A Heuristic for Commodity Procurement in the Presence of Price Discounting

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Abstract

Procuring commodities is difficult due to the fluctuating prices intrinsic to the value of commodities. These price fluctuations can allow a firm to benefit from buying for future demand, as well as current demand, when prices are low. Additionally, obtaining large volumes at one time may allow a firm to take advantage of volume price discounts. We provide a heuristic to determine how much to buy at each purchasing opportunity in order to maximize expected profit. We compare our method with existing methods through simulation by using real plywood data from BlueLinx, a two-stage distributor of building products. We find that our heuristic performs better than existing methods for all tested settings of volume surcharge, no discounts, and volume discounts.

Key words: Procurement, Commodities, Newsvendor, Volume Discounts

1. Introduction

A two-stage distributor is a company that purchases bulk commodities and then sells smaller truckloads to customers as requested. The customers do not procure commodities directly because they 1) do not purchase enough volume at one time to satisfy the minimum mill quantity requirement, or 2) they do not want to give up the flexibility of shipment size and destination that is absorbed by the two-stage distributor.

The two-stage distributor earns a stable portion of its margin from the value added by absorbing lead-time, breaking bulk, and, in the case of Bluelinx, brand premium. However, a highly variable portion of either profit or loss is derived from the purchase price of commodities rather than the selling price. In this paper, we model a two-stage distributor that has a purchasing opportunity at a known price with forecasts for future demands and known distributions for future prices. The distributor needs to decide if they will buy more than the minimum requirement to satisfy demand in the current period and to maximize expected profits.

We tie together two existing methods for determining order up to levels and we use simulated sales data from BlueLinx for the years 2001-2005 to demonstrate the effectiveness of our proposed heuristic.

1.1 BlueLinx Purchasing Environment

The conditions for the data simulation are based on discussions about procurement practices with the current and former directors of supply chain procurement at BlueLinx Corporation.

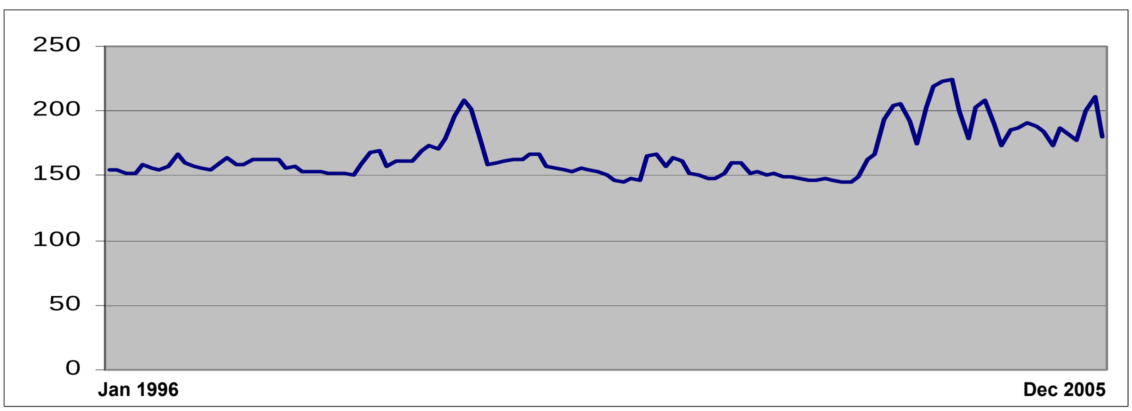
Condition 1: BlueLinx is a price taker. It exists in a highly fragmented market where the largest player comprises only 10% of the total market and where there are many small players with no influence at all. Moreover, the company has little price flexibility. Selling prices cannot be raised to cover prior high priced purchases; rather, selling prices are an adder on top of current market prices.

Condition 2: Demand is stochastic with known distribution, but it is not constant in time. The shape of the demand curve can be derived from historical sales while the reasonable forecast error can be calculated using Holt-Winters with additive seasonality.

Condition 3: The demand forecast is unbiased. That is, tracking signals demonstrate that the Holt-Winters forecasting method is unbiased for the products at BlueLinx.

Condition 4: The purchase price of plywood fluctuates often as one can see in Figure 2 below.

Figure 2: Historical Plywood Prices 1996-2005



Condition 5: The lead-time for international sourcing is significant. Either plywood is purchased internationally, with significant lead times, or it comes from domestic mills at a price premium.

Condition 6: There are no viable substitute products. Builders have specifications calling for certain materials and so they will not use different grades or variants. If the company is out of a particular commodity, it cannot fill demand with a substitute product; for example, a builder requiring Oriented Strand Board (OSB) would not substitute plywood for their application.

1.2 Forecasts

We use Holt-Winters with additive seasonality to forecast future demand. Given the seasonal nature of building products, this method fits the invoice data well. We apply the seasonal indices to the plywood that BlueLinx had computed based on the prior four years of sales. The smoothing parameters are kept constant during the simulation as the company does not frequently adjust them in their forecasting process.

We are able to fit autoregressive functions to historical prices for plywood that achieved normally distributed errors and low Mean Absolute Deviations. ARIMA (3,0,0) worked best for the wood price data. The plywood data is from January 1996 through May 2001. Our simulation starts with June 2001 so that we can be certain the price forecast parameters are based only on the historical data that would have been available at the time. For wood, the last known price -.1*(two periods ago) + .1 * (three periods ago) produces the lowest error.

2. Literature and Modeling Review

Price discounts are discussed in the literature in several ways. One stream of literature addresses only the largest valid Economic Order Quantity (EOQ) and the price breaks associated with that EOQ (Hadley and Whitin, 1963). For example, Rubin, Dilts and Barron (1983) show that if the EOQ falls within a certain quantity discount interval, only EOQs associated with discount prices higher than this interval need be checked. Fogarty and Hoffman (1983) quantify the discount problem by trading off the Return On Investment (ROI) tied up in excess inventory compared to the extra investment obtained from taking advantage of the volume discount for the EOQ. Krupp (1985) extends this research by computing the ROI not just for the EOQ but for all quantity discount intervals. Patterson (1989) extends Krupps's contribution by considering the rate of return for the cost savings that results from increasing the current order size to the next price break quantity. To demonstrate the effectiveness of our method we focus on a single price break point, with no lot sizing restrictions or discounting.

Two types of uncertainty are prominent in buying and selling commodities: 1) purchase price uncertainty and 2) demand uncertainty. Golabi (1985) and Magirou (1982) assume that demand is deterministic but that forward buying can be done based upon the current realized purchase price compared to future expected prices (that are partly determined by taking into account holding costs). Demand uncertainty is addressed in a paper by Gavirneni (2004) where he uses a modified newsvendor to cover non-perishable demand. The holding cost is included in the cost of overage. The ratio also takes into account the difference between the current realized price versus the expected price. We will discuss these two methods and give an example of each in the following sections.

2.1 Forward Buying with Deterministic Future Demand

Golabi (1985) proposes a method whereby material for future periods is bought as long as the marginal cost is less than the marginal savings. His recursive heuristic yields a series of decreasing thresholds corresponding to the number of periods to buy forward. Ordering prices in each period are random with a known distribution. Magirou (1987) comments on the similarity between his 1982 paper and Golabi's 1985 work. Golabi's equation accounts for the probability that the next period price will be less than the current price plus the benefit of locking in the prior price minus the holding costs of one period. As there was a typographical error in the Golabi publication, Equation (1) below is the corrected equation (9) from Golabi's paper that specifies the next price point such that forward buying *n* periods is optimal. A_n is the threshold price per unit such that buying *n* periods ahead is optimal. If the current purchase price is less than or equal to A_n, then it is optimal to buy for the current period and *n* periods ahead. Let *x* be the purchase price and F(*x*) is the known cumulative price distribution for each period (in equation 1 below dF(*x*) is equivalent to f(*x*)*dx* the probability density function). *h* is the cost to hold one unit of stock for one period. A_0 is the highest possible purchase price since Golabi assumes all demand must be met for the current period (period 0). Each additional threshold price is computed according to equation (1) below recursively..

$$A_{n+1} = \int_0^{A_n} x dF(x) + \int_{An}^{\infty} A_n dF(x) - h \qquad \textbf{(1)}$$

Given the probability that the price falls in the future, the first integral in equation 1 above is the opportunity cost of buying now versus buying at a possible lower cost next period. The second integral is the benefit of locking in at the A_n purchase cost. The final term is the holding cost for buying inventory in period n for use in period $n+1$. We will now illustrate this heuristic with stationary, uniform price distributions. Assume that the price at any buying opportunity can be \$25, \$50, \$75 or \$100 – each with equal probability. Assume the holding cost for one period is \$5.

Since we must buy to cover demand in the current period (0), A_0 equals the highest possible price. Thus $A_0 = 100$, the highest possible purchase price of our distribution.

A_1 is the expected price that is lower than or equal to A_0 plus the benefit of locking in at the price A_0 minus the one period holding cost h.

The expected price lower or equal to A_0 is \$25 * 25% + \$50 * 25% + \$75 * 25% + \$100 * 25% = \$62.50

The expected benefit of locking in the price of A_0 is \$0 (since the price cannot go higher than \$100). A_1 = \$62.50 + \$0 - \$5 = \$57.50. If the current purchase price is \$57.50 or lower, we should buy for the current period demand plus the demand for next period.

Likewise, A_2 = \$18.75 + \$28.75 - \$5 = \$42.50, A_3 = \$6.25 + \$31.88 - \$5 = \$33.13,

A_4 = \$6.25 + \$24.84 - \$5 = \$26.09, and A_5 = \$19.57 which is below the possible price range for the distribution so we can be certain that we will never buy for more than four periods in advance.

This method seeks to answer how many periods in advance we should buy to satisfy all predicted demand and to minimize total expect costs. Given our price distribution, Table 1 shows the current purchasing prices that make sense for forward buying without any volume discounts.

Table 1: Price Thresholds to Forward Buy Without Discounts

	Calculated Value	$ 25.00	$ 50.00	$ 75.00	$ 100.00
A0	$ 100.00	Yes	Yes	Yes	Yes
A1	$ 57.50	Yes	Yes		
A2	$ 42.50	Yes			
A3	$ 33.13	Yes			
A4	$ 26.09				

We now demonstrate the calculations under volume price discounts. We again assume that the list price at any buying opportunity can be $25, $50, $75 or $100 – each with equal probability. Assume the holding cost for one period is $5. We have the following threshold price breaks shown in Table 2.

Table 2: Quantity Discount Thresholds

Threshold	% Discount
250	10%
300	15%
350	20%

As an example, assume the forecasts for the next 6 months are as shown in Table 3 where 0 is the current month, 1 denotes one period in the future, etc.:

Table 3: Example Monthly Demand Forecasts

Period	**0**	**1**	**2**	**3**	**4**	**5**
Demand	150	110	105	90	175	190

To account for volume discounts and surcharges, we modify (1) as below, where Δd is the change in discount rate from the last threshold A_n.

$$A_{n+1} = \int_{0}^{\frac{A_n}{1-\Delta d}} x dF(x) + \int_{\frac{An}{1-\Delta d}}^{\infty} \frac{A_n}{1-\Delta d} dF(x) - h \quad \textbf{(2)}$$

Using (2) we can create new price thresholds as shown in Table 4, where *Cum. Dem* is the cumulative volume of demand, *d* is the percent discount or surcharge, and *Δd* is the change in percent discount from the last threshold.

Table 4: Forward Buying with Discounts

	A0 Current Period	A1 Buy 1 Period Ahead	A2 Buy 2 Period Ahead	A3 Buy 3 Period Ahead	A4 Buy 4 Period Ahead	A5 Buy 5 Period Ahead
	\$ 100.00	**\$ 64.44**	**\$ 51.64**	**\$ 39.98**	**\$ 31.23**	**\$ 24.67**
\$ 25.00	YES	YES	YES	YES	YES	
\$ 50.00	YES	YES	YES			
\$ 75.00	YES					
\$ 100.00	YES					
Cum. Dem	150	260	365	455	630	820
d	0%	10%	20%	20%	20%	20%
Δd	0%	10%	10%	0%	0%	0%

Now it is beneficial, with a current price of \$50, to buy 2 periods ahead compared with 1 period in the base case shown in Table 2. Notice that at a quantity of 300 we have a 15% discount, but also a 20% discount at 20%. Since the additional demand of 105 for period 2 takes the total to 365, we apply just the 20% discount for *d*, for a net change of 10% (20% - 10%) on the *A2* row.

As expected, the price thresholds are higher starting in period 1 since we are eligible for a 10% discount off the entire purchase. Notice that even though there are no future price discounts for additional quantities starting in period 3, the price thresholds A3, A4 and A5 are all higher than in the base case. This is due to the lowered realized price threshold in period 0 as we cross price thresholds when buying ahead for period 1 and period 2.

As an extreme example of surcharges, the next example in Table 5 shows a 30% surcharge for purchases of 200 or more at one time.

Table 5: Forward Buying with Surcharge

	Calculated Value	$ 25.00	$ 50.00	$ 75.00	$ 100.00	Cum Dem.	d	Δd
A0	$ 100.00	Yes	Yes	Yes	Yes	150	0%	0%
A1	$ 43.08	Yes				260	10%	10%
A2	$ 35.29	Yes				365	20%	10%
A3	$ 27.72	Yes				455	20%	0%

Notice that the threshold to buy for period 1 has lowered significantly from the base case from $57.50 to $43.08. Again, all additional forward buys are lowered as well. In the base case it was possible to buy four periods ahead, while this surcharge example makes only three periods feasible (since the fourth period $22.04 threshold is not possible given the lowest possible price of $25 per unit). Now for a current price to buy of $50, it does not make sense to buy beyond the current period. The current price would have to be $43.08 or lower to recommend buying ahead one period.

The second stream of literature on procurement focuses on buying safety stock to account for demand uncertainty. Gavirneni (2004) recognizes that demand is uncertain according to a known distribution rather than a single point. Therefore, he applies a modified newsvendor ratio to the demand distribution. This ratio is applied to the demand distribution to offer the optimal amount to procure given the demand distribution. The notation for this method is:

p	Selling price
c	Actual cost to purchase in the current period
$\bar{c}$	Expected cost to purchase in the current period
h	Holding cost for one period
y	Order up to level
Φ^{-1}	Inverse CDF of demand

Note that the selling price p stays the same regardless of our purchase discount. Our ability to gain cost discounts through volume purchases does not affect the market selling price for the goods. The order up to y is given by (3) below.

$$y = \Phi^{-1}\left(\frac{p-c}{p+h-c}\right) \quad \textbf{(3)}$$

The current purchase cost (c) is \$50 and the selling price (p) for this item is \$101. Holding is still \$5 per period. Using the forecast error, a distribution can be generated about the point forecast. For the current period 0, we again use the point forecast of 150 from Table 2. For the sake of clarity, we will assume that the demand distribution is calculated as uniform centered on the point forecast of 150, ~U(50,350).

The critical ratio is calculated in (4):

$$\left(\frac{p-c}{p+h-c}\right) = \left(\frac{101-50}{101+5-50}\right) = \left(\frac{51}{56}\right) = 91\% \quad \textbf{(4)}$$

The order up to quantity y is given by (5) below:

$$y = \Phi^{-1}(.91) \quad \textbf{(5)}$$

Therefore, y = (350 – 50) * .91 + 50 = 323 units for the uniform distribution in our example. The standard newsvendor suggests we buy enough units to have 323 units in the current period instead of just the 150 point forecast. The extra 173 units are safety stock and may be used to fill future demand as well, but they have been calculated solely based on the demand distribution in the current period.

Notice that, from Table 2, we are buying enough to qualify for a 20% price discount. We now recalculate using Gavirneni's modified newsvendor equation with the new price of $40 per unit replacing the $50 original price, as shown below in (6).

$$\left(\frac{p-c}{p+h-c}\right)=\left(\frac{101-40}{101+5-40}\right)=\left(\frac{61}{66}\right)=92\% \qquad \textbf{(6)}$$

Now we would order up to y = (350 – 50) * .92 + 50 = 326 units.

3.2 Proposed Heuristic

Our proposed method, which combines the work of Golabi and Gavirneni, is named GOGA out of respect for their prior contributions. We use the price breaks per Golabi in (2) to determine how many periods to forward buy. For the current period, the newsvendor from Gavirneni is used to account for uncertainty in the demand. For forward buys, only the mean forecasted demand is bought, just as Golabi suggests for all periods including the current period. In essence, we use Gavirneni's method for the current period, and then we use Golabi's method for forward buys.

We assume that Holt-Winters with additive seasonality is used to produce point forecasts for future sales of the commodity. A forecasting method with seasonality needs to be used given the clear seasonality of the consumption of building products. Wood is used extensively in construction, which is clearly a seasonal activity. (For a thorough discussion on the Holt-Winters forecasting method see Chatfield [1978).) Given our work with the two stage distributor BlueLinx, we believe that additive seasonality is appropriate for the forecast models.

For our combined GOGA method, we establish the forward buys using Golabi's method and then, using any applicable price discount, we compute Gavirneni's modified newsvendor equation. If this new total quantity to purchase exceeds another price discount threshold, we recalculate the forward buys and safety stock using these methods recursively. Assuming the forecasts in Table 3, we can use the three periods to forward buy previously calculated from Golabi's method. Next, the 20% discounted price is used in Gavirneni's modified newsvendor equation to find the same 92% percent as we found in (6) above. Recall that Gavirneni's method computes the total amount required in the current period. Therefore, we take the 326 units from Gavirneni's calculation and then add the next three periods for forward buying under Golabi's method. The total amount to order up to is 326 + 110 + 105 + 90 = 631 units for GOGA, compared to 455 for Golabi, 326 for Gavirneni, and 150 without applying any forward buying or safety stock.

3. Problem Statement and Proposed Heuristic

3.1 Problem

A company that procures commodities knows the current prices at which to purchase. At each ordering opportunity a company needs to decide how much to order to cover both current demand and possible future demand. The tradeoff to such a decision is that the company buys more and thus incurs holding costs (warehouse space, capital tied up, etc.) to offset the possibility of paying higher purchase prices closer to when demand will be realized.

We use a two stage distributor, BlueLinx Corporation, as our data source. They have generously supplied us with historical sales, forecast indices, and forecast method with parameter values. Unfilled demand is assumed to be lost to competitors in their environment.

The costs and revenue are incremented according to the steps below each period:

1. Order up to inventory level Y calculated with new equation (2).
2. Determine amount to procure x = min (Y-z, 0) where z is the current onhand inventory.
3. If x >= Threshold volume then apply discount to cost C, and then recalculate steps 1 and 2.
4. C * min (Y-z , 0) added to total cost. Assume the additional units arrive immediately, so current onhand $z = Y$.
5. Demand D_i is realized. Sell max (Di , z). Total Revenue has c * (1 + ε) * max (D_i , z) added to it, where ε is the profit margin. We assume that in each period we maintain a constant percent margin beyond the current purchase cost that is valid for commodities. Onhand inventory z is decremented by max (D_i , z).
6. Each remaining unit incurs a holding percentage h of current cost c. Total cost has h * c * z added to it.

7. After the 48 month horizon is finished, steps 5 and 6 iterate until all stock is depleted.

4. Simulation Method

To test the effectiveness of our proposed method we conducted a simulation to compare the results with no forward buys and no safety stock, safety stock via the newsvendor, Golabi (1985), and Gavirneni (2004). The data was randomly generated using bootstrapping ([Demirel and Willemain, 2002), (Davidson and Hinkley, 1997]].

We ran 100 replications of four years of daily demand. For each replication we ran nine parameter combinations of holding cost and profit margin. Four purchase discount levels were tried for each of the nine parameter combinations. Discounts of 1%, 2% and 5% of the purchase price were tried. Additionally, the no discount case and a 1% surcharge were tested. While the surcharge sounds counterintuitive, once a firm has purchased its allocation of a commodity it may have to pay higher prices to get additional product. Since discounts of 2% are considered large by many players in the commodity business, 5% is used as a fairly extreme value whereas 1% and 2% are more common discount levels.

We chose 100 replications because it gave us a relative error of 3.5%. The objective was to maximize profits given that purchase prices fluctuate from period to period. Selling prices are a percent profit ε of the selling period purchase price regardless of the price at which the inventory was bought. Holding costs of 20% are typical for companies wishing to include obsolescence, shrinkage and other miscellaneous costs. A company looking solely at the costs would use a number around 8% or 9% for holding. We also included 14% as a number between these two extremes.

Plywood gross margins, as reported on the internet for firms in this industry, range from 14% to 22% in the last few years. We chose to use 14%, 18%, and 22% as the profit margins to test the wood data. The parameters used are shown in Table 6 below.

Table 6: Parameter Levels

Holding h	8%	14%	20%		
Plywood profit margin ε	14%	18%	22%		
Price Discount	-1%	0%	1%	2%	5%

We used the actual invoiced daily demand for plywood over four years. To create a new test case, the number of selling days in a month were calculated; subsequently, the same number of random draws with replacement were performed to make a new possible sales month for the item. Since every day had sales except weekends and holidays, we did not need to determine time between orders; we only needed to determine the quantity of each selling day per month. Note that ordering costs were omitted, as they are insignificant compared to the material purchase costs.

Five methods were used for each test data set: 1) no forward buy or safety stock purchased, 2) safety stock using the normal newsvendor ratio, 3) Golabi's (1985) method for forward buys, 4) Gavirneni's (2004) newsvendor method for safety stock, and 5) our proposed method, called GOGA, which has forward buys and the newsvendor model for safety stock. Each run included no volume discount, 1% volume surcharge, 1% volume discount, 2% volume discount, and 5% volume discount.

5. Results

For the 100 simulation runs of the wood product, the profits for each of the parameter settings for the five methods are shown below in Table 7 for a zero percent discount (effectively, a no volume discount scenario). Note that the four year profit has been adjusted so that the lowest profit method shows $1000 and all other methods are scaled accordingly.

Table 7: Comparison of Buying Methods for Wood

	ε = 14%	ε = 18%	ε = 22%	
h = 8%	$ 1,000	$ 1,000	$ 1,000	No forward buys/SS
	$ 1,264	$ 1,294	$ 1,319	Newsvendor SS
	$ 1,287	$ 1,232	$ 1,198	Golabi
	$ 1,454	$ 1,432	$ 1,411	Gavirneni
	$ 1,564	**$ 1,517**	**$ 1,508**	GOGA
h = 18%	$ 1,000	$ 1,000	$ 1,000	No forward buys/SS
	$ 1,207	$ 1,244	$ 1,269	Newsvendor SS
	$ 1,099	$ 1,080	$ 1,068	Golabi
	$ 1,318	$ 1,316	$ 1,327	Gavirneni
	$ 1,373	**$ 1,367**	**$ 1,369**	GOGA
h = 20%	$ 1,000	$ 1,000	$ 1,000	No forward buys/SS
	$ 1,165	$ 1,204	$ 1,231	Newsvendor SS
	$ 1,000	$ 1,000	$ 1,000	Golabi
	$ 1,232	**$ 1,253**	**$ 1,269**	Gavirneni
	$ 1,232	**$ 1,253**	**$ 1,269**	GOGA

For the highest holding cost (20%), it was not optimal to ever forward buy. Therefore, the GOGA and Gavirneni methods are equally effective. For lower holding costs, the profit from our GOGA method was increased by using the forward buys from Golabi as one can see in Table 8 below.

Table 8: Profit % improvement over no FB/no SS

	Newsvendor	Golabi	Gavirneni	GOGA
h = 8%	29.2%	23.9%	43.2%	53.0%
h = 18%	24.0%	8.2%	32.0%	37.0%
h = 20%	20.0%	0.0%	25.1%	25.1%
ε = 14%	21.2%	12.9%	33.5%	39.0%
ε = 18%	24.7%	10.4%	33.4%	37.9%
ε = 22%	27.3%	8.9%	33.6%	38.2%

The results above demonstrate that, by taking into account demand uncertainty, the normal newsvendor equation and Gavirneni's modified newsvendor equation give additional profit beyond no safety stock. Forward buying under Golabi is most beneficial when holding costs are lowest because these low holding costs allow more speculative stock to be purchased. However, at the holding cost of 20%, no forward buying was done and thus Golabi's methods are equal to the method with no forward buying and no safety stock. Our GOGA method combines the modified newsvendor of Gavirneni to handle demand uncertainty with Golabi's view of price uncertainty to forward buy. When the distribution of future prices is known, this information can be exploited for forward buys along with the demand uncertainty, thereby making the GOGA method superior for commodity forward buys in comparison to existing methods. This base case is developed and the results are discussed in detail in Manikas, Chang and Ferguson (2006).

We now examine price discounts for volume purchases. The threshold quantity was set to twice the average monthly invoiced sales over the four year data set from BlueLinx. This amount allows forward buying and safety stock buying to be even more advantageous over buying the mean forecast every period. Discounts are typically small for commodity purchases and they can be an immediate price break, year end rebate, or other more favorable terms. Since immediate price breaks and favorable terms are easily translated into a single percent discount off the current purchase price, we focus on those price breaks here. We first examine the case where, at a threshold, the price drops by 1%. If this threshold is met, all units purchased are at the discounted price. There are no additional tiers of further discounting. Table 9 below shows the improvement over no forward buying and no safety stock from the various methods with a 1% discount. The results of holding and margin combinations are listed in the appendix.

Table 9: Profit % improvement over no FB/no SS with 1% Discount

	Newsvendor	Golabi	Gavirneni	GOGA
h = 8%	30.3%	26.3%	41.9%	53.8%
h = 18%	24.9%	9.0%	31.2%	36.7%
h = 20%	20.9%	0.0%	24.7%	24.7%
ε = 14%	22.3%	14.2%	32.1%	38.8%
ε = 18%	25.7%	11.4%	32.7%	38.0%
ε = 22%	28.1%	9.7%	33.1%	38.4%

We also consider the effects of a 2% discount and a 5% discount to meet the threshold purchase quantity in one period as shown in tables 10 and 11 respectively.

Table 10: Profit % improvement over no FB/no SS with 2% Discount

	Newsvendor	Golabi	Gavirneni	GOGA
h = 8%	31.3%	32.1%	43.9%	57.6%
h = 18%	25.8%	9.7%	32.2%	38.2%
h = 20%	21.7%	0.0%	22.7%	22.7%
ε = 14%	23.4%	16.8%	32.3%	40.0%
ε = 18%	26.6%	13.5%	33.0%	39.0%
ε = 22%	28.8%	11.5%	33.5%	39.5%

Table 11: Profit % improvement over no FB/no SS with 5% Discount

	Newsvendor	Golabi	Gavirneni	GOGA
h = 8%	34.3%	40.8%	50.0%	69.0%
h = 18%	28.6%	11.8%	38.3%	45.8%
h = 20%	24.3%	0.0%	26.8%	26.8%
ε = 14%	26.8%	21.4%	39.2%	49.6%
ε = 18%	29.3%	17.0%	38.3%	46.5%
ε = 22%	31.2%	14.2%	37.7%	45.6%

There is a possibility that exceeding a certain threshold will actually cause prices to increase. Given the typical high capacity nature of mills it may allocate a certain volume of stock to the buyer, and any excess will have to be met by witholding stock from another customer. It may also be the case that the excess quantity has to be bought from an alternative supply source, which is equivalent to a volume surcharge. In table 12, we show the results for a 1% surcharge for meeting the threshold.

Table 12: Profit % improvement over no FB/no SS

	Newsvendor	Golabi	Gavirneni	GOGA
h = 8%	28.2%	21.5%	35.0%	51.5%
h = 18%	23.0%	7.5%	29.0%	33.0%
h = 20%	19.2%	0.0%	25.0%	25.0%
ε = 14%	20.1%	11.6%	27.8%	36.7%
ε = 18%	23.8%	9.4%	29.7%	36.3%
ε = 22%	26.5%	8.0%	31.5%	36.5%

At quantities less than half the average monthly demand, the methods all take advantage of discounts and the results trend toward the base case with no discount as shown in Table 7. Similarly, at thresholds greater than four times the average monthly demand, none of the methods trigger discounts and the results again trend toward the base case with no discount.

The GOGA method performs as well as or better than the other methods tested here for a wide range of profit margins and holding costs, even in the presence of volume price discounts or surcharges. In all situations, the new GOGA heuristic is likely to increase profits of plywood buying and selling.

6. Conclusions and Discussion

Given the fluctuating prices of commodities, forward buys and safety stock make sense in certain situations. Golabi's method works well to determine the number of periods to forward buy given current and expected future prices. However, the quantity to procure in the current period should not be the point forecast but rather it should include safety stock based upon the holding costs and price differential in the current period versus expected price. By utilizing both Gavirneni's application of the newsvendor equation to the normal distribution and Golabi's price thresholds for forward buys, our GOGA heuristic achieves better results overall than either of those methods alone achieves.

Forward buys as outlined in Golabi's method clearly increases profits over not doing so, as shown in the Tables 8-12. However, as holding costs increase and profit margins decrease, this benefit is reduced or nullified. The modified newsvendor equation of Gavirneni provides advantages over buying no safety stock. Tables 8-12 show that over the range of price discounts, holding costs, and profit margins tested, using Gavirneni's modified newsvendor formula is advantageous to total profit. Our GOGA method combines aspects of these two methods for maximum profit improvement.

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Appendix

Table 13: 1% Discount Results

	ε = 14%		ε = 18%		ε = 22%		
h = 8%	$	1,000	$	1,000	$	1,000	No forward buys/SS
	$	1,276	$	1,304	$	1,328	Newsvendor SS
	$	1,317	$	1,255	$	1,216	Golabi
	$	1,436	$	1,420	$	1,401	Gavirneni
	$	**1,572**	**$**	**1,525**	**$**	**1,517**	GOGA
h = 18%	$	1,000	$	1,000	$	1,000	No forward buys/SS
	$	1,218	$	1,253	$	1,277	Newsvendor SS
	$	1,108	$	1,087	$	1,074	Golabi
	$	1,301	$	1,310	$	1,326	Gavirneni
	$	**1,366**	**$**	**1,365**	**$**	**1,371**	GOGA
h = 20%	$	1,000	$	1,000	$	1,000	No forward buys/SS
	$	1,175	$	1,213	$	1,238	Newsvendor SS
	$	1,000	$	1,000	$	1,000	Golabi
	$	**1,227**	**$**	**1,250**	**$**	**1,265**	Gavirneni
	$	**1,227**	**$**	**1,250**	**$**	**1,265**	GOGA

Table 14: 2% Discount Results

	ε = 14%		ε = 18%		ε = 22%		
h = 8%	$	1,000	$	1,000	$	1,000	No forward buys/SS
	$	1,288	$	1,314	$	1,336	Newsvendor SS
	$	1,386	$	1,311	$	1,265	Golabi
	$	1,464	$	1,439	$	1,415	Gavirneni
	$	**1,620**	**$**	**1,561**	**$**	**1,546**	GOGA
h = 18%	$	1,000	$	1,000	$	1,000	No forward buys/SS
	$	1,229	$	1,262	$	1,284	Newsvendor SS
	$	1,117	$	1,094	$	1,079	Golabi
	$	1,311	$	1,320	$	1,335	Gavirneni
	$	**1,384**	**$**	**1,379**	**$**	**1,384**	GOGA
h = 20%	$	1,000	$	1,000	$	1,000	No forward buys/SS
	$	1,185	$	1,222	$	1,245	Newsvendor SS
	$	1,000	$	1,000	$	1,000	Golabi
	$	**1,195**	**$**	**1,231**	**$**	**1,255**	Gavirneni
	$	**1,195**	**$**	**1,231**	**$**	**1,255**	GOGA

Table 15: 5% Discount Results

	ε = 14%	ε = 18%	ε = 22%	
h = 8%	$ 1,000	$ 1,000	$ 1,000	No forward buys/SS
	$ 1,325	$ 1,343	$ 1,362	Newsvendor SS
	$ 1,497	$ 1,395	$ 1,331	Golabi
	$ 1,547	$ 1,497	$ 1,456	Gavirneni
	$ 1,763	**$ 1,672**	**$ 1,635**	GOGA
h = 18%	$ 1,000	$ 1,000	$ 1,000	No forward buys/SS
	$ 1,262	$ 1,289	$ 1,307	Newsvendor SS
	$ 1,144	$ 1,115	$ 1,096	Golabi
	$ 1,383	$ 1,380	$ 1,386	Gavirneni
	$ 1,479	**$ 1,451**	**$ 1,444**	GOGA
h = 20%	$ 1,000	$ 1,000	$ 1,000	No forward buys/SS
	$ 1,216	$ 1,247	$ 1,267	Newsvendor SS
	$ 1,000	$ 1,000	$ 1,000	Golabi
	$ 1,245	**$ 1,272**	**$ 1,288**	Gavirneni
	$ 1,245	**$ 1,272**	**$ 1,288**	GOGA

Table 16: 1% Surcharge Results

	ε = 14%	ε = 18%	ε = 22%	
h = 8%	$ 1,000	$ 1,000	$ 1,000	No forward buys/SS
	$ 1,252	$ 1,284	$ 1,310	Newsvendor SS
	$ 1,258	$ 1,209	$ 1,179	Golabi
	$ 1,332	$ 1,347	$ 1,371	Gavirneni
	$ 1,545	**$ 1,508**	**$ 1,491**	GOGA
h = 18%	$ 1,000	$ 1,000	$ 1,000	No forward buys/SS
	$ 1,195	$ 1,234	$ 1,261	Newsvendor SS
	$ 1,090	$ 1,073	$ 1,062	Golabi
	$ 1,272	$ 1,291	$ 1,307	Gavirneni
	$ 1,325	**$ 1,329**	**$ 1,337**	GOGA
h = 20%	$ 1,000	$ 1,000	$ 1,000	No forward buys/SS
	$ 1,155	$ 1,196	$ 1,224	Newsvendor SS
	$ 1,000	$ 1,000	$ 1,000	Golabi
	$ 1,231	**$ 1,252**	**$ 1,267**	Gavirneni
	$ 1,231	**$ 1,252**	**$ 1,267**	GOGA