

# Moduli Stabilization with String Gas and Fluxes

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**ABSTRACT:** The question of how to stabilize all of the modulus fields coming from compactifications of string theory on internal manifolds is one of the outstanding challenges for string cosmology. Here, in the simple example of a toroidal compactification, we study the dynamics of the moduli fields corresponding to the size and shape of the torus along with the ambient flux. Whereas it is known that a string gas containing states with non-vanishing winding and momentum numbers can stabilize the volume modulus, we show that a gas of long strings winding all internal directions coupled to background fluxes can stabilize the remaining moduli.

**KEYWORDS:** String Gas Cosmology Model, String Phenomenology, moduli stabilization.

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## 1. Introduction

Critical superstring theory [1, 2] is consistent only in ten space-time dimensions. One possibility to restore consistency with our observed four-dimensional space-time is to assume that the six spatial dimensions which are not seen experimentally are compactified on a manifold with string-scale volume. The size and shape of the compact internal manifold, along with any flux in the compactified space, can be parametrized in terms of scalar fields on the observed four-dimensional space-time. These are the string theoretic moduli fields. In order to avoid conflicts with observations, there must be a mechanism which fixes these moduli fields.

In the context of the low energy effective field theory coming from string theory, it has recently been shown [3] (see also [4]) that the presence of fluxes can stabilize many but not all of the moduli fields. In particular, all of the complex structure moduli can be stabilized, which includes the shape associated with the internal manifold. It has, however, proven very difficult to stabilize the total volume of the internal manifold using this framework. On the other hand, consider “string gas cosmology,” an approach to superstring cosmology pioneered in [5] (see also [6]) and further developed in [7], in which a gas of strings containing, in addition to the usual effective field theory degrees of freedom, string winding and momentum modes is coupled to

a background space-time described by dilaton gravity. It has recently been shown that this combined action of string winding and momentum modes can stabilize the radion modulus field at the self-dual radius [8, 9]. Thus, it is reasonable to conjecture that it might be possible to stabilize all of the string moduli fields by including fluxes in the analysis.

In this paper, we consider a very simple toy model with moduli associated with a two dimensional toroidal compactification with flux. By populating the torus with a string gas carrying flux quanta, in addition to momentum and winding charges, we find that all of the moduli in the problem can be stabilized dynamically with the exception of the dilaton which we fix by hand in this paper<sup>1</sup>. We develop a way to incorporate strings winding all internal directions (with the internal momenta consistent with T-duality). Moduli stabilization is achieved at the classical level with extended winding strings and momentum modes alone. We analyse the quantum fluctuation of the system and show that both the shape moduli of the torus as well as the flux moduli are stabilized dynamically. This is in contrast to the analyses done in the context of the low energy field theory action coming from supergravity. Here we study the fully time-dependent equations and not just the effective potential. Dilaton stabilization using tools of string gas cosmology will be studied in a followup paper.

## 2. Background and Toy Model

Before turning to a review of string gas cosmology and to the formulation of the toy model which we will study, let us briefly summarize the status of moduli stabilization in the field theory limit of string theory, the limit which is most often used as the starting point for “string cosmology” (e.g. the papers following up on [10, 11] which study the construction of metastable de Sitter solutions and of inflationary solutions to the equations of motion). As was realized in [3] (see also earlier work in [12]), the presence of fluxes can stabilize the “shape moduli” of string compactifications. If the internal manifold is a torus, then the angles of the torus are shape moduli, as are the ratios of the radii of the individual toroidal directions. A heuristic argument for the stabilization of the angle modulus is as follows [13]: for fixed values of the two radii of the torus, the flux energy will be minimal if the volume is largest, i.e. if the angle between the two cycles of the torus is  $\pi/2$ . Similarly, for fixed volume of the torus, the flux energy will be minimal if the two cycles have identical length. Hence, the other shape modulus of the torus will also be stabilized by the flux. On the other hand, the same argument would lead to the total volume of the torus increasing without bound. Fluxes alone cannot stabilize the volume modulus. In

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<sup>1</sup>The dilaton can be likewise trapped at a particular value by turning on both Neveu-Schwarz and Ramond-Ramond fluxes. Since we are interested in the stabilization of the complex moduli in this paper, we turn on only one kind of flux for simplicity and thus have to fix the dilaton by hand.

the original constructions of [3] the dependence on the internal volume drops out of the potential all together, leading to so-called no-scale models. This situation was improved in [10], where non-perturbative corrections were invoked. However, as discussed in [14], it is not enough to find a local minimum of the potential, one must also ensure that the moduli do not overshoot such a minimum. As observed in [15], this remains a challenge for the KKLT models where the non-perturbative potential is generically very shallow. Progress is being made on the former issue, and constructions which also stabilize the volume modulus have recently been obtained making use of additional inputs (see e.g. [16]). The latter problem of dynamical stabilization still remains largely neglected.

“String gas cosmology” is an approach to combining string theory and cosmology which makes crucial use of degrees of freedom and symmetries which are specific to string theory (as opposed to point particle field theory). The key degrees of freedom are string winding modes, and the new symmetry is target space duality (T-duality). The background space-time is described in terms of dilaton gravity and matter is taken to be a gas of strings (branes can also be included [17]) containing all degrees of freedom which are energetically allowed. It is assumed that the background space contains stable cycles (generalizations were discussed in [18]). For simplicity, space is often taken to be the nine-dimensional torus  $T^9$ . The initial conditions are chosen to correspond to a hot small universe. Specifically, all spatial dimensions are of string scale. In the absence of string interactions, the combination of string momentum and winding modes keeps all spatial dimensions stabilized at the self-dual radius ( $R = 1$  in string units, where  $R$  is the radius of each of the tori). The momentum modes whose energies are quantized in integers of  $1/R$  prevent space from contracting to a singularity while the winding modes whose energies are quantized in units of  $R$  prevent space from expanding without bound. Thus, string gas cosmology provides a nonsingular cosmological model <sup>2</sup>.

As argued in [5], string intersections will not allow the disappearance of winding number in more than three spatial dimensions (in higher dimensions the intersection probability of string world sheets vanishes), assuming that the net winding number density vanishes. Numerical support for this argument was provided in [22]. The evolution of the three large spatial dimensions in string gas cosmology was studied in detail in [23], demonstrating that three spatial dimensions can indeed become large (see [24] for some caveats). Thus, another major success of string gas cosmology is that it has the potential to explain why there are only three large spatial dimensions.

Once the three large spatial dimensions are expanding, the combined action of the string winding and momentum modes stabilizes the radii  $R_i$  of all other tori to the self-dual radius [8]. If the value of  $R_i$  for some  $i = 4, \dots, 9$  starts off at a value different

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<sup>2</sup>For further works on string gas cosmology see [19]. It is important to note that the conclusions depend crucially on having fundamental strings or 1-branes in the spectrum of states, and thus might not extend to the 11-d supergravity corner of the M-theory space [20] - but see [21].

from the self-dual radius, it will perform damped oscillations about the self-dual radius, the damping term coming from the expansion of the three large dimensions. The dilaton, however, is not yet stabilized. How to stabilize the dilaton is in fact one of the major challenges for string gas cosmology (see e.g. [25] for a discussion). A second major challenge is the flatness problem - how to make the three large spatial dimensions (which begin at a temperature close to the string scale with string size) sufficiently large to contain our observed universe. Let us, for the moment, assume that the dilaton has been stabilized. After the time of stabilization, the background dynamics is described by the Einstein equations coupled to the string gas. In this context, it has been shown [9] that the radii of the extra dimensions which are still wrapped with winding strings remains stabilized at the self-dual radius. Crucial to the analysis of [9] is the inclusion of states with both winding and momentum quantum numbers which become massless at the self-dual radius (see also [26, 27] and in a different context [28] for a discussion of these states).

These results imply that string gas cosmology has the potential to stabilize the volume modulus, the one modulus which has proven problematic to stabilize in the context of the effective field theory limit of string theory. Since string gases stabilize each  $R_i$  at the self-dual radius, the shape moduli corresponding to the ratios of  $R_i$ 's are also automatically stabilized. On the other hand, string gas cosmology to date has not addressed the issue of the stabilization of the shape moduli which correspond to the angles of a torus nor the moduli associated with flux.

However, the same heuristic arguments (see [13] for a recent review) which indicate that fluxes can stabilize the shape moduli of string compactifications also apply in the context of string gas cosmology. Thus, what we do in this paper is to add fluxes to the existing framework of string gas cosmology. We study the simplest toy model in which one angle and flux modulus are free. Our results show that these moduli are indeed fixed by the string gas carrying nonzero flux. Thus, merging string gas cosmology with fluxes appears to lead to the *dynamical* fixing of all moduli resulting from the string compactification.

After this brief review of string gas cosmology and why we expect that by introducing fluxes into the scenario one will be able to stabilize all of the moduli fields, we will turn to the formulation of the toy model in which we will study moduli stabilization. Our starting point is Type II superstring theory on the background manifold

$$\mathcal{M} = \mathcal{R} \times T^9, \tag{2.1}$$

where  $T^9$  is a nine-dimensional spatial torus. The radii of the individual toroidal directions are denoted by  $R_i$ . The background fields are the space-time metric  $G_{MN}$ , the dilaton  $\phi$ , and the antisymmetric tensor field  $B_{MN}$ . These are the fields which are massless in perturbative superstring theory (see [1] for a review). The equations of motion are the string  $\beta$ -functional and will be discussed in the next section.

The starting point of string gas cosmology is to couple the background fields to a matter sector consisting of all string degrees of freedom treated in the ideal gas (i.e. homogeneous) approximation. Initial conditions are chosen to correspond to an isotropic string-scale universe, i.e.  $R_i = R = 1$  in string units. Following the arguments of [5, 23] we assume that three of the spatial dimensions become large since in those dimensions the winding modes can annihilate. Previous work has shown that the combined action of string winding and momentum modes will stabilize the other radii at the self-dual radius.

The background contains many moduli fields: the radii  $R_i$  of the individual tori, the angles  $\theta_{ij}$  between the  $i$ 'th and  $j$ 'th toroidal direction, the flux on the torus  $B_{ij}$ , and the dilaton. As discussed above, string gas cosmology without fluxes leads to a stabilization of the overall volume and of the ratio of radii. Thus, the moduli to focus on are the angles  $\theta_{ij}$ , the flux  $B_{ij}$ , and the dilaton.

In the absence of string interactions (intersections), all spatial dimensions remain small. The self-dual field configuration in this case also corresponds to a fixed dilaton. Thus, the only moduli left to worry about are the angles and fluxes. It is sufficient to focus on one particular angle and flux. Although easily generalizable, for simplicity we will study compactifications of the form  $\mathbb{R}^{1,3} \times T^2$  with metric

$$ds^2 = -(dx^0)^2 + (d\vec{x})^2 + G_{mn}dx^m dx^n, \quad (2.2)$$

where  $x^0 \equiv t$  is the physical time. We want to focus on the dynamics associated with the  $T^2$  compactification manifold, so we take Minkowski space as the solution for the “large” dimensions which represents a solution of the equations of motion ignoring flux in the large dimensions (this is easily generalized). It will prove useful to introduce Greek indices to indicate time along with the compact coordinates, i.e.  $x^\mu = (t, x^m)$ . The metric of  $T^2$  is parameterized by

$$G_{mn} = \begin{pmatrix} R^2 & R^2 \sin \theta(t) \\ R^2 \sin \theta(t) & R^2 \end{pmatrix} \quad (2.3)$$

where  $\theta$  is the angle of the torus ( $\theta = 0$  corresponds to a rectangular torus). Note that we are using the *dimensionful* metric for the torus, so the two coordinates,  $x^m$  (which we will later denote by  $x$  and  $y$ ), run from 0 to  $2\pi$ . We turn on flux on the  $T^2$  associated with the antisymmetric tensor field  $B_{mn}$  given by

$$B_{mn} = \begin{pmatrix} 0 & b(t) \\ -b(t) & 0 \end{pmatrix} \quad (2.4)$$

In this toy model, the moduli fields to be determined will be the angle  $\theta$  and flux  $b(t)$  with the radion moduli fixed by hand at the value which correspond to the T-dual symmetry point. We wish to study small fluctuations about  $\theta = 0$  and  $b = 0$  to demonstrate the stability of this point in the presence of a string gas with flux.

### 3. The Equations of Motion

Consistency of the string sigma model requires Weyl invariance at the quantum level which in turn implies the vanishing of the  $\beta$ -functionals of the string fields, namely of the metric  $G_{\mu\nu}$ , the rank-two tensor gauge potential  $B_{\mu\nu}$ , and the dilaton  $\phi$  [29]. For a constant dilaton background they become:

$$\beta_{\mu\nu}^G = R_{\mu\nu} + \frac{1}{4}H_{\mu\kappa\sigma}H_{\nu}{}^{\kappa\sigma} \quad (3.1)$$

$$\beta_{\mu\nu}^B = e^{-2\phi}D^\kappa H_{\kappa\mu\nu} \quad (3.2)$$

$$\beta^\phi = \frac{D-26}{6\alpha'} + R + \frac{1}{12}H_{\kappa\mu\nu}H^{\kappa\mu\nu} . \quad (3.3)$$

Here,  $R_{\mu\nu}$  is the Ricci tensor,  $R$  the Ricci scalar,  $D^\kappa$  denotes the covariant derivative,  $H_{\mu\kappa\sigma}$  is the field strength of  $B_{\mu\nu}$ ,  $D$  is the number of space-time dimensions, and  $\alpha'$  is the string Regge slope parameter<sup>3</sup>. In the presence of matter the  $\beta$ -functionals no longer vanish. They are determined by the matter sources, specifically by the stress-energy tensor  $T_\nu^\mu$  and by the current  $J_{\mu\nu}$  of matter.

The Einstein equations in the string frame are obtained by combining the  $\beta$ -functions (3.1) and (3.3) in the following way:

$$\beta_{\mu\nu}^G - \frac{1}{2}G_{\mu\nu}\beta^\phi = e^{-2\phi}T_{\mu\nu} , \quad (3.4)$$

where  $T_{\mu\nu}$  is measured in the string frame. Taking the trace of the Einstein equations one obtains one more equation:

$$-\frac{1}{2}R + \frac{1}{8}H^2 + \frac{3}{2}c = e^{-2\phi}T_\mu^\mu \quad (3.5)$$

which is different from naively setting  $\beta^\phi = e^{-2\phi}T_\mu^\mu$ .

The flux obeys a Maxwell-like equation given by

$$D^\kappa H_{\kappa\mu\nu} = J_{\mu\nu} \quad (3.6)$$

where the current  $J_{\mu\nu}$  is determined by varying the matter action  $S_{matter}$  with respect to  $B_{\mu\nu}$

$$J^{\mu\nu} = \frac{-2}{\sqrt{-G_D}} \frac{\partial S_{matter}}{\partial B_{\mu\nu}} , \quad (3.7)$$

where  $G_D$  denotes the determinant of the full space-time metric. We denote the components of  $T_{\mu\nu}$  by

$$T_{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & p & \tau \\ 0 & \tau & p \end{pmatrix} \quad (3.8)$$

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<sup>3</sup>We follow the conventions in Green, Schwarz and Witten, chapter 3.

where  $\epsilon$  is the energy density and  $p$  the pressure (density). In the presence of a nontrivial angle modulus, we must add an off-diagonal component  $\tau$  in the spatial part of  $T_{\mu\nu}$  for consistency.

Plugging our ansatz for the metric into the Einstein equations (3.4), we obtain component by component the results

$$\text{tt} : -\frac{1}{4}\dot{\theta}^2 - \frac{1}{2}c + \frac{\dot{b}^2}{4G} = e^{-2\phi}\epsilon \quad (3.9)$$

$$\text{xx} : -\frac{S_\theta}{C_\theta}\ddot{\theta} + \frac{1}{4}\dot{\theta}^2 + \frac{1}{2}c - \frac{\dot{b}^2}{4G} = \frac{e^{-2\phi}}{R^2}p \quad (3.10)$$

$$\text{xy} : -\frac{1}{2}\left(\frac{1+S_\theta^2}{S_\theta C_\theta}\right)\ddot{\theta} + \frac{1}{4}\dot{\theta}^2 + \frac{1}{2}c - \frac{\dot{b}^2}{4G} = \frac{e^{-2\phi}}{R^2 S_\theta}\tau \quad (3.11)$$

where we have denoted  $c \equiv \frac{26-D}{6\alpha'}$ . To shorten the expressions, we have used  $S_\theta \equiv \sin\theta$  and  $C_\theta \equiv \cos\theta$ . From these equations one can derive a consistency condition:

$$(1 + S_\theta^2)p - 2\tau S_\theta + \epsilon R^2 C_\theta^2 = 0. \quad (3.12)$$

## 4. Adding Matter and Fluxes

We proceed to compute the source terms for the Einstein equations. We are following the usual approach in string gas cosmology (first used in [7]) of treating the string matter sources as an ideal gas characterized by a homogeneous energy-momentum tensor. In string gas cosmology it is crucial to consider situations in which the winding strings fall out of thermal equilibrium [5]. Hence, we must use the internal energy of the system instead of the one-loop free energy as the string-matter source. We will consider a gas of strings with specified momentum and winding numbers, plus a homogeneous flux. We will denote the number density of strings by  $\rho$  (not to be confused with an energy density for which we use the symbol  $\epsilon$ ).

The internal energy, denoted by  $E$ , can be obtained from one of the Virasora constraints (see Appendix (6.7)) - the lack of dynamics for the world sheet metric requires the world-sheet stress tensor of the string to be identically zero. Here we will simply state the result, with a detailed derivation given in the appendix. The energy of a string in the presence of a non-trivial B-field is given by

$$E^2 = \int_0^{2\pi} d\sigma [G_{mn}p^m p^n + G_{mn}\dot{X}^m \dot{X}^n - 2p^n B_{np}\dot{X}^p + G^{mp}B_{pq}\dot{X}^q B_{nm}\dot{X}^m] + N + \tilde{N} - 2 \quad (4.1)$$

where  $m, n = 1, 2$  refer to the compactification manifold  $T^2$  and the integral runs along the length of the string. The right- and left-handed oscillatory modes are denoted  $N$  and  $\tilde{N}$ , respectively,  $p^m = G^{ml}n_l - B_l^m\omega^l$ , and  $n_l$  and  $\omega^l$  are the momentum



and winding numbers resulting from the compactness of the torus  $T^2$ . In the ideal gas approximation, we take the matter contribution to the action to be

$$S_{matter} = \rho \int \sqrt{-G_D} E, \quad (4.2)$$

where we recall that  $\rho$  is the number density of strings. From this we obtain the stress-energy tensor  $T_{\mu\nu}$ , and the string current  $J_{\mu\nu}$ :

$$T_{\mu\nu} \rho^{-1} = -E G_{\mu\nu} + \frac{1}{E} \frac{\partial E^2}{\partial G^{\mu\nu}} \quad (4.3)$$

$$J^{\mu\nu} \rho^{-1} = \frac{1}{E} \frac{\partial E^2}{\partial B_{\mu\nu}} \quad (4.4)$$

The values of the string quantum numbers are constrained by the second Virasoro constraint (see Appendix (6.7)), namely the level matching condition:

$$n_i \omega^i = \tilde{N} - N. \quad (4.5)$$

Among the states that obey this condition, there are preferred states, which are massless at the self-dual radius (and in the absence of flux). These have quantum numbers given by

$$4(N-1) + \langle n+w, n+w \rangle = 0, \quad (4.6)$$

where  $\langle \ , \ \rangle$  in the second term indicates a scalar product, and  $n$  and  $w$  are vectors with components  $n_i$  and  $w_i$ , respectively. These states will dominate the ensemble of string states if the initial conditions are set up in a thermal-like state. We will focus on the contribution of states with  $N=1$ ,  $\tilde{N}=0$  and  $n_i = -w_i = \pm 1$ . We take all strings to have the same momentum and winding quantum numbers in a T-dual ensemble. With equal probability, we will have any of the following possibilities

$$\begin{array}{cccc} n_1 & \omega_1 & n_2 & \omega_2 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & 1 \end{array} \quad (4.7)$$

Summing over these states, the average internal energy of the system becomes:

$$\langle E^2 \rangle = \frac{2}{R^2 \cos^2 \theta} + 2R^2 + \frac{b^2}{R^4 \cos^4 \theta} + 2N. \quad (4.8)$$

(Note that if we had only kept a subset of these states, we would have introduced by hand an asymmetry and obtained terms of order  $\theta$  in the expression for the internal energy.) From this one finds the average contribution of the strings to the flux,

$$\begin{aligned} \langle J_{xy} \rangle &= - \langle J_{yx} \rangle = \frac{R^2}{E} b(t), \\ \langle J_{xx} \rangle &= \langle J_{yy} \rangle = 0 \end{aligned} \quad (4.9)$$

## 5. Analysis

A rectangular torus ( $\theta = 0$ ) with  $R = 1$  (self-dual radius) is a solution of the equations of motion for fixed dilaton and vanishing flux. We wish to study linear fluctuation around  $\theta = 0$  and  $b = 0$  and show that the solution is a stable fixed point. To do this, it is sufficient to expand the expression for internal energy (4.8) to second order in  $\theta$  and to drop all higher order terms. We obtain

$$E^2 = \frac{2}{R^2}(1 + \theta^2) + 2R^2 + \frac{b^2}{R^4}(1 + 2\theta^2 + \dots) + 2N \quad (5.1)$$

In this limit the expressions for the stress-energy tensor  $T_{\mu\nu}$  and the string source  $J_{\mu\nu}$  simplify:

$$T_{xx} = T_{yy} = -E + \frac{1}{E}(2b^2 + (2 + 4b^2)\theta^2) \quad (5.2)$$

$$T_{xy} = -E \sin \theta + \frac{4}{E}(1 + b^2)\theta \quad (5.3)$$

$$J_{xy} = \frac{1}{E}b. \quad (5.4)$$

We have set  $R = 1$ , the self-dual radius.

Inserting these results for the energy-momentum tensor into (3.11) and (3.9) we obtain

$$\begin{aligned} \text{xy : } & -\frac{1}{2}\left(\frac{1 + \sin^2 \theta}{\sin \theta \cos \theta}\right)\ddot{\theta} + \frac{1}{4}\dot{\theta}^2 + \frac{1}{2}c - \frac{\dot{b}^2}{4G} \\ & = e^{-2\phi}\left(-E \sin \theta + \frac{4}{E}(1 + b^2)\right) \end{aligned} \quad (5.5)$$

and

$$\text{tt : } -\frac{1}{4}\dot{\theta}^2 - \frac{1}{2}c + \frac{\dot{b}^2}{4G} = e^{-2\phi}E \quad (5.6)$$

The dependence on  $\dot{b}$  and on  $c$  can be removed by combining the two above formulas, yielding the following equation of motion for  $\theta$

$$\ddot{\theta} + 8(1 + b^2)K^{-\frac{1}{2}}e^{-2\phi}\theta = 0, \quad (5.7)$$

where  $K \equiv 4 + b^2 + 2N$ . This is the equation for a stable harmonic oscillator. The restoring force receives contributions from both the momentum modes and the flux. This is because they both prefer the torus to be at maximum volume, that is at  $\theta = 0$ . The xx-equation and yy-equation are satisfied at the classical level and the lowest order of perturbations is quadratic, which we ignore.

Now let us turn our attention to the equation of motion for the flux ((3.6)) with source given by (5.4)

$$\ddot{b} + \frac{\sin \theta}{\cos \theta}\dot{\theta}\dot{b} = -J_{xy} \quad (5.8)$$

which is satisfied by the classical values of the fields. We now study the quantum fluctuation. At linear level the equation becomes:

$$\ddot{b} + (4 + 2N)^{-\frac{1}{2}}b = 0. \quad (5.9)$$

In other words the flux also performs harmonic oscillations about the classical value,  $b = 0$ . This is consistent with the results of [31] and [32] which have studied time-dependent solutions in the context of the low energy effective field theory actions and found that the only solution for  $b = \text{const}$  consistent with our metric ansatz is  $b = 0$ .

## 6. Discussion and Conclusions

In this paper we studied a simple two-dimensional toroidal background for string gas cosmology. We developed a way to incorporate long strings winding all internal directions. Moduli stabilization was achieved, at the classical level, by these long strings carrying equal winding and momentum charges without resorting to fluxes. At the quantum level we have shown that in the presence of fluxes, the angle between the cycles, the only shape modulus in this problem, is stabilized at a value which maximizes the area given fixed radii of the torus. Meanwhile the flux also executes harmonic oscillation around its classical value. It is already known that the combined action of string winding and momentum modes stabilizes the ratio of the radii and the total volume. Hence, we have shown that, in this example, all moduli (except for the dilaton which we have to freeze by hand because there is only Neveu-Schwarz flux in the problem) are stabilized. Our analysis is based on the solution of the actual dynamical equations of motion rather than simply by a study of the static effective potential.

We expect that our main conclusion—namely that in the context of string gas cosmology all moduli modulo the dilaton can be stabilized dynamically—extends to more general backgrounds. We also expect that a more detailed analysis of the action of branes in string gas cosmology will lead to ways to also stabilize the dilaton. Work on these topics is in progress.

Phrased differently, our work indicates that the key ingredient missing in the low energy effective field theory approach to moduli stabilization is the inclusion of string winding modes.

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## Appendix: Energy of String Gas

Here for completeness we present the standard method for obtaining the energy of a string gas in the presence of flux (see e.g. [30]). The worldsheet action for the string is

$$\mathcal{L} = \frac{1}{2} \int d^2\sigma \left( \sqrt{h} h^{ab} G_{MN} + \epsilon^{ab} B_{MN} \right) \partial_a X^M \partial_b X^N \quad (6.1)$$

where  $M, N = 0, 1, \dots, 9$  run over all space-time dimensions and

$$h = h_{\tau\sigma} h_{\tau\sigma} - h_{\tau\tau} h_{\sigma\sigma}. \quad (6.2)$$

The generalized momenta are

$$\begin{aligned} \Pi_M &= \frac{\delta \mathcal{L}}{\delta \dot{X}^M} \\ &= \sqrt{h} G_{MN} (h^{\tau\tau} \dot{X}^N + h^{\tau\sigma} \dot{X}^{\sigma N}) + B_{MN} \dot{X}^N, \end{aligned} \quad (6.3)$$

where  $\partial_\tau X^m \equiv \dot{X}^m$  and  $\partial_\sigma X^m \equiv \dot{X}^{\sigma m}$ . Now, solving for  $\dot{X}$

$$\dot{X}^m = \frac{G^{mn}}{\sqrt{h} h^{\tau\tau}} \left( \Pi_n - B_{np} \dot{X}^p \right) - \frac{h^{\tau\sigma}}{h^{\tau\tau}} \dot{X}^{\sigma m} \quad (6.4)$$

Eliminating  $\dot{X}$  from the action, after a tedious computation, a simple answer emerges:

$$\mathcal{L} = \frac{1}{2} \int d^2\sigma \frac{1}{\sqrt{h} h^{\tau\tau}} \left( \Pi^2 - \dot{X}^2 + \dot{X}^m B_m^p B_{pq} \dot{X}^q \right). \quad (6.5)$$

Note that we have made use of (6.2) to eliminate  $h^{\sigma\sigma}$ . We then Legendre-transform to arrive at the Hamiltonian

$$\begin{aligned} \mathcal{H} &= \Pi_m \dot{X}^m - \mathcal{L} \\ &= \Pi_m \left( \frac{G^{mn}}{\sqrt{h} h^{\tau\tau}} (\Pi_n - B_{np} \dot{X}^p) - \frac{h^{\tau\sigma}}{h^{\tau\tau}} \dot{X}^{\sigma m} \right) - \mathcal{L} \\ &= \frac{1}{2\sqrt{h} h^{\tau\tau}} \left[ \Pi^2 + \dot{X}^2 - 2\Pi_m G^{mn} B_{np} \dot{X}^p \right. \\ &\quad \left. - 2\dot{X}^m B_{mn} G^{np} B_{pq} \dot{X}^q \right] - \frac{h^{\tau\sigma}}{h^{\tau\tau}} \Pi \cdot \dot{X} \end{aligned} \quad (6.6)$$

The independent components of the worldsheet metric above play the role of Lagrange multipliers, and thus a variation with respect to them gives the constraint equations:

$$0 = \Pi^2 + \dot{X}^2 - 2\Pi^m B_{mp} \dot{X}^p - 2\dot{X}^m B_{mn} G^{np} B_{pq} \dot{X}^q \quad (6.7)$$

$$0 = \Pi \cdot \dot{X} \quad (6.8)$$

In order to get the physical spectrum we will introduce light-cone coordinates,  $X^\pm = (2)^{-1/2}(X^0 \pm X^1)$ , which leaves us with  $D - 2$  physical degrees of freedom. The first constraint in (6.7) allows us to obtain the energy, while the second expresses the longitudinal coordinates in terms of transverse physical degrees of freedom. Fixing the gauge of the world-sheet theory, namely  $h_{\tau\tau} = -1$ ,  $h_{\sigma\sigma} = 1$ , and  $h_{\sigma\tau} = 0$  the canonical momenta (6.3) become,

$$\Pi_M = p_M - G_{MN} \dot{X}^N + B_{MN} \dot{X}^N, \quad (6.9)$$

In this paper we are interested in the effects of a string gas living on the compact torus  $T^2$ . These directions correspond to the world-sheet fields  $X^m$ . The most general solution respecting our background is given by

$$\begin{aligned} X^\pm &= x^\pm + \frac{1}{\sqrt{2}} E\tau, \\ X^2 &= x^2, \quad X^3 = x^3, \\ X^m &= x^m + \omega^m \sigma + p^m \tau + \text{oscillators}, \end{aligned} \quad (6.10)$$

where  $n_l$  and  $\omega^l$  are the momentum and winding numbers respectively, resulting from the compactness of the torus and we define  $p^m = G^{ml} n_l - 2B_l^m \omega^l$ . Plugging this ansatz into the first constraint in (6.7) and making use of (6.9) one finds the expression for the energy,

$$\begin{aligned} E^2 &= \int_0^{2\pi} d\sigma [G_{mn} p^m p^n + G_{mn} \dot{X}^m \dot{X}^n - 2p^n B_{np} \dot{X}^p \\ &\quad + G^{np} B_{pq} \dot{X}^q B_{nm} \dot{X}^m + N + \tilde{N} - 2], \end{aligned} \quad (6.11)$$

where we integrate over the length of the string, assuming the energy is not localized.

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