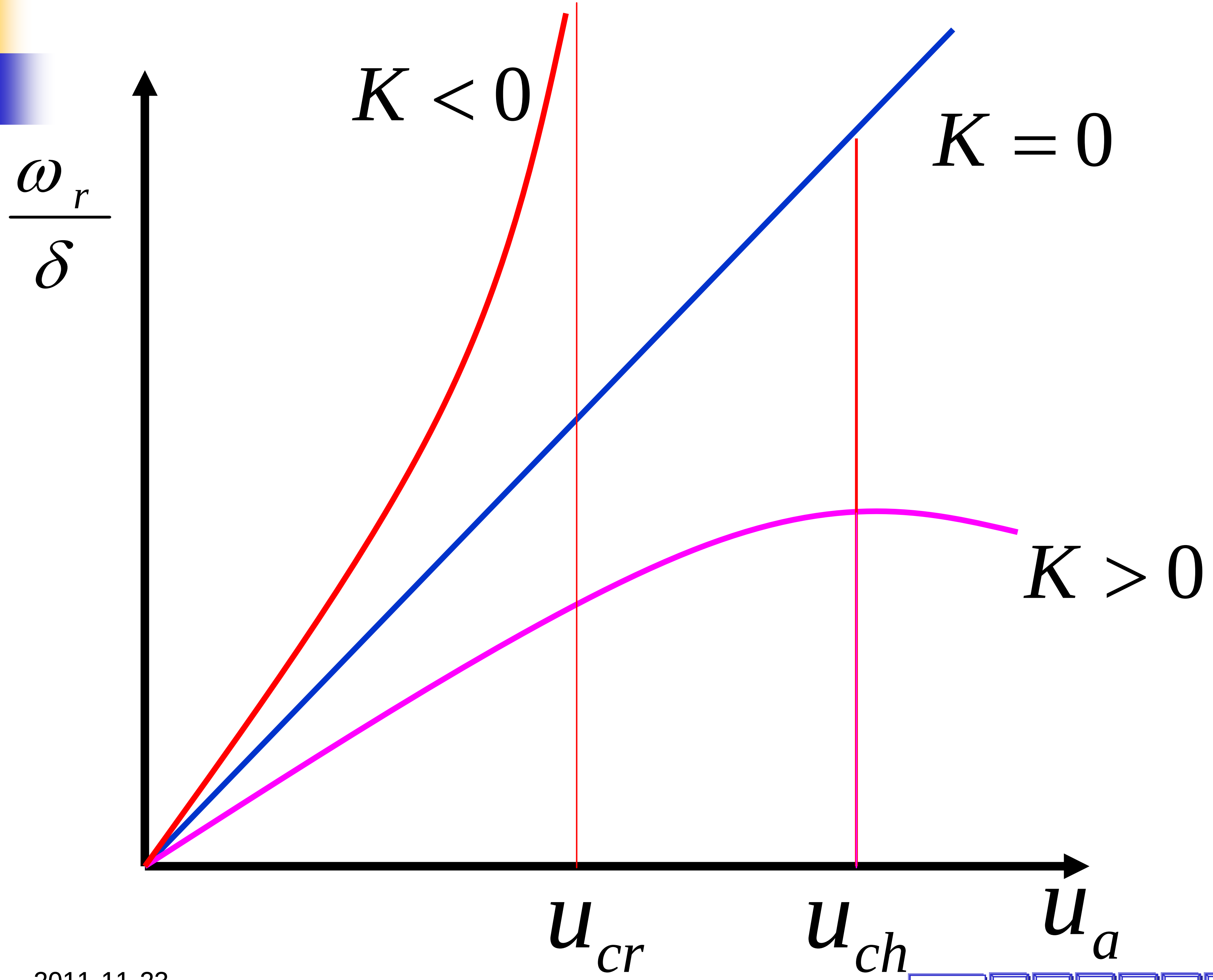
$$\left.\frac{\omega_r}{\delta}\right)_{K<0} > \left.\frac{\omega_r}{\delta}\right)_{K=0}$$

当$u_{cr}=\sqrt{-\frac{1}{K}}$时，$\left.\frac{\omega_r}{\delta}\right)\to\infty$

$$\left.\frac{\omega_r}{\delta}\right)_s = \frac{u/L}{1+Ku^2} > \frac{u}{L}$$



Critical velocity


$K<0$
$K=0$
$K>0$
$\frac{\omega_r}{\delta}$
u_{cr}
u_{ch}
u_a

过度转向特性的问题

- 过度转向汽车车速达到临界车速时将失去稳定性。此时只要一个很小的转角δ，横摆角速度增益ω_r/δ就趋于无穷大。
- 因纵向速度是优先值，根据纵向速度与角速度的关系可知，汽车转向半径迅速变得极小。这样，汽车必定发生激转，导致侧滑或侧翻的发生。

乘用车　Passenger Car

$$a_y = 0.3 \sim 0.4g, K \approx 0.0020 \sim 0.0035\,\mathrm{s}^2/\mathrm{m}^2$$

$$a_y = 0.4g, u = 22.35\,\mathrm{m/s}^1, \frac{\omega_r}{\delta} = 0.16 \sim 0.33\mathrm{s}^{-1}$$

3 稳态响应的参数

$\alpha_1-\alpha_2$与a_y的关系

$$F_{Y1}=\frac{mL_2}{L}a_y$$

$$F_{Y2}=\frac{mL_1}{L}a_y$$

$$K=\frac{m}{L^2}\left(\frac{L_1}{k_2}-\frac{L_2}{k_1}\right)=\frac{ma_y}{L^2a_y}\left(\frac{L_1}{k_2}-\frac{L_2}{k_1}\right)$$

$$=\frac{1}{La_y}\left(\frac{F_{Y2}}{k_2}-\frac{F_{Y1}}{k_1}\right)=\frac{1}{La_y}(\alpha_1-\alpha_2)$$

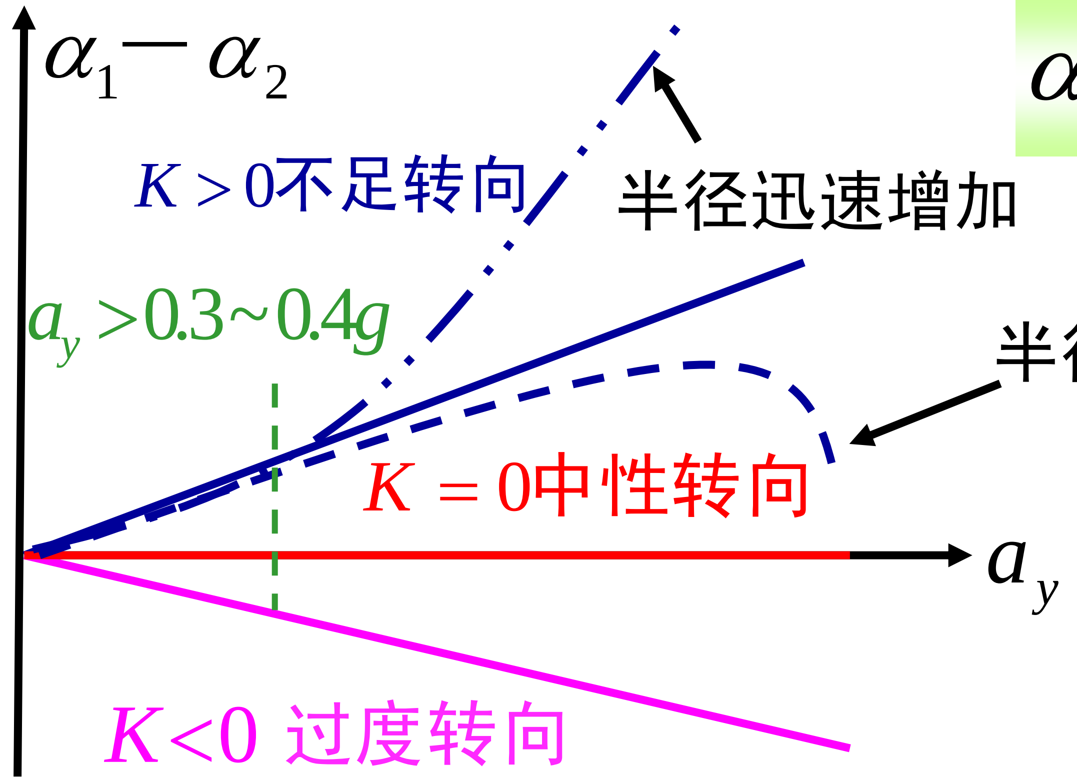
$$\alpha_1-\alpha_2=Ka_yL$$

$$\alpha_1-\alpha_2>0\Rightarrow K>0$$

$$\alpha_1-\alpha_2=0\Rightarrow K=0$$

$$\alpha_1-\alpha_2<0\Rightarrow K<0$$

m: vehicle mass



$$\alpha_1 - \alpha_2 = K a_y L$$

$a_y > 0.3 \sim 0.4g$，$\alpha_1 - \alpha_2$与a_y不再为线性关系。

α和ω_r急剧变化，出现半径迅速增加或减小的现象。

a_y对$\alpha_1 - \alpha_2$关系用斜率表示，斜率$>0 \Rightarrow$不足转向;

斜率$=0 \Rightarrow$中性转向;斜率$<0 \Rightarrow$过度转向。

$\alpha_1 - \alpha_2$与R的关系

$$L\frac{\omega_r}{u} + LKu\omega_r = \delta$$

$$\frac{\omega_r}{\delta} = \frac{u/L}{1+Ku^2} \Rightarrow \omega_r + Ku^2\omega_r = \delta u / L \rightarrow / \frac{u}{L}$$

$$u\omega_r = a_y, R = u/\omega_r$$

$$Ka_yL = \alpha_1 - \alpha_2$$

$$\delta = \frac{L}{R} + LKa_y = \frac{L}{R} + (\alpha_1 - \alpha_2)$$

$$\alpha_1 = 0, \alpha_2 = 0$$

$$R = \frac{L}{\delta - (\alpha_1 - \alpha_2)}$$，若u很小，则$R_0 = \frac{L}{\delta}$ 阿克曼几何关系

$u\uparrow \Rightarrow \alpha_1 - \alpha_2$发生变化或不变

$$R = \frac{L}{\delta - (\alpha_1 - \alpha_2)} \qquad R_0 = \frac{L}{\delta}$$

$$\alpha_1 - \alpha_2 = 0 \quad \Rightarrow \quad \delta - (\alpha_1 - \alpha_2) = \delta, \quad R = R_0$$

$$\alpha_1 - \alpha_2 > 0 \quad \Rightarrow \quad \delta - (\alpha_1 - \alpha_2) < \delta, \quad R > R_0$$

$$\alpha_1 - \alpha_2 < 0 \quad \Rightarrow \quad \delta - (\alpha_1 - \alpha_2) > \delta, \quad R < R_0$$

转向半径比值

Steering sensitivity

ω_r yaw angular velocity

$$\frac{\omega_r}{\delta} = \frac{u/L}{1+Ku^2}$$

$$\frac{u}{\omega_r} = \frac{(1+Ku^2)L}{\delta}$$

$$R = (1+Ku^2)R_0$$

$$\frac{R}{R_0} = 1+Ku^2$$

$$R_0 = \frac{L}{\delta}$$

L: wheel base

R: curve radius

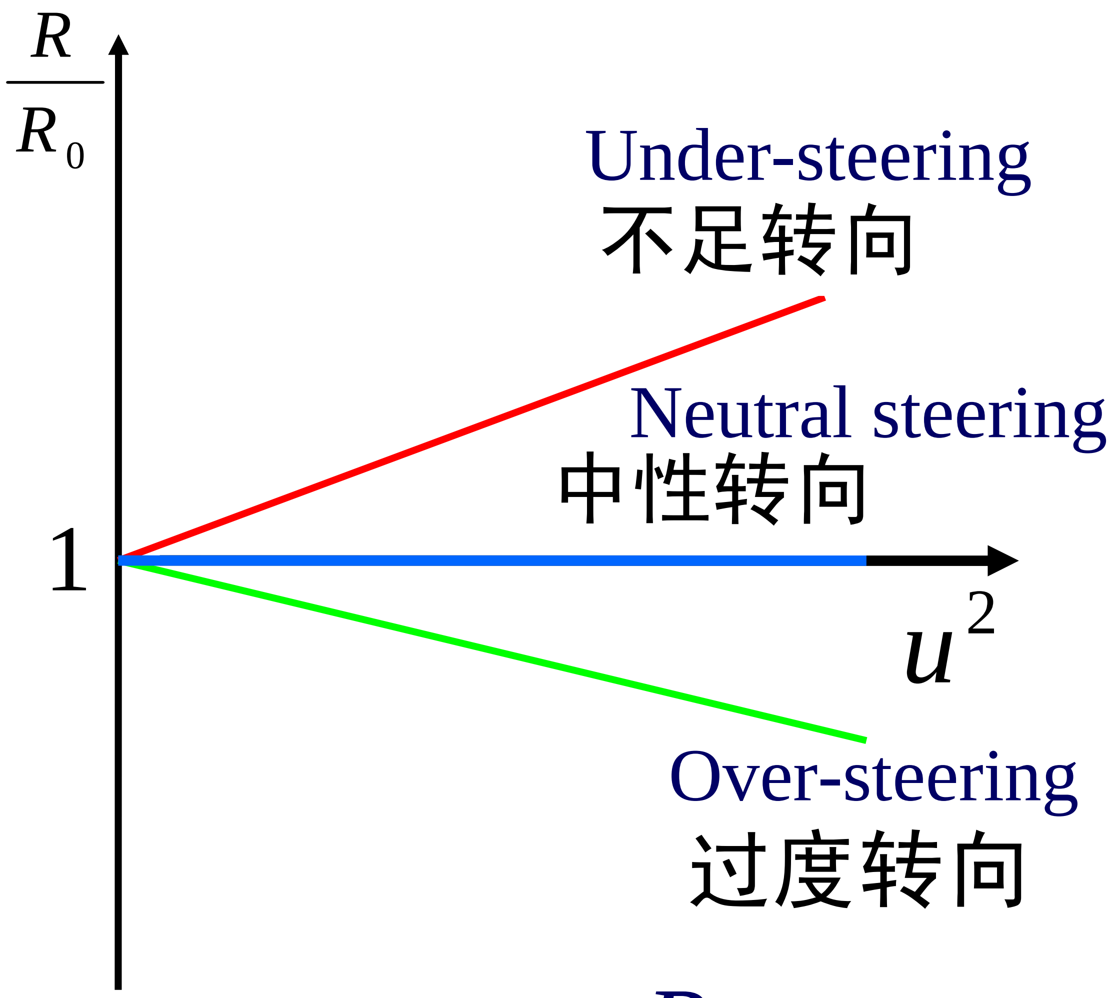
δ : cornering angle

$$\frac{R}{R_0} = 1 + Ku^2$$

$$K > 0 \Rightarrow \frac{R}{R_0} > 1$$

$$K = 0 \Rightarrow \frac{R}{R_0} = 1$$

$$K < 0 \Rightarrow \frac{R}{R_0} < 1$$



Relation: $\frac{R}{R_0} - u^2$

静态裕度

静态储备系数

$$\alpha_1 = \alpha_2$$

使汽车前后轮产生相同侧偏角的侧向力作用点 $\Rightarrow$ 中性转向点 C_n

静态裕度 $$SM = \frac{L_1' - L_1}{L}$$

static margin

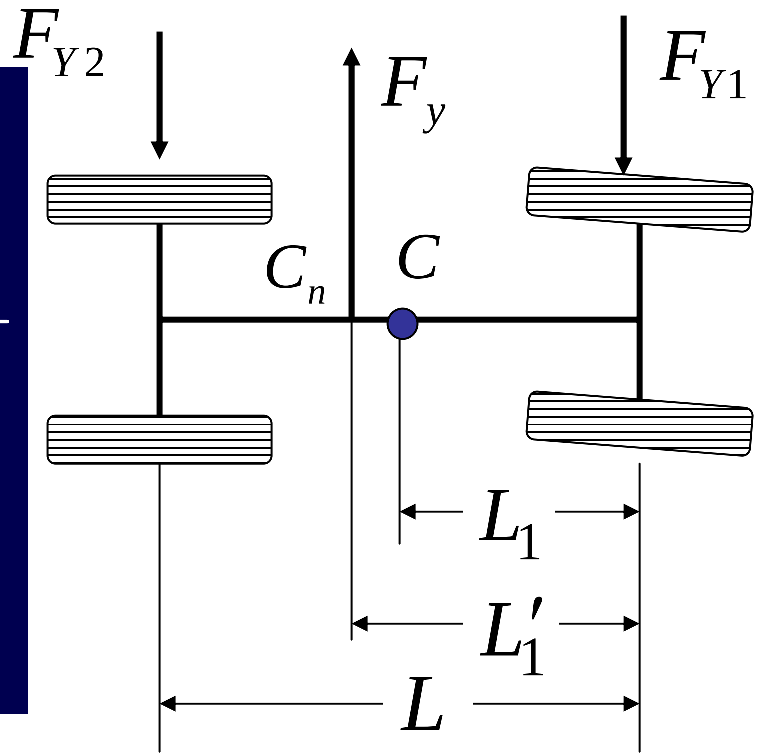
L_1' – 中性转向点至前轴的距离;

L_1 – 质心至前轴的距离。

同时对前轴取矩，则

$$L_1' = \frac{F_{Y2}L}{F_{Y1}+F_{Y2}} = \frac{k_2\alpha L}{k_1\alpha + k_2\alpha}$$

$$= \frac{k_2 L}{k_1 + k_2} \qquad \alpha_1 = \alpha_2 = \alpha$$



$$SM = \frac{L_1' - L_1}{L} = \frac{k_2}{k_1+k_2} - \frac{L_1}{L} = \frac{k_2L_2 - k_1L_1}{L}$$

$$F_y = -F_Y$$

$$SM = 0 \Rightarrow L_1' - L_1 = 0 \Rightarrow \alpha_1 = \alpha_2$$

$$F_Y = F_{Y1} + F_{Y2}$$

$$SM > 0 \Rightarrow L_1' - L_1 > 0 \Rightarrow \alpha_1 > \alpha_2$$

$$SM < 0 \Rightarrow L_1' - L_1 < 0 \Rightarrow \alpha_1 < \alpha_2$$