The Rational Part of QCD Amplitude II: the Five-Gluon

Xun Su*[1], Zhi-Guang Xiao†[1,2], Gang Yang‡[1] and Chuan-Jie Zhu§[1,3]

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[1]Institute of Theoretical Physics, Chinese Academy of Sciences
P. O. Box 2735, Beijing 100080, P. R. China

[2]The Interdisciplinary Center of Theoretical Studies, Chinese Academy of Sciences, P. O. Box 2735, Beijing 100080, P. R. China

[3]Center of Mathematical Science, Zhejiang University
Hangzhou 310027, P. R. China

Abstract

The rational part of the 5-gluon one-loop amplitude is computed by using the newly developed method for computing the rational part directly from Feynman integrals. We found complete agreement with the previously well-known result of Bern, Dixon and Kosower obtained by using the string theory method. Intermediate results for some combinations of Feynman diagrams are presented in order to show the efficiency of the method and the local cancellation between different contributions.

*E-mail: ghkcn@itp.ac.cn
†E-mail: zhgxiao@itp.ac.cn
‡E-mail: yangg@itp.ac.cn
§E-mail: zhucj@itp.ac.cn

1 Introduction

In a previous paper [1], we developed a method for computing the rational part of the one-loop amplitude directly from the Feynman integral. The purpose of this paper is to apply this method to compute the rational part of the 5-gluon one-loop amplitude. The result agrees with the well-known result of Bern, Dixon and Kosower [2] obtained first by using string-inspired method. (Other 5-parton amplitudes were later computed by using either standard Feynman diagrammatic technique [3] or supersymmetric decomposition and perturbative unitarity [4].)

The computation of the multi-particle one-loop amplitude in QCD is a very difficult problem. Even for 4-parton amplitude the computation is quite non-trivial [5]. For 5-gluon amplitude a new method was developed [6] by using string theory.

The constant effort to calculate multi-leg one-loop amplitudes lies in the application to the forthcoming experimental program at CERN's Large Hadron Collider (LHC), as there are lots of processes with many particles as final states [7]. We refer the reader to [1] for a discussion and extensive references for the recent efforts in computing the multi-leg one-loop amplitudes and the recent developments inspired by twistor string theory [8, 9].

In order to compute multi-leg one-loop amplitude in QCD, it is a good strategy [10] to decompose the QCD amplitude into simpler ones by using supersymmetric decomposition:

$$A^{QCD} = A^{N=4} - 4A^{N=1 \text{ chiral}} + A^{N=0 \text{ or scalar}}, \tag{1}$$

where A^{QCD} denotes an amplitude with only a gluon circulating in the loop, $A^{N=4,1}$ have the full $N = 4, 1$ multiplets circulating in the loop, and $A^{N=0}$ has only a complex scalar in the loop.

By using the general properties of the one-loop amplitude, Bern, Dunbar, Dixon and Kosower proved that the supersymmetric amplitudes $A^{N=4,1}$ are completely determined by 4-dimensional unitarity [10], i.e. the amplitude is completely cut-constructible and the rational part is vanishing (see [10, 1] for more detail explanation). For MHV helicity configurations, explicit results were obtained for $A^{N=4}$ in [10]. The recent development of using MHV vertices to compute one-loop amplitudes leads to many new results for the cut-constructible part [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. In particular, Bedford, Brandhuber, Spence and Travaglini [11, 15] applied the

the MHV vertices to one-loop calculations. Britto, Buchbinder, Cachazo, Feng and Mastrolia [20, 21, 22] developed efficient technique for evaluating the rational coefficients in an expansion of the one-loop amplitude in terms of scalar box, triangle and bubble integrals (the cut-constructible part, see [1] for details). By using their technique, it is much easier to calculate the coefficient of box integrals without doing any integration. Recently, Britto, Feng and Mastrolia completed the computation of the cut-constructible terms for all the 6-gluon helicity amplitudes [22].

In order to complete the QCD calculation for the 6-gluon helicity amplitudes, the remaining challenge is to compute the rational part of the helicity amplitudes with scalar circulating in the loop. In general, we need an efficient and powerful method to compute the rational part of any amplitude.

As we reviewed in [1], there are various approaches [23, 24, 25, 26, 27, 28] to compute the rational part. In particular, Bern, Dixon and Kosower [27, 28] developed the bootstrap recursive approach which has leads to quite general results [29, 30, 31]. In this paper we will use the approach as developed in [1] and apply it to compute the rational part of the 5-gluon one-loop amplitude. In another paper [32] we will compute the rational part of the 6-gluon amplitude in QCD (see also [31]) which are the last missing pieces for the complete partial helicity amplitudes of the 6-gluon one-loop QCD amplitude.

This paper is organized as follows: in Sect. 2, we recall briefly the Feynman diagrams and the Feynman rules, tailored for our computation of the 5-gluon amplitude. Some simple reduction formulas are recalled briefly in Sect. 3. In Sect. 4 we summarized all the integral formulas we will use in this paper. Then following 2 sections present the results for the rational part of the two independent MHV helicity configurations.

2 Notation, the Feynman diagrams and the Feynman rules

A word about notation: we use the same notation as given in [1]. We use $\epsilon_{i(i+1)\cdots(i+n)}$ to denote the composite polarization vectors for sewing trees to the loop.

For the purpose of this paper we consider only the Feynman diagrams and Feynman rules for the one-loop gluon amplitude with scalar circulating in the loop. We don't follow the usual convention of differentiating different particles by different kinds of lines because there are only two kinds of particles: gluons and scalars, and scalars only appear in the loop.

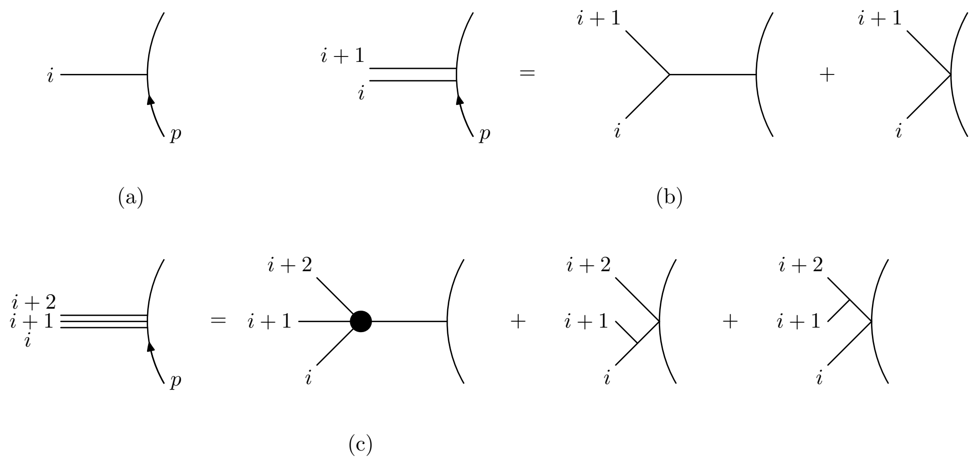


Figure 1: The Feynman rules for sewing trees to loop. The blob denotes an expansion of tree amplitude.

For explicit calculation of the one-loop amplitude by the usual Feynman diagram technique, we can first collect all terms with the same loop structure into one entity. Generally a few cyclicly consecutive external lines are joined in tree diagrams and connected to the same point on the loop (sewing trees to loop). We denote the sum of all these contributions by $P_{i(i+1)\cdots(i+m-1)}$ for m such external lines. For $m = 1, 2, 3$, the relevant Feynman diagrams were shown in Fig. 1. Explicitly we have:

$$P_i(p) = (\epsilon_i, p) = (\epsilon_i, p - k_i), \tag{2}$$

$$P_{i(i+1)}(p) = (\epsilon_{i(i+1)}, p) - \frac{1}{2}(\epsilon_i, \epsilon_{i+1}), \tag{3}$$

$$P_{i(i+1)(i+2)}(p) = (\epsilon_{i(i+1)(i+2)}, p) - \frac{1}{2}\left((\epsilon_{i(i+1)}, \epsilon_{i+2}) + (\epsilon_i, \epsilon_{(i+1)(i+2)})\right), \tag{4}$$

where $\epsilon_{\ldots}$'s are composite polarization vectors introduced in [1]. The computation of these composite polarization vectors is a simplified version of the general recursive calculation of the tree-level n-gluon amplitudes [33].

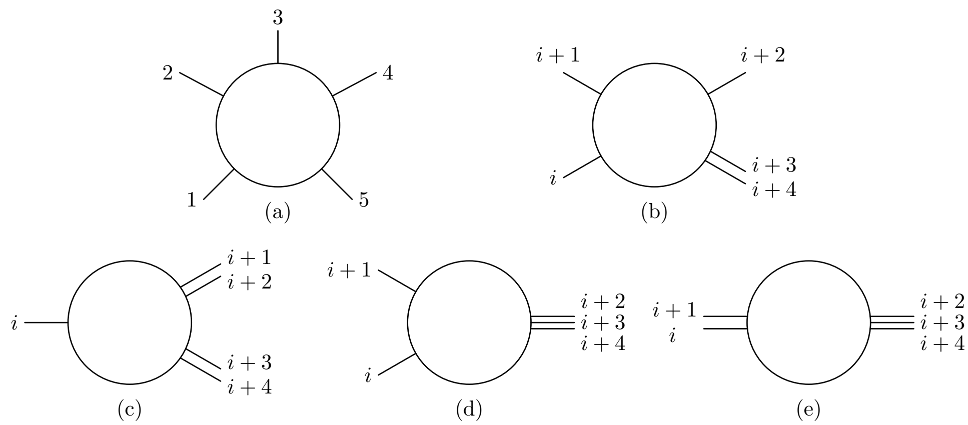


Figure 2: All the possible one-loop Feynman diagrams for 5 gluons. The index i runs from 1 to 5 if there is an index i.

Considering all these different tree diagrams just as the same diagram and denoting them by multiple parallel lines attached to the loop, we have only 21 different Feynman diagrams for the 5-gluon one-loop amplitude (with scalars circulating the loop). Some representative diagrams are given in Fig. 2. The counting goes as follows:

- 1 pentagon diagram, the diagram (a);
- 5 box diagrams because there are 5 different ways of combining two consecutive external lines, diagram (b) with $i = 1, \cdots, 5$;
- 10 triangle diagrams which are further divided into 5 two mass triangle diagrams (diagram (c)) and 5 one mass triangle diagrams (diagram (d));
- 5 bubble diagrams (e).

It is straightforward to compute the rational part from each diagram for a given helicity configuration. In the next two sections we will review all the formulas needed.

3 Tensor reduction of the one-loop amplitude

There is a vast literature on this subject [34, 35, 36]. The tensor reduction relations we will use for our calculation of the 5-gluon amplitude is quite simple. It was based on the BDK trick [28] of multiplying and dividing by spinor square roots. For 5-gluon amplitude it is not so important to make a clever choice of reference momenta. For 6-gluon amplitude it is important to make a specific choice of the reference momenta to make all tensor reduction simple enough to obtain relatively compact analytic results.

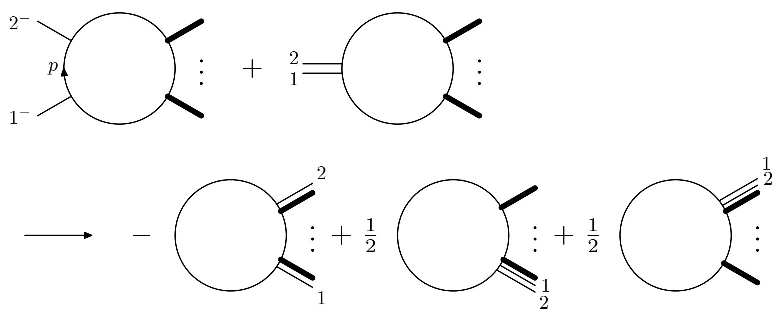


Figure 3: For two adjacent same helicity, the tensor reduction for the combination of two diagrams is even simpler.

For tensor reduction with only 2 neighboring same helicity external gluons, it is possible to chose the reference momenta to be each other's momenta, i.e. $\epsilon_1 = \lambda_1\tilde{\lambda}_2$, $\epsilon_2 = \lambda_2\tilde{\lambda}_1$.[1] The tensor reduction is done by considering the contributions from 2 diagrams together and the result formula is shown pictorially in Fig. 3. The exact algebraic formula is:

$$\begin{aligned} \frac{(\epsilon_1, p+k_1)(\epsilon_1, p)}{(p+k_1)^2 p^2 (p-k_2)^2)} &+ \frac{(\epsilon_{12}, p+k_1) - (\epsilon_1, \epsilon_2)/2}{(p+k_1)^2 (p-k_2)^2} \\ &= -\frac{1}{p^2} + \frac{1/2}{(p+k_1)^2} + \frac{1/2}{(p-k_2)^2}. \end{aligned} \tag{5}$$

For a different choice of reference momenta,

$$\epsilon_4 = \lambda_5\tilde{\lambda}_4, \qquad \epsilon_5 = \eta\tilde{\lambda}_5, \tag{6}$$

[1] An overall constant is omitted for the polarization vector which can be easily restated at the end of calculation. See [37, 38] for details.

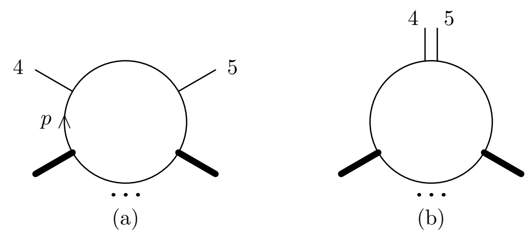


Figure 4: A combination of 2 diagrams with the same adjacent helicity.

the tensor reduction of the 2 diagrams shown in Fig. 4 is given as follows:

$$\begin{aligned} A_a + A_b &= \frac{P_4(p)\, P_5(p-k_4)}{p^2\,(p-k_4)^2\,(p-k_{45})^2} + \frac{P_{45}(p)}{p^2(p-k_{45})^2} \\ &= -\frac{(\eta\tilde{\lambda}_4, p)}{p^2\,(p-k_4)^2} + \frac{\langle \eta\, 5\rangle}{2\,\langle 4\, 5\rangle}\left[\frac{1}{(p-k_{45})^2} - \frac{1}{p^2}\right]. \end{aligned} \tag{7}$$

Setting $\eta = \lambda_4$, the above equation agrees with eq. (5).

4 A summary of the rational part for triangle and box integrals

For quick reference we list here the explicit formulas for the rational part of some Feynman integrals which are needed to compute the 5-gluon amplitude. The derivation can be found in [1].

Firstly, for the bubble integral we have:

$$\begin{aligned} I_2(\epsilon_1, \epsilon_2) &= \int \frac{\mathrm{d}^D p}{i\pi^{D/2}}\, \frac{(\epsilon_1, p)(\epsilon_2, p)}{p^2(p+K)^2} \\ &= \frac{1}{18}\left((\epsilon_1, K)\,(\epsilon_2, K) - 2\, K^2\,(\epsilon_1, \epsilon_2)\right). \end{aligned} \tag{8}$$

where K is the sum of momenta on one side of the bubble diagram. The above result first appeared in [2].

For 2 mass triangle integral with external momenta $\{k_1, K_2, K_3\}$ and $k_1^2 = 0$, the degree 2 integral is:

$$I_3(\epsilon_1, \epsilon_2) \equiv \int \frac{\mathrm{d}^D p}{i\pi^{D/2}}\, \frac{(\epsilon_1, p)\,(\epsilon_2, p)}{p^2(p-k_1)^2(p+K_3)^2}$$

$$
\begin{aligned}
&= \frac{1}{2}\,(\epsilon_1,\epsilon_2) + \frac{(K_2^2+K_3^2)}{2(K_2^2-K_3^2)^2}\,(\epsilon_1,k_1)\,(\epsilon_2,k_1) \\
&+ \frac{((\epsilon_1,K_2)\,(\epsilon_2,k_1)-(\epsilon_1,k_1)\,(\epsilon_2,K_3))}{2(K_2^2-K_3^2)}, \qquad (9)
\end{aligned}
$$

$$
I_3(\epsilon_1,\epsilon_2) = \frac{1}{2}\,(\epsilon_1,\epsilon_2) + \frac{(\epsilon_1,K_2)\,(\epsilon_2,k_1)}{2(K_2^2-K_3^2)}, \qquad (\epsilon_1,k_1)=0, \qquad (10)
$$

and the degree 3 integral is:

$$
\begin{aligned}
I_3(\epsilon_i) &\equiv \int \frac{\mathrm{d}^D p}{i\pi^{D/2}}\,\frac{(\epsilon_1,p)\,(\epsilon_2,p-k_1)\,(\epsilon_3,p)}{p^2\,(p-k_1)^2\,(p+K_3)^2} \\
&= \frac{1}{36}\Big((\epsilon_2,4\,K_2-7\,k_1)\,(\epsilon_1,\epsilon_3)-(2\leftrightarrow 3)+4(\epsilon_1,K_2)\,(\epsilon_2,\epsilon_3)\Big) \\
&- \frac{(K_2^2+K_3^2)}{6\,(K_2^2-K_3^2)^2}\,(\epsilon_1,K_2)\,(\epsilon_2,k_1)\,(\epsilon_3,k_1) \\
&- \frac{(\epsilon_1,K_2)\,((\epsilon_2,k_1)\,(\epsilon_3,K_3)-(\epsilon_2,K_2)\,(\epsilon_3,k_1))}{6\,(K_2^2-K_3^2)} \\
&- \frac{(K_2^2+K_3^2)}{12\,(K_2^2-K_3^2)}\,((\epsilon_1,\epsilon_2)\,(\epsilon_3,k_1)+(\epsilon_1,\epsilon_3)\,(\epsilon_2,k_1)), \qquad (11)
\end{aligned}
$$

where ϵ_1 satisfies the physical condition $(\epsilon_1,k_1)=0$ and $\epsilon_{2,3}$ are arbitrary 4-dimensional polarization vectors.

For box integrals we need only 2 mass easy and 1 mass box integrals. The 1 mass box integrals can be obtained simply by setting the mass of one massive external line to 0 from the 2 mass easy box integral. The four external momenta of the 2 mass easy box diagram are denoted as $\{k_1,K_2,k_3,K_4\}$.

For degree 3 two mass easy box integrals, we have:

$$
I_4^{2me}(\lambda_3\tilde{\lambda}_1,\epsilon_2,\epsilon_3) = \frac{\langle 3|K_2|1]}{2}\left[\frac{(\epsilon_2,k_3)\,(\epsilon_3,k_3)}{(K_2^2-t)(K_4^2-s)} - (k_3\to k_1,\,s\leftrightarrow t)\right], \qquad (12)
$$

where $s=(k_1+K_2)^2$ and $t=(K_2+k_3)^2$. If two of the polarization vectors satisfy the physical condition, i.e. $(\epsilon_1,k_1)=0$ and $(\epsilon_3,k_3)=0$, we have

$$
\begin{aligned}
I_4^{2me}(\epsilon_1,\epsilon_2,\epsilon_3) &= -\frac{(\epsilon_1,k_3)\,(\epsilon_3,k_1)}{2\,(k_1,k_3)}\left[\frac{(\epsilon_2,k_3)}{K_2^2-t}+\frac{(\epsilon_2,k_1)}{K_4^2-t}\right] \\
&-\frac{(\epsilon_1,K_2)\,(\epsilon_2,k_1)\,(\epsilon_3,k_1)}{2\,(K_2^2-s)\,(K_4^2-t)} - \frac{(\epsilon_1,k_3)\,(\epsilon_2,k_3)\,(\epsilon_3,K_4)}{2\,(K_2^2-t)\,(K_4^2-s)}. \qquad (13)
\end{aligned}
$$

In order to give the formulas for the rational part of the degree 4 polynomial, we define:

$$I_4(\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4) \equiv \int \frac{\mathrm{d}^D p}{i\pi^{D/2}} \frac{(\epsilon_1,p)\,(\epsilon_2,p-k_1)\,(\epsilon_3,p-K_{12})\,(\epsilon_4,p+K_4)}{p^2(p-k_1)^2(p-K_{12})^2(p+K_4)^2}, \quad (14)$$

where $K_{12}=k_1+K_2$. For two mass easy case, we have

$$\begin{aligned} I_4(\lambda_3\tilde\lambda_1,\epsilon_2,\lambda_1\tilde\lambda_3,\epsilon_4) &= -\frac{1}{4}\left(\frac{K_2^2+s}{K_2^2-s}+\frac{K_4^2+t}{K_4^2-t}\right)(\epsilon_2,k_1)(\epsilon_4,k_1) \\ &\quad -\frac{1}{4}\left(\frac{K_2^2+t}{K_2^2-t}+\frac{K_4^2+s}{K_4^2-s}\right)(\epsilon_2,k_3)(\epsilon_4,k_3)-\frac{5}{9}(k_1,k_3)(\epsilon_2,\epsilon_4) \\ &\quad + \frac{4}{9}\Big((\epsilon_2,k_1)(\epsilon_4,k_3)+(\epsilon_2,k_3)(\epsilon_4,k_1)\Big), \qquad (15) \\ I_4(\lambda_1\tilde\lambda_3,\epsilon_2,\lambda_1\tilde\lambda_3,\epsilon_4) &= \frac{5}{9}\langle 1|\epsilon_2|3]\,\langle 1|\epsilon_4|3] \\ &\quad +\frac{\langle 1|K_2|3]^2}{3}\left[\frac{(\epsilon_2,k_1)\,(\epsilon_4,k_1)}{(K_2^2-s)\,(K_4^2-t)}+\frac{(\epsilon_2,k_3)\,(\epsilon_4,k_3)}{(K_2^2-t)\,(K_4^2-s)}\right]. \qquad (16) \end{aligned}$$

Other cases can be either obtained by conjugation or relabelling $k_{1,3}$.

The 3 mass triangle and 2 mass hard box integrals are not needed for the computation of the 5-gluon amplitude. The rational part for these integrals can be found in [1, 32].

5 MHV: $A_5(1^-2^-3^+4^+5^+)$

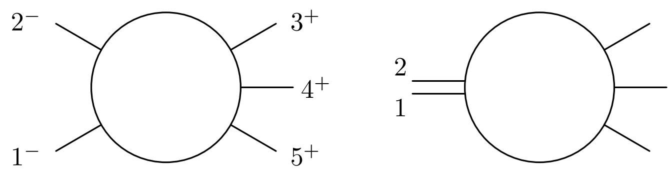
Now we begin the computation of the rational part of the 5-gluon amplitude. In this section we compute the rational part for the helicity configuration $(1^-2^-3^+4^+5^+)$. For this helicity configuration we choose the following polarization vectors:

$$\epsilon_1=\frac{\lambda_1\tilde\lambda_2}{[1\,2]}, \qquad \epsilon_2=\frac{\lambda_2\tilde\lambda_1}{[2\,1]}, \qquad (17)$$

$$\epsilon_3=\frac{\lambda_4\tilde\lambda_3}{\langle 4\,3\rangle}, \qquad \epsilon_5=\frac{\lambda_4\tilde\lambda_5}{\langle 4\,5\rangle}, \qquad \epsilon_4=\frac{\eta\tilde\lambda_4}{\langle \eta\,4\rangle}. \qquad (18)$$

In order to keep the symmetry under $1\leftrightarrow 2$, $3\leftrightarrow 5$ manifest, we leave the reference momentum for ϵ_4 arbitrary, i.e. η is an arbitrary (holomorphic) spinor. The final result should be independent of η. In the following formulas for R_i's, we will omit all the denominators of the polarization vectors.

We classify all the 21 diagrams into 7 sets. They are shown in Figs. 5 to 11. Now let us present the results of the rational parts for these 7 sets of Feynman diagrams.



(a) (b)

Figure 5: The first diagram is the only 5-point diagram. Its combination with the pinched $k_{1,2} \to k_{12}$ 4-point diagram leads to triangle diagrams only by making use of the reduction formula (5).

For the two Feynman diagrams given in Fig. 5, by using the tensor reduction formula given in eq. (5), they are reduced to just triangle diagrams. Because the other three external lines have the same helicities, these triangle diagrams can be further reduced to bubble diagrams which can be computed easily. We note that some extra terms must be added for tensor reduction of the box or triangle integrals [1]. The final result for the rational part is exceptionally simple and is given as follows:

$$R_1 = -\frac{1}{18}\left((\epsilon_4, \epsilon_5)\epsilon_3 + (\epsilon_3, \epsilon_4)\epsilon_5, k_1 - k_2\right). \tag{19}$$

For the four Feynman diagrams shown in Fig. (6), we can use the same tensor reduction formula as above. Because we use the tensor reduction formula for box integral, we should add an extra term. The rational part of these four diagrams is:

$$\begin{aligned} R_2 &= \frac{1}{18}\left((\epsilon_{34}, k_2)(\epsilon_5, k_1) + (\epsilon_3, k_2)(\epsilon_{45}, k_1)\right) \\ &+ \frac{1}{18}\left(2s_{51} - s_{34} - 3s_{12}\right)(\epsilon_{34}, \epsilon_5) \\ &+ \frac{1}{18}\left(2s_{23} - s_{45} - 3s_{12}\right)(\epsilon_3, \epsilon_{45}). \end{aligned} \tag{20}$$

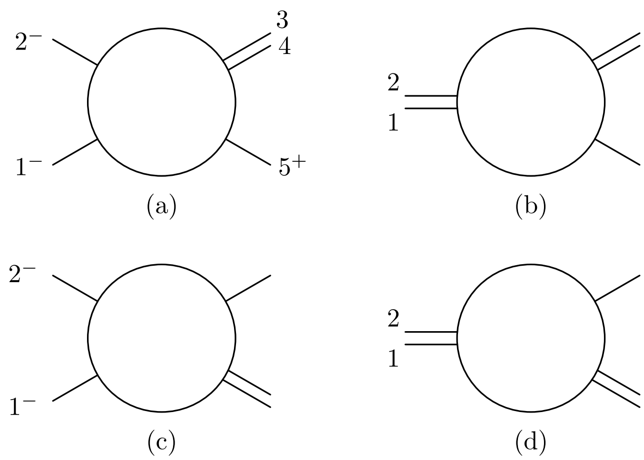


Figure 6: These 4 Feynman diagrams are reduced as the 2 diagrams in Fig. 5.

The four diagrams shown in Fig. 7 can also be reduced simply. Because we leave the reference momentum of ϵ_4 arbitrary, the reduction is just to triangle integrals by using eq. (7). The rational part is:

$$\begin{aligned} R_3 &= -\frac{1}{9}\, s_{23}(\tilde{\epsilon}_{23}, \epsilon_{51}) - \frac{\langle \eta\, 4\rangle}{18\,\langle 3\, 4\rangle}\, s_{51}(\epsilon_2, \epsilon_{51}) - \frac{1}{6}\,(\eta\tilde{\lambda}_3, k_4)\,(\epsilon_2, \epsilon_{51}) \\ &+ \frac{1}{9}\, s_{51}(\tilde{\epsilon}_{51}, \epsilon_{23}) - \frac{\langle \eta\, 4\rangle}{18\,\langle 5\, 4\rangle}\, s_{23}(\epsilon_1, \epsilon_{23}) - \frac{1}{6}\,(\eta\tilde{\lambda}_5, k_4)\,(\epsilon_1, \epsilon_{23}), \quad (21) \end{aligned}$$

where $\tilde{\epsilon}_{23} = \epsilon_{23}|_{\epsilon_3 \to \eta\tilde{\lambda}_3}$ and $\tilde{\epsilon}_{51} = \epsilon_{51}|_{\epsilon_5 \to \eta\tilde{\lambda}_5}$, i.e.:

$$\tilde{\epsilon}_{23} = \frac{[1\,3]}{[2\,3]}\,\eta\tilde{\lambda}_3 - \frac{\langle 2\,\eta\rangle}{\langle 2\,3\rangle}\,\lambda_2\tilde{\lambda}_1 + \frac{\langle 2\,\eta\rangle\,[3\,1]}{2\,\langle 2\,3\rangle\,[3\,2]}\,(k_2 - k_3), \quad (22)$$

$$\tilde{\epsilon}_{51} = \frac{\langle 1\,\eta\rangle}{\langle 1\,5\rangle}\,\lambda_1\tilde{\lambda}_2 - \frac{[2\,5]}{[1\,5]}\,\eta\tilde{\lambda}_5 + \frac{\langle 1\,\eta\rangle\,[5\,2]}{2\,\langle 1\,5\rangle\,[5\,1]}\,(k_5 - k_1). \quad (23)$$

The Feynman diagram shown in Fig. 8 is the only 2 mass triangle diagram which doesn't combine with a higher point diagram. This actually gives rise to spurious pole terms which can only be cancelled by the contribution from cut-constructible part. The rational part is computed by using eq. (11):

$$R_4 = \frac{1}{36}\,(7(\epsilon_4, \epsilon_{51})(\epsilon_{23}, k_4) - 7(\epsilon_4, \epsilon_{23})(\epsilon_{51}, k_4) + 4(\epsilon_{23}, \epsilon_{51})(\epsilon_4, k_{51}))$$

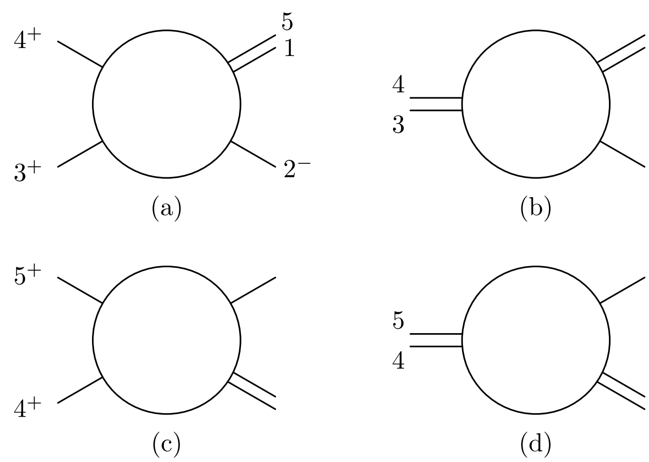


Figure 7: The 3rd set of Feynman diagrams. They can also be reduced easily because of the same adjacent helicity 3^+4^+ or 4^+5^+.

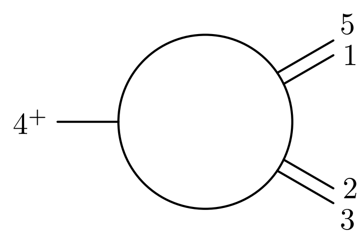


Figure 8: The only 2 mass triangle diagram which doesn't combine with a higher point diagram.

$$
\begin{aligned}
- & \frac{s_{51}+s_{23}}{12(s_{51}-s_{23})}\left((\epsilon_4,\epsilon_{51})(\epsilon_{23},k_4)+(\epsilon_4,\epsilon_{23})(\epsilon_{51},k_4)\right) \\
- & \frac{s_{51}+s_{23}}{6(s_{51}-s_{23})^2}(\epsilon_4,k_{51})(\epsilon_{51},k_4)(\epsilon_{23},k_4) \\
- & \frac{1}{4}(\epsilon_4,(\epsilon_5,\epsilon_1)\epsilon_{23}+(\epsilon_2,\epsilon_3)\epsilon_{51}) \\
- & \frac{1}{4}\,\frac{(\epsilon_4,k_{51})\left((\epsilon_5,\epsilon_1)\epsilon_{23}+(\epsilon_2,\epsilon_3)\epsilon_{51},k_4\right)}{s_{51}-s_{23}}. \qquad (24)
\end{aligned}
$$

The 2 Feynman diagrams shown in Fig. 9 are reduced to tadpole (one-point) diagrams which are zero in dimensional regularization. The only con-

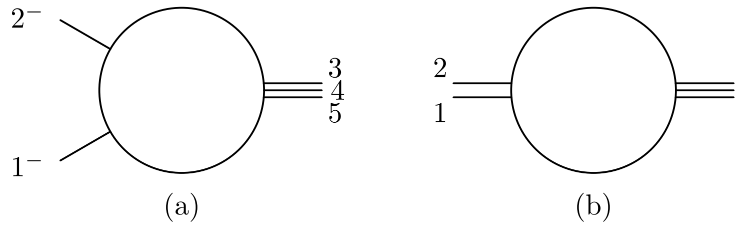


Figure 9: This set has 2 Feynman diagrams. Their reduction leads to tadpole (one-point) diagrams which are zero in dimensional regularization.

tribution is from the extra terms and the result is:

$$R_5 = -\frac{1}{6}\, s_{12}(\epsilon_{345}, k_1) + \frac{1}{4}\, s_{12}\left((\epsilon_3, \epsilon_{45}) + (\epsilon_{34}, \epsilon_5)\right). \tag{25}$$

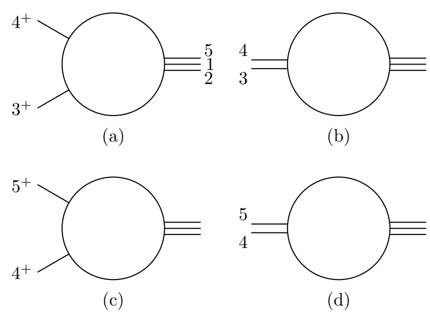


Figure 10: This set has 4 Feynman diagrams. Tensor reduction by making use of eq. (7) leads to bubble diagrams.

The 4 Feynman diagrams shown in Fig. 10 are reduced to bubble diagrams by making use of eq. (7) because the 2 massless external lines $k_{3,4}$ or $k_{4,5}$ have the same adjacent helicity. The rational part is:

$$\begin{aligned} R_6 &= (\eta\tilde{\lambda}_3, k_4)\left[\frac{1}{4}\left((\epsilon_5, \epsilon_{12}) + (\epsilon_{51}, \epsilon_2)\right) + \frac{1}{6}\,(\epsilon_{512}, k_4)\right] \\ &+ (\eta\tilde{\lambda}_5, k_4)\left[\frac{1}{4}\left((\epsilon_1, \epsilon_{23}) + (\epsilon_{12}, \epsilon_3)\right) - \frac{1}{6}\,(\epsilon_{123}, k_4)\right]. \end{aligned} \tag{26}$$

The last 4 diagrams shown in Fig. 11 are two pairs of Feynman diagrams with different adjacent helicities. By explicit computation we found that

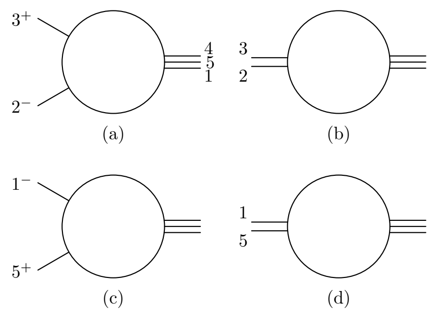


Figure 11: The last set has two pairs of Feynman diagrams with different adjacent helicities. Each combination is identically zero.

each combination is identically zero. This result is true for more general case where the composite polarization vector can be arbitrary.

Having computed all the 21 diagrams separately, the complete rational part is obtained by adding them together. We have

$$
\begin{aligned}
R &= \frac{1}{[1\,2]^2\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle \eta\,4\rangle}\,\Big[-\frac{1}{18}\,((\epsilon_4,\epsilon_5)\epsilon_3+(\epsilon_3,\epsilon_4)\epsilon_5,k_1-k_2) \\
&+ \frac{1}{18}\,((\epsilon_{34},k_2)(\epsilon_5,k_1)+(\epsilon_3,k_2)(\epsilon_{45},k_1)) \\
&+ \frac{1}{18}\,(2s_{51}-s_{34}-3s_{12})\,(\epsilon_{34},\epsilon_5)+\frac{1}{18}\,(2s_{23}-s_{45}-3s_{12})\,(\epsilon_3,\epsilon_{45}) \\
&- \frac{1}{9}\,s_{23}(\tilde{\epsilon}_{23},\epsilon_{51})-\frac{\langle\eta\,4\rangle}{18\,\langle 3\,4\rangle}\,s_{51}(\epsilon_2,\epsilon_{51})-\frac{1}{6}\,(\eta\tilde{\lambda}_3,k_4)\,(\epsilon_2,\epsilon_{51}) \\
&+ \frac{1}{9}\,s_{51}(\tilde{\epsilon}_{51},\epsilon_{23})-\frac{\langle\eta\,4\rangle}{18\,\langle 5\,4\rangle}\,s_{23}(\epsilon_1,\epsilon_{23})-\frac{1}{6}\,(\eta\tilde{\lambda}_5,k_4)\,(\epsilon_1,\epsilon_{23}) \\
&- \frac{1}{6}\,s_{12}(\epsilon_{345},k_1)+\frac{1}{4}\,s_{12}\,((\epsilon_3,\epsilon_{45})+(\epsilon_{34},\epsilon_5)) \\
&+ (\eta\tilde{\lambda}_3,k_4)\,\left[\frac{1}{4}\,((\epsilon_5,\epsilon_{12})+(\epsilon_{51},\epsilon_2))+\frac{1}{6}\,(\epsilon_{512},k_4)\right] \\
&+ (\eta\tilde{\lambda}_5,k_4)\,\left[\frac{1}{4}\,((\epsilon_1,\epsilon_{23})+(\epsilon_{12},\epsilon_3))-\frac{1}{6}\,(\epsilon_{123},k_4)\right] \\
&+ \frac{1}{36}\,(7(\epsilon_4,\epsilon_{51})(\epsilon_{23},k_4)-7(\epsilon_4,\epsilon_{23})(\epsilon_{51},k_4)+4(\epsilon_{23},\epsilon_{51})(\epsilon_4,k_{51})) \\
&- \frac{s_{51}+s_{23}}{12(s_{51}-s_{23})}\,((\epsilon_4,\epsilon_{51})(\epsilon_{23},k_4)+(\epsilon_4,\epsilon_{23})(\epsilon_{51},k_4))
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{s_{51}+s_{23}}{6(s_{51}-s_{23})^2}\,(\epsilon_4,k_{51})(\epsilon_{51},k_4)(\epsilon_{23},k_4) \\
&- \frac{1}{4}(\epsilon_4,(\epsilon_5,\epsilon_1)\epsilon_{23}+(\epsilon_2,\epsilon_3)\epsilon_{51}) \\
&- \frac{1}{4}\,\frac{(\epsilon_4,k_{51})\,((\epsilon_5,\epsilon_1)\epsilon_{23}+(\epsilon_2,\epsilon_3)\epsilon_{51},k_4)}{s_{51}-s_{23}}\Bigg]. \qquad (27)
\end{aligned}
$$

For the same helicity configuration the rational part obtained by Bern, Dixon and Kosower as given in [2] is:

$$
\begin{aligned}
\tilde{R} &= \frac{2}{9}\,\frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle} - \frac{1}{3}\,\frac{\langle 3\,5\rangle\,[3\,5]^3}{[1\,2]\,[2\,3]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,[5\,1]} \\
&+ \frac{1}{3}\,\frac{\langle 1\,2\rangle\,[3\,5]^2}{[2\,3]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,[5\,1]} + \frac{1}{6}\,\frac{\langle 1\,2\rangle\,[3\,4]\,\langle 4\,1\rangle\,\langle 2\,4\rangle\,[4\,5]}{s_{23}\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,s_{51}} \\
&- \frac{1}{6}\,\frac{[3\,4]\,\langle 4\,1\rangle\,\langle 2\,4\rangle\,[4\,5]\,(\langle 2\,3\rangle\,[3\,4]\,\langle 4\,1\rangle + \langle 2\,4\rangle\,[4\,5]\,\langle 5\,1\rangle)}{s_{23}\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,s_{51}} \\
&\quad \times \frac{s_{51}+s_{23}}{(s_{51}-s_{23})^2} \qquad (28)
\end{aligned}
$$

We didn't find an easy way to prove that these results (R and $\tilde{R}$) agree with each other. With the help of Mathematica one can easily check the following result:

$$
R = \frac{1}{2}\,\tilde{R}. \qquad (29)
$$

This shows that the rational part computed directly from Feynman integrals agrees with the well-known result of Bern, Dixon and Kosower [2].

6 MHV: $A_5(1^-2^+3^-4^+5^+)$

For this helicity configuration we choose the following polarization vectors:

$$
\epsilon_1 = \frac{\lambda_1\tilde{\lambda}_5}{[1\,5]}, \qquad \epsilon_3 = \frac{\lambda_3\tilde{\lambda}_4}{[3\,4]}, \qquad (30)
$$

$$
\epsilon_4 = \frac{\lambda_5\tilde{\lambda}_4}{\langle 5\,4\rangle}, \qquad \epsilon_5 = \frac{\lambda_4\tilde{\lambda}_5}{\langle 4\,5\rangle}, \qquad \epsilon_2 = \frac{\eta\tilde{\lambda}_2}{\langle \eta\,2\rangle}. \qquad (31)
$$

The reference momentum of ϵ_2 is arbitrary. As before, in the following formulas for R_i's, we will omit all the denominators of the polarization vectors.

The 21 Feynman diagrams are classified into 6 sets. Apart from the last set, they are shown in Figs. 12 to 16. Let's compute the rational part from each set in turn.

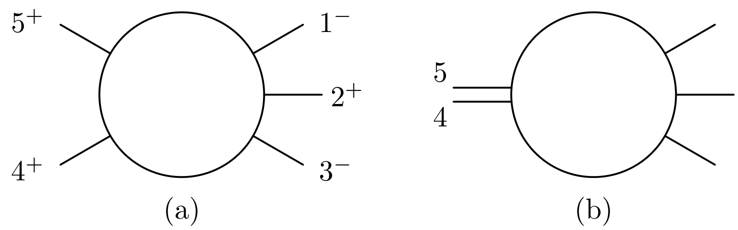


Figure 12: 6 Feynman diagrams with the same adjacent helicity.

The 1st set consists of 2 Feynman diagrams which includes the only pentagon diagram. Tensor reduction is easy by using eq. (5) which gives rise to triangle diagrams. The rational part of the reduced integrals can be computed by using eqs. (9) to (11) and we have:

$$\begin{aligned} R_1 &= -\frac{1}{18}((\epsilon_2,\epsilon_3)\epsilon_1 + (\epsilon_1,\epsilon_2)\epsilon_3 + (\epsilon_1,\epsilon_3)\epsilon_2, k_4 - k_5) \\ &+ \frac{1}{12}(\epsilon_1,\epsilon_2)(\epsilon_3, k_2 - k_{45}) - \frac{1}{12}(\epsilon_2,\epsilon_3)(\epsilon_1, k_2 - k_{45}) \\ &+ \frac{s_{34}+s_{51}}{12(s_{34}-s_{51})}(((\epsilon_2,\epsilon_3)\epsilon_1 + (\epsilon_2,\epsilon_1)\epsilon_3, k_2) \\ &+ \frac{s_{34}+s_{51}}{6(s_{34}-s_{51})^2}(\epsilon_2, k_{34})(\epsilon_3, k_2)(\epsilon_1, k_2). \end{aligned} \tag{32}$$

The 2 set of Feynman diagrams consists of 4 diagrams. They are reduced identically as in the above. We only need to add an extra term because of the box reduction in $D = 4$. The rational part from set 2 is:

$$\begin{aligned} R_2 &= \left(-\frac{1}{6}s_{45} + \frac{1}{9}s_{51} - \frac{1}{18}s_{23}\right)(\epsilon_1, \epsilon_{23}) \\ &+ \left(-\frac{1}{6}s_{45} + \frac{1}{9}s_{34} - \frac{1}{18}s_{12}\right)(\epsilon_{12}, \epsilon_3) \\ &+ \frac{1}{18}((\epsilon_{23}, k_4)(\epsilon_1, k_5) + (\epsilon_3, k_4)(\epsilon_{12}, k_5)). \end{aligned} \tag{33}$$

The two diagrams shown in Fig. 14 are the most complicated for the 5-gluon amplitude. There is no simple reduction formulas to simplify the

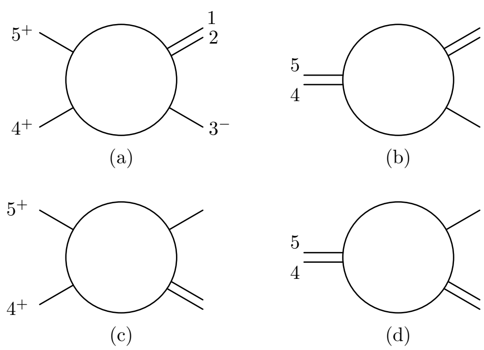


Figure 13: This set consists of 4 Feynman diagrams which can be reduced easily because of the same adjacent helicity.

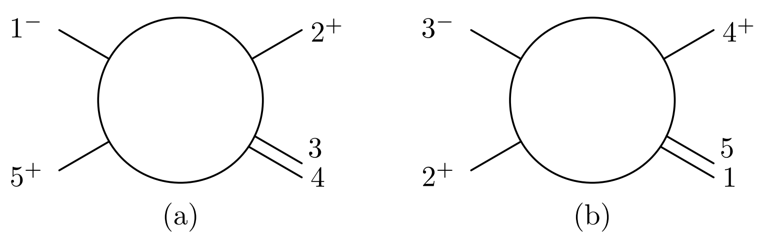


Figure 14: 2 Feynman diagrams which are computed by using the 2 mass easy formulas as given in Sect. 4.

computation. One needs to use the 2 mass easy formulas as given in Sect. 4. The explicit result of the rational part from the the second diagram is:

$$
\begin{aligned}
R_3^{(2)} &= \frac{\langle \eta\, 4\rangle \langle 5\, 2\rangle}{18\, \langle 2\, 4\rangle \langle 4\, 2\rangle} \left[(\epsilon_3, k_2)(\epsilon_{51}, k_4) - 2(k_2, k_4)(\epsilon_3, \epsilon_{51})\right] \\
&+ \frac{\langle \eta\, 4\rangle}{\langle 2\, 4\rangle} \left[\frac{1}{9}\left((\epsilon_4, \epsilon_{51})\epsilon_3 + (\epsilon_3, \epsilon_{51})\epsilon_4, k_2\right) + \frac{(\epsilon_4, k_{51})\,(\epsilon_{51}, k_4)\,(\epsilon_3, k_2)}{6\,(s_{51} - s_{23})}\right] \\
&+ \frac{\langle 5\, 2\rangle}{\langle 4\, 2\rangle} \left[\frac{1}{12}\left((\epsilon_2, \epsilon_3)\,\epsilon_{51} - (\epsilon_2, \epsilon_{51})\,\epsilon_3, k_2\right)\right. \\
&\quad + \frac{1}{18}(2(\epsilon_3, \epsilon_{51})\epsilon_2 - (\epsilon_2, \epsilon_3)\epsilon_{51}, k_4) \\
&\quad - \frac{s_{34} + s_{51}}{12\,(s_{34} - s_{51})}\left((\epsilon_2, \epsilon_3)\,\epsilon_{51} + (\epsilon_2, \epsilon_{51})\,\epsilon_3, k_2\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{(s_{34}+s_{51})}{6\,(s_{34}-s_{51})^2}\,(\epsilon_2,k_{51})\,(\epsilon_3,k_2)\,(\epsilon_{51},k_2)\Bigg] \\
+\;& \frac{\langle \eta\, 2\rangle\,\langle 5\, 4\rangle}{\langle 4\, 2\rangle\,\langle 2\, 4\rangle}\left[-\frac{5}{9}(\epsilon_3,\epsilon_{51})(k_2,k_4)+\frac{4}{9}(\epsilon_3,k_2)(\epsilon_{51},k_4)\right. \\
&\left.+\frac{1}{4}(\epsilon_3,k_2)(\epsilon_{51},k_2)\left(1+\frac{s_{34}+s_{51}}{s_{34}-s_{51}}\right)\right]. \qquad (34)
\end{aligned}
$$

The rational part from the first diagram in Fig. 14 can be obtained from the above result by symmetry transformation:

$$
R_3(1) = -R_3(2)|_{1\leftrightarrow 3,4\leftrightarrow 5}. \qquad (35)
$$

We also set $R_3 = R_3(1) + R_3(2)$.

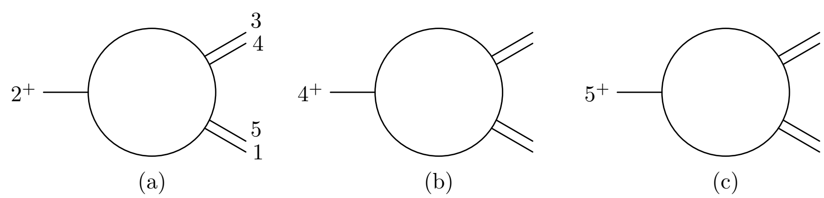


Figure 15: The 4th set consists of 3 two mass triangle diagrams.

The rational part of the 3 two mass triangle diagrams shown in Fig. 15 can be directly computed by using eqs. (11) and (10). The rational part from the first diagram in Fig. 15 is denoted by $R_4^{(0)}$ and is given by:

$$
\begin{aligned}
R_4^{(0)} &= \frac{1}{36}\left(7(\epsilon_2,\epsilon_{34})(\epsilon_{51},k_2)-7(\epsilon_2,\epsilon_{51})(\epsilon_{34},k_2)+4(\epsilon_{34},\epsilon_{51})(\epsilon_2,k_{34})\right) \\
&- \frac{s_{34}+s_{51}}{12(s_{34}-s_{51})}\left((\epsilon_2,\epsilon_{34})(\epsilon_{51},k_2)+(\epsilon_2,\epsilon_{51})(\epsilon_{34},k_2)\right) \\
&- \frac{s_{34}+s_{51}}{6(s_{34}-s_{51})^2}\,(\epsilon_2,k_{34})(\epsilon_{34},k_2)(\epsilon_{51},k_2). \qquad (36)
\end{aligned}
$$

The rational part from the third Feynman diagram in Fig. 15 is denoted by $R^{(1)}$ and we have:

$$
\begin{aligned}
R_4^{(1)} &= \frac{1}{36}\left(7(\epsilon_5,\epsilon_{12})(\epsilon_{34},k_5)-7(\epsilon_5,\epsilon_{34})(\epsilon_{12},k_5)+4(\epsilon_{12},\epsilon_{34})(\epsilon_5,k_{12})\right) \\
&- \frac{s_{12}+s_{34}}{12(s_{12}-s_{34})}\left((\epsilon_5,\epsilon_{12})(\epsilon_{34},k_5)+(\epsilon_5,\epsilon_{34})(\epsilon_{12},k_5)\right)
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{s_{12}+s_{34}}{6(s_{12}-s_{34})^2}\,(\epsilon_5,k_{12})(\epsilon_{12},k_5)(\epsilon_{34},k_5) \\
&- \frac{1}{4}(\epsilon_1,\epsilon_2)\left[(\epsilon_5,\epsilon_{34}) + \frac{(\epsilon_5,k_{12})(\epsilon_{34},k_5)}{s_{12}-s_{34}}\right]. \qquad (37)
\end{aligned}
$$

The rational part from the second diagram in Fig. 15 is denoted by $R_4^{(2)}$ and it can be obtained from $R_4^{(1)}$ by symmetry operation. In total we have:

$$
R_4 = R_4^{(0)} + R_4^{(1)} - (R_4^{(1)}|_{1\leftrightarrow 3,4\leftrightarrow 5}). \qquad (38)
$$

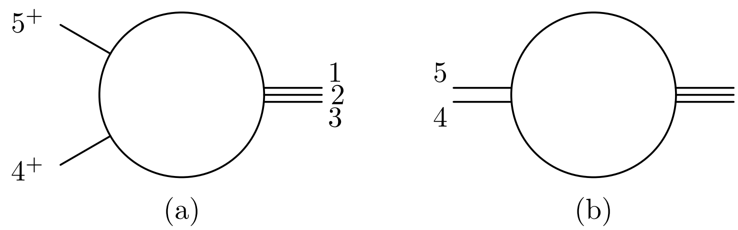


Figure 16: 2 Feynman diagrams with the same adjacent helicity $k_{4,5}$.

The 5th set consists of 2 Feynman diagrams which are reduced to tadpole. The rational part is:

$$
R_5 = -\frac{1}{6}\,(\epsilon_{123},k_4)\,s_{45} + \frac{1}{4}\,((\epsilon_1,\epsilon_{23}) + (\epsilon_{12},\epsilon_3))\,s_{45}. \qquad (39)
$$

Up to now we have computed 13 Feynman diagrams. The rest 8 Feynman diagrams consists of 4 one mass triangle diagrams and 4 bubble diagrams. As we showed explicitly in the last section, they are identically zero for each pair of triangle and bubble diagrams.

The final result for the rational part is:

$$
R \;=\; -\frac{1}{[1\,5]\,[3\,4]\,\langle 4\,5\rangle^2\,\langle \eta\,2\rangle}\sum_{i=1}^{5} R_i. \qquad (40)
$$

In comparison, the result obtained by Bern, Dixon and Kosower [2] from string theory is:

$$
\tilde{R} \;=\; \frac{2}{9}\,\frac{\langle 1\,3\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle} + \frac{1}{3}\,\frac{[2\,4]^2\,[2\,5]^2}{[1\,2]\,[2\,3]\,[3\,4]\,\langle 4\,5\rangle\,[5\,1]}
$$

$$
\begin{aligned}
&- \frac{1}{3}\frac{\langle 1\,2\rangle\,\langle 4\,1\rangle^2\,[2\,4]^3}{\langle 4\,5\rangle\,\langle 5\,1\rangle\,\langle 2\,4\rangle\,[2\,3]\,[3\,4]\,s_{51}} + \frac{1}{3}\frac{\langle 3\,2\rangle\,\langle 5\,3\rangle^2\,[2\,5]^3}{\langle 5\,4\rangle\,\langle 4\,3\rangle\,\langle 2\,5\rangle\,[2\,1]\,[1\,5]\,s_{34}} \\
&+ \frac{1}{6}\frac{\langle 1\,3\rangle^2\,[2\,4]\,[2\,5]}{s_{34}\,\langle 4\,5\rangle\,s_{51}} - \frac{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle^2\,[2\,4]^2}{\langle 4\,5\rangle\,\langle 5\,1\rangle\,\langle 2\,4\rangle^2\,s_{51}}\left[\frac{1}{s_{51}-s_{23}} - \frac{1}{s_{34}-s_{51}}\right] \\
&+ \frac{\langle 3\,2\rangle\,\langle 2\,1\rangle\,\langle 1\,5\rangle\,\langle 5\,3\rangle^2\,[2\,5]^2}{\langle 5\,4\rangle\,\langle 4\,3\rangle\,\langle 2\,5\rangle^2\,s_{34}}\left[\frac{1}{s_{34}-s_{51}} - \frac{1}{s_{12}-s_{34}}\right] \\
&+ \frac{1}{3}\frac{\langle 2\,3\rangle^2\,\langle 4\,1\rangle^3\,[2\,4]^3}{\langle 4\,5\rangle\,\langle 5\,1\rangle\,\langle 2\,4\rangle\,s_{23}\,s_{51}}\frac{s_{51}+s_{23}}{(s_{51}-s_{23})^2} \\
&- \frac{1}{3}\frac{\langle 2\,1\rangle^2\,\langle 5\,3\rangle^3\,[2\,5]^3}{\langle 5\,4\rangle\,\langle 4\,3\rangle\,\langle 2\,5\rangle\,s_{12}\,s_{34}}\frac{s_{12}+s_{34}}{(s_{12}-s_{34})^2} \\
&+ \left[\frac{1}{6}\frac{\langle 1\,3\rangle\,[2\,4]\,[2\,5]\,(\langle 1\,5\rangle\,[5\,2]\,\langle 2\,3\rangle - \langle 3\,4\rangle\,[4\,2]\,\langle 2\,1\rangle)}{\langle 4\,5\rangle}\right. \\
&\quad \left. + \frac{1}{3}\frac{\langle 1\,2\rangle^2\,\langle 3\,4\rangle^2\,\langle 4\,1\rangle\,[2\,4]^3}{\langle 4\,5\rangle\,\langle 5\,1\rangle\,\langle 2\,4\rangle} - \frac{1}{3}\frac{\langle 3\,2\rangle^2\,\langle 1\,5\rangle^2\,\langle 5\,3\rangle\,[2\,5]^3}{\langle 5\,4\rangle\,\langle 4\,3\rangle\,\langle 2\,5\rangle}\right] \\
&\quad \times \frac{s_{34}+s_{51}}{s_{34}\,s_{51}\,(s_{34}-s_{51})^2}. \qquad (41)
\end{aligned}
$$

As in the other MHV case, we didn't find an easy way to prove that these results (R and $\tilde{R}$) agree with each other. With the help of Mathematica one can easily check the following result:

$$R = \frac{1}{2}\,\tilde{R}. \qquad (42)$$

This shows that the rational part computed directly from Feynman integrals agrees with the well-known result of Bern, Dixon and Kosower [2].

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