

## § 1.1 二阶、三阶行列式

(一)二阶行列式

(二)三阶行列式

## (一)二阶行列式

### 二阶行列式

我们用记号  $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$  表示代数和  $a_{11}a_{22}-a_{12}a_{21}$ , 称为二阶行列式, 即

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

### 对角线法则

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$-a_{12}a_{21}$        $+a_{11}a_{22}$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

**例 1**  $\begin{vmatrix} 5 & -1 \\ 3 & 2 \end{vmatrix} = 5 \times 2 - (-1) \times 3 = 13$

**例 2** 设  $D = \begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix}$ . 问: (1) 当  $\lambda$  为何值时  $D=0$ . (2) 当  $\lambda$  为

何值时  $D \neq 0$ .

**解:**  $D = \begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} = \lambda^2 - 3\lambda = \lambda(\lambda - 3)$

(1) 当  $\lambda=0$  或  $\lambda=3$  时,  $D=0$ .

(2) 当  $\lambda \neq 0$  且  $\lambda \neq 3$  时,  $D \neq 0$ .

## (二)三阶行列式

### 三阶行列式

我们用记号  $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$  表示代数和

$$a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}-a_{11}a_{23}a_{32}-a_{12}a_{21}a_{33}-a_{13}a_{22}a_{31}$$

称为三阶行列式, 即

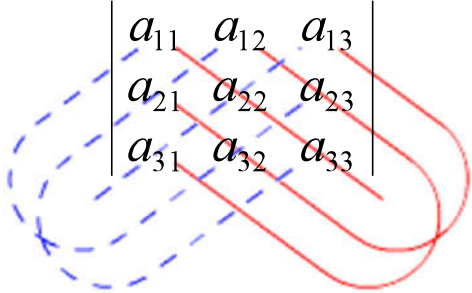
$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32}-a_{12}a_{21}a_{33}-a_{13}a_{22}a_{31}$$

## (二)三阶行列式

### 三阶行列式

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$

### 对角线法则


$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$      $a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}$

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$

例 3

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix}$$

$$= 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 - 1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1)$$

$$= -10 - 48 = -58$$

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$

**例 4** 实数  $a, b$  满足什么条件时有  $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 0 & 1 \end{vmatrix} = 0$ ?

**解:**  $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 0 & 1 \end{vmatrix} = a^2 + b^2$

$a, b$  为实数, 若要  $a^2 + b^2 = 0$ , 则  $a$  与  $b$  须同时等于零. 因此, 当  $a=0$  且  $b=0$  时, 给定的行列式等于零.

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$

**例 5**  $D = \begin{vmatrix} a & 1 & 0 \\ 1 & a & 0 \\ 4 & 1 & 1 \end{vmatrix} > 0$  的充分必要条件是什么?

**解:**  $D = \begin{vmatrix} a & 1 & 0 \\ 1 & a & 0 \\ 4 & 1 & 1 \end{vmatrix} = a^2 - 1$

当且仅当  $a^2 - 1 > 0$ , 即  $|a| > 1$  时,  $D > 0$ , 因此可得  $D > 0$  的充分必要条件是  $|a| > 1$ .