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The Inner Branes

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Abstract: WZW models are abstract conformal field theories with an infinite dimensional symmetry which accounts for their integrability, and at the same time have a sigma model description of closed string propagation on Lie group manifolds which endows the models with an intuitive geometric meaning. We exploit this dual algebraic and geometric property of WZW models to provide an explicit example of new type of field-dependent reflection matrix for open-strings in Nappi-Witten model–a pp-wave background supported by a null NS three-form flux. This reflection matrix, corresponding to a space-filling D-brane, arises from using an inner automorphism to glue the left-moving with the right-moving chiral currents at the boundary. Different choices of the inner automorphisms correspond to different background gauge field configurations. The reflection matrix is such that half of the infinite dimensional affine Kac-Moody symmetry present in the closed-string theory is preserved by a unique combination of the left and the right chiral currents.

Keywords: D-branes, Current Algebra, WZW model, String Sigma Model.

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Contents

Since their discovery [1] studying D-branes has been a very important aspect of research in string theory. While D-branes have simple interpretation as being the hypersurface confining the endpoints of the open strings from the sigma model point of view, it is much harder to study them in an abstract conformal field theory that does not necessary have any geometric interpretation for the fields as coordinates of a target spacetime. Important progress has been made throughout the years [4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 31] especially in the case of D-branes in WZW models [7, 8, 9, 10, 13, 14, 15, 16, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31].

WZW models [2] are very special. Not only do they have a sigma model description in which the fields take values in a group manifold of a particular Lie algebra, they are also conformal field theories with an infinite dimensional affine Kac-Moody symmetry, where the current algebra is built upon the very same Lie algebra. In this way they nicely relate the algebraic bootstrap approach of the conformal field theory to the more intuitive geometric approach associated with the sigma model analysis. While closed string propagation on these Lie group manifolds is much studied and well understood, much less is known about the properties of open strings dynamics on these manifolds. The known class of D-branes were obtained by gluing the left-moving and the right-moving worldsheet currents in the closed string theory with a field independent "reflection" matrix, $\mathcal{R}$, *i.e.* $\mathcal{J}_L = \mathcal{R}\mathcal{J}_R$, [4, 5, 6]. We would like to propose using inner automorphisms as general gluing condition, giving rise to field-dependent gluing matrices, $\mathcal{R}$.[1] Different choices of the inner automorphisms

[1]We speak of inner automorphisms of adjoint action by an arbitrary group element, g. In the case of identity, e, the gluing condition is again $\mathcal{J}_L + \mathcal{J}_R = 0$, coincided with the previously studied case.

are related to different configurations of the background gauge fields coupled to the endpoints of the open strings. However choosing a consistent background gauge field for the open-strings is far from obvious within the conformal field theoretical framework. The intuition gained from the geometric approach serves well in this regard. Sigma model analysis of the boundary conditions also straightforwardly endows the D-brane with a geometric interpretation. The underlying symmetry of the D-branes is, on the other hand, more easily seen by studying the symmetry preserved by the boundary conformal theory. The algebraic and geometric approaches compliment each other and provide a complete picture of the D-branes.

We exploit this unique gift of WZW theories to uncover these new type of field dependent gluing conditions, illustrating our techniques using the Nappi-Witten WZW model [3], a pp-wave background supported by a null NS three-form flux. We show, in this note, that it is in fact easy to generalize to field-dependent gluing matrices without compromising the geometric data of the D-branes. A very special choice of the inner automorphism, adjoint action by the group element, e^{uJ}, gives rise to a space-filling brane in NW model. On a non-compact manifold the usual restriction on impossibility of space-filling branes does not apply and indeed we find a D3-brane whose worldvolume spanning all four directions. These are presented in Section 1 and Section 2. The reflection matrix relates the left-moving currents with the right-moving ones. This unique combination of the left and the right chiral currents is shown, in Section 3, to preserve half of the infinite dimensional affine Kac-Moody symmetry present in the closed-string theory.

Nappi-Witten model receives a lot of attention lately because it is one of the solvable string models in the plane-polarized gravitational waves [32, 33, 34, 35]. Following this realization D-branes in these backgrounds have been extensively studied (See [36] for a sample of literature.). Nappi-Witten model, describing closed string propagation in a pp-wave background supported by light-like NS fluxes, has the added merit of being a WZW model which eventually led to its complete and covariant solution, via a Wakimoto "free-field" realization [37], in which string vertex operators were constructed and scattering amplitudes for an arbitrary number of tachyon given. (See also [38] for a different way of deriving the three and four point amplitudes by taking a pp-wave contraction of the $SU(2)\times U(1)$ amplitudes.) In this note we will use a space filling D3-brane in NW model to illustrate our techniques. Emphasis is placed on the interplay of the geometric and algebraic approaches. Using the covariant free field realization introduced in [37] the fact that the free fields obey Neumann boundary condition at the boundaries is made apparent (in Section 3).

1. Sigma Model Analysis

We shall start with the sigma model description. The action is given by [3]:

$$\mathcal{S} = \frac{1}{2\pi}\int \partial_+ u\,\partial_- v + \partial_+ v\,\partial_- u + \partial_+ a\,\partial_- \bar{a} + \partial_+ \bar{a}\,\partial_- a$$
$$+ i\,H\,(\bar{a}\,\partial_- a \partial_+ u - a\partial_- \bar{a}\,\partial_+ u) \qquad (1.1)$$

where the background metric, G, and the gauge invariant combination of Neveu-Schwarz B-field and gauge field, $\mathcal{F} = B + F$, are given by

$$ds^2 = du\,dv + da\,d\bar{a} + 2\,i\,H\,(\bar{a}\,da - a\,d\bar{a})du \qquad (1.2)$$
$$\mathcal{F} = 2\,i\,H\,(\bar{a}\,da - a\,d\bar{a}) \wedge du \qquad (1.3)$$

H being a constant. Notice however that we are using a different gauge from that of [3]. In going from the closed-string theory to the open-string theory, one has to specify a background gauge field, F, in addition to the NS-background, in order to fully specify an open string background. It is the gauge invariant combination, $\mathcal{F} = B + F$, that couples to the endpoints of the open-strings. Compare (eq. 2.6) in [37], used here for the open strings, and (eq. 3.4) in [37], for the closed strings: we had introduced a constant gauge field, $F = du \wedge dv + da \wedge d\bar{a}$. This necessity of using a different gauge from the original one adopted by Nappi and Witten in studying open string was also noticed by the authors of [39, 40] when they studied other aspects of the Nappi-Witten model. We will also set $H = 1$ in the later part of the paper.

We now vary the above action and study the boundary conditions at the endpoints of the string, $\sigma = 0$ and $\sigma = \pi$:

$$\delta v\;(-\partial_+ u + \partial_- u) = 0 \qquad (1.4)$$
$$\delta u\;(-\partial_+ v + \partial_- v + i\,H\,(\bar{a}\,\partial_- a - a\,\partial_- \bar{a})) = 0 \qquad (1.5)$$
$$\delta \bar{a}\;(-\partial_+ \bar{a} + \partial_- \bar{a} - i\,H\,\bar{a}\partial_+ u) = 0 \qquad (1.6)$$
$$\delta a\;(-\partial_+ a + \partial_- a + i\,H\,a\partial_+ u) = 0 \qquad (1.7)$$

A Dirichlet boundary condition corresponds to $\delta X = 0$, where X can be any one of the fields above. We are looking for a solution that satisfies Neumann boundary condition in all four directions. So we demand instead that the four expressions inside the parenthesis vanish.

Neumann condition is equivalent to the requirement that no worldsheet momentum flows out of the boundary of the worldsheet. The worldsheet momentum flowing across a small element, $d\vec{l} = (d\tau,\, d\sigma)$, of the boundary of the worldsheet is given by

$$dP^\mu = P^\mu_\tau\, d\sigma - P^\mu_\sigma\, d\tau \qquad (1.8)$$

has to vanish in order for the endpoints of the string to be free. The components of the worldsheet momentum are defined canonically by

$$P^\mu_\tau = \frac{\delta \mathcal{L}}{\delta \partial_\tau X^\mu} \qquad\qquad P^\mu_\sigma = \frac{\delta \mathcal{L}}{\delta \partial_\sigma X^\mu} \tag{1.9}$$

To read off the gluing condition for the Neumann directions, say in the "open-string picture" *i.e.* an open segment terminating at $\sigma = 0$ and $\sigma = \pi$ at the D-brane ($d\sigma = 0$), we demand no momentum outflow at all values of τ. We must have

$$P_\sigma = 0 \ . \tag{1.10}$$

Using the definition of the canonical momentum, P_σ, one can show that the vanishing P_σ is identical to requiring all four expressions inside the parenthesis in (eq.1.4), (eq.1.5), (eq.1.6), (eq.1.7) to vanish. This unambiguously established the "Neumannity" of the above boundary conditions. Thus the generalized Neumann boundary conditions in curved space are

$$\textbf{open string channel}: \quad P_\sigma = 0 \tag{1.11}$$

$$\textbf{closed string channel}: \quad P_\tau = 0 \tag{1.12}$$

in the open-string and closed-string channels, respectively. As we shall see below, if we express this boundary condition in terms of the free fields we find that they satisfy Neumann boundary condition at $\sigma = 0, \pi$, with a phase that is characteristic of an orbifolded field.

In general, starting with a generic string sigma model (in conformal gauge):

$$\mathcal{L} = \int G_{\mu\nu}\, \partial X^\mu \cdot \partial X^\nu + \epsilon^{ab} \mathcal{F}_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \tag{1.13}$$

Upon variation of the sigma model action there is a contribution from the boundary terms. Consistency of the open-string background requires that such terms vanish. In the presence of the B-field and the gauge field coupling to the end-points of the open strings the boundary terms read

$$\delta X^\mu \left[G_{\mu\nu}(X)\, \partial_\sigma X^\nu + \mathcal{F}_{\mu\nu}(X)\, \partial_\tau X^\nu \right] |_\sigma = 0 \tag{1.14}$$

The boundaries of the worldsheet are set at $\sigma = 0$ and $\sigma = \pi$. Writing the above using the left-moving and right-moving worldsheet derivatives, $\partial_\pm = \frac{1}{\sqrt{2}}(\partial_\tau \pm \partial_\sigma)$, we have

$$\partial_+ X^\mu = \left[(G + \mathcal{F})^{-1} (G - \mathcal{F}) \right]_{\mu\nu} \partial_- X^\nu \ . \tag{1.15}$$

provided that $(G + \mathcal{F})$ is invertible, which it will be for the case at hand.

2. Currents and their Gluing Conditions

Nappi-Witten model is also a WZW model in which case there are infinite-dimensional conserved currents on the closed string worldsheet. In fact there are two independent sets of such currents, one for the left-movers and one for the right-movers, generating the $\mathbf{G}(z) \times \mathbf{G}(\bar{z})$ isometry of the loop group. The insertion of boundary implies that the left-moving and right-moving currents are no longer independent. They are related to each other at the boundary by a "reflection matrix," $\mathcal{R}$:

$$J_L^a(z) = \mathcal{R}^a_{\ b} \; J_R^b(\bar{z}) \; . \tag{2.1}$$

$\mathcal{R}^a_b$ are the component of a z dependent linear map acting on the Lie algebra. $\mathcal{R}$ encodes both Dirichlet and Neumann boundary conditions. Comparing with the expression of boundary condition (eq.1.15) from the sigma model analysis we have

$$\mathcal{R}^a_b = \left[(G + \mathcal{F})^{-1} (G - \mathcal{F}) \, C \right]^a_{\ b} \tag{2.2}$$

where G_{ij}, B_{ij} denotes the components of the metric and the B-field in terms of left invariant form fields, $ds^2 = G_{ij}\,(g\,dg^{-1})^i\,(g\,dg^{-1})^j$, $B = B_{ij}\,(g\,dg^{-1})^i \wedge (g\,dg^{-1})^j$. C^a_b denotes the adjoint map

$$T_a \, C^a_b \equiv Ad(g) \cdot T_b = g T_b g^{-1} \; . \tag{2.3}$$

This relation between the background field, $\mathcal{F}$, and the $\mathcal{R}$ field is invertible as long as $C + \mathcal{R}$ does not have a kernel, in which case we can write

$$\mathcal{F} = G\,(C - \mathcal{R})\,(C + \mathcal{R})^{-1}. \tag{2.4}$$

In other words this equation serves to relate the algebraic data with the geometric data of WZW models. If a metric, G, and a background field, $\mathcal{F}$ are given then $\mathcal{R}$ can be determined from it; and vice versa. The insight gained from the sigma model analysis will enable us to find a consistent background for open strings easily. This gives us considerable advantage over the purely algebraic manipulation as the consistency of a particular open-string background is not obvious in the closed-string picture.

Using this recipe we can directly compute the reflection matrix $\mathcal{R}$ in Nappi-Witten model. For the space-filling D3-brane[2] it turns out to be

$$\vec{\mathcal{J}}_L = \mathcal{R} \vec{\mathcal{J}}_R = -Ad(e^{uJ}) \vec{\mathcal{J}}_R \; . \tag{2.5}$$

[2]The currents are vectors on the tangent space of the group manifold, $\vec{\mathcal{J}} \equiv \mathcal{J}^+ \hat{J}^- + \mathcal{J}^- \hat{J}^+ + \mathcal{T}\,\hat{J} + \mathcal{J}\,\hat{T}$, where $\hat{J}^-$, $\hat{J}^+$, $\hat{J}$, $\hat{T}$, the generators of the algebra, act as the basis for the tangent vectors of the group manifold. To save typing we will drop the hats for the generators and only use the calligraphy letters to denote chiral current components.

where $Ad(g)$ denotes the adjoint action by the group element g, $Ad(g)\,X = gXg^{-1}$. The boundary conditions (eq.2.1) are then explicitly given by

$$\begin{aligned} \mathcal{J}_L^{\pm} + e^{\pm iu}\mathcal{J}_R^{\pm} &= 0 \\ \mathcal{J}_L + \mathcal{J}_R &= 0 \\ \mathcal{T}_L + \mathcal{T}_R &= 0\ . \end{aligned} \tag{2.6}$$

The gluing matrices were usually taken to be outer automorphisms of the algebra, which give rise to a large body of D-branes we know and love. Although the possibility of using inner automorphisms, and hence field-dependent gluing matrices, has been noticed by many researchers [10, 14, 20, 27, 30] and the constraint equation that $\mathcal{R}$ has to satisfy in order to be an inner automorphism has been written down in [30], in many conformal field analysis, due to the lack of geometric intuition, the components of the gluing matrices are quickly taken to be field-independent. Adjoint action by a group element is of course an inner automorphism of the algebra. The element J lives in the Cartan of the Nappi-Witten algebra and it is the only element in the Cartan subalgebra that has nontrivial action on the rest of the algebra. In a sense what we have proposed is the most natural generalization of the constant gluing matrix.

Recall that the Nappi-Witten algebra[3] is

$$[J, J_+] = iJ_+, \qquad [J, J_-] = -iJ_-, \qquad [J_+, J_-] = iT\ . \tag{2.8}$$

We shall parametrize a generic group element by

$$g = e^{aJ_+ + \bar{a}J_-}\, e^{uJ + vT} \tag{2.9}$$

and use the definitions of $\mathcal{J}_+ = -\partial_+ g g^{-1}$ and $\mathcal{J}_- = g^{-1}\partial_- g$ to obtain the following sets of chiral currents:

$$\begin{aligned} -\mathcal{J}_L^+ &= \partial_+ a - i\, a\partial_+ u & \mathcal{J}_R^+ &= e^{-iu}\partial_- a \\ -\mathcal{J}_L^- &= \partial_+ \bar{a} + i\,\bar{a}\partial_+ u & \mathcal{J}_R^- &= e^{+iu}\partial_- \bar{a} \\ -\mathcal{J}_L &= \partial_+ v - \tfrac{i}{2}\left[\bar{a}\,\partial_+ a - a\,\partial_+\bar{a}\right] - a\,\bar{a}\,\partial_+ u & \mathcal{J}_R &= \partial_- v + \tfrac{i}{2}\left[\,\bar{a}\partial_- a - a\partial_-\bar{a}\,\right] \\ -\mathcal{T}_L &= \partial_+ u & \mathcal{T}_R &= \partial_- u \end{aligned} \tag{2.10}$$

Let us remark that the minus sign in the definition of the left-moving current is not arbitrary: it is to ensure that the left-moving and right-moving currents obey the same Kac-Moody algebra.

[3] This is a centrally-extended two-dimensional Euclidean algebra as it is manifest in the following way of writing it:

$$[J, P_1] = P_2, \qquad [J, P_2] = -P_1, \qquad [P_1, P_2] = T, \qquad [T, *] = 0 \tag{2.7}$$

In the note we have defined $J_+ = \frac{1}{\sqrt{2}}(P_1 - iP_2)$ and $J_- = \frac{1}{\sqrt{2}}(P_1 + iP_2)$.

Now it is straightforward exercise to plug in the definitions of the left and right currents (eq.2.10) above into the gluing conditions (eq.2.5) and obtain the following boundary conditions:

$$-\partial_+ u + \partial_- u = 0 \tag{2.11}$$

$$-\partial_+ a + \partial_- a + ia\,\partial_+ u = 0 \tag{2.12}$$

$$-\partial_+ \bar{a} + \partial_- \bar{a} - i\bar{a}\,\partial_+ u = 0 \tag{2.13}$$

$$-\partial_+ v + \partial_- v + a\,\bar{a}\,\partial_+ u + \frac{i}{2}\left[\bar{a}\,(\partial_+ + \partial_-)\,a - a\,(\partial_+ + \partial_-)\,\bar{a}\right] = 0 \tag{2.14}$$

Compare these with the boundary conditions obtained earlier, (eq.1.4), (eq.1.5), (eq.1.6), (eq.1.7), one sees that they indeed describe a D3-brane! We have made use of (eq.1.6) and (eq.1.7) to write (eq.1.5) into the form of (eq.2.14). So by now we have succeeded in showing that if we allow more general gluing conditions than those have been considered in the literature we can have a space-filling D-brane in Nappi-Witten model. The geometry of this D-brane is completely determined by the boundary conditions imposed on the open string endpoints. In our case it is that of the plane-polarized gravitational waves supported by the null Neveu-Schwarz three-from fluxes. Whereas the consistency of the open string background is apparent from the sigma-model analysis, the symmetry preserved by the underlying D-brane is most easily seen from the algebraic study which we shall turn to in the following section. This concludes the classical analysis. We shall turn our attention to the quantum algebra, the OPEs, of the currents at the boundary.

3. Current Algebra

The gluing matrix relates the left-moving currents with the right-moving ones as in (eq.2.6). This particular linear combination of the chiral currents, dictated by the Neumann boundary condition, will be shown in this section, to obey the same Kac-Moody algebra as the original closed-string theory in the bulk. This combination of chiral currents form a close algebra: an OPE of any two such "boundary" currents does not take one outside of this set of four boundary currents. The other linear combination with a relative minus sign between the left and the right chiral currents, corresponding to Dirichlet boundary condition, would not form a close algebra. Alternatively in the closed string picture, we can construct boundary operators, Q^a, as the conserved charges of these boundary currents. Q^a will have to annihilate the boundary state [41] representing this D3-brane. Any commutators $[Q^a, Q^b]$ will have to annihilate this very boundary state. This again implies that it is consistent to impose Neumann boundary condition on all of these four directions. The underlying D3-brane, or the boundary state, thus preserves half of infinite dimensional $\mathbf{G}(z) \times \mathbf{G}(\bar{z})$ symmetries of the WZW model. This is analogous to the BPS property of D-branes–preserving half of the spacetime supersymmetry. Given that the gluing condition is actually an inner automorphism of the algebra it is perhaps not surprising to see part of the original symmetry get preserved after all.

To prove our claims let us recall that the bulk currents satisfy the OPE:

$$\mathcal{J}(z)\,\mathcal{J}^{\pm}(w) \sim \pm i\,\frac{\mathcal{J}^{\pm}(z)}{z-w} \tag{3.1}$$

$$\mathcal{J}^{+}(z)\,\mathcal{J}^{-}(w) \sim \frac{1}{(z-w)^2} + i\,\frac{\mathcal{T}(z)}{z-w} \tag{3.2}$$

$$\mathcal{T}(z)\,\mathcal{J}(w) \sim \frac{1}{(z-w)^2}\ . \tag{3.3}$$

These OPEs are easily realized using the Wakimoto "free-field" representation introduced in [37]. For this purpose we parametrize the group element by

$$g = e^{\gamma J^-}\,e^{uJ+\tilde{v}T}\,e^{\bar{\gamma}J^+} \tag{3.4}$$

where γ, $\bar{\gamma}$, u and $\tilde{v}$ are the new coordinates on the group manifold. They are related to the previous (geometric) coordinates, u, v, a and $\bar{a}$ by

$$\gamma = \bar{a}, \qquad \bar{\gamma} = e^{-iu}\,a, \qquad \tilde{v} = v + \frac{i}{2}\,a\bar{a} \tag{3.5}$$

Utilizing these free fields the chiral currents are easily obtained from their definitions:

$$\begin{array}{ll} -\mathcal{J}_L^{+}(z) = \partial\gamma_L + i\,\gamma_L\,\partial u_L & \mathcal{J}_R^{+}(\bar{z}) = \bar{\beta}_R \\ -\mathcal{J}_L^{-}(z) = \beta_L & \mathcal{J}_R^{-}(\bar{z}) = \bar{\partial}\bar{\gamma}_R + i\,\bar{\gamma}_R\,\bar{\partial}u_R \\ -\mathcal{J}_L(z) = \partial\tilde{v}_L - i\,\beta_L\,\gamma_L & \mathcal{J}_R(\bar{z}) = \bar{\partial}\tilde{v}_R - i\,\bar{\beta}_R\,\bar{\gamma}_R \\ -\mathcal{T}_L(z) = \partial u_L & \mathcal{T}_R(\bar{z}) = \bar{\partial}u_R \end{array} \tag{3.6}$$

where $\beta = e^{iu}\,\partial\bar{\gamma}$ and $\bar{\beta} = e^{iu}\,\bar{\partial}\gamma$. The free fields are contracted according to the following rules:

$$\begin{aligned} u(z)\,\tilde{v}(w) &\sim \ln(z-w) \\ \beta(z)\,\gamma(w) &\sim \frac{1}{z-w}\ . \end{aligned} \tag{3.7}$$

We can now proceed to verify that the linear combinations of the chiral currents in the left hand side of (eq.2.6) are consistent, *i.e.* the algebra closes onto itself.

$$\begin{aligned} &[\mathcal{J}_L^{+}(z) + e^{iu(z,\bar{z})}\mathcal{J}_R^{+}(\bar{z})][\mathcal{J}_L^{-}(w) + e^{-iu(w,\bar{w})}\mathcal{J}_R^{-}(\bar{w})] \\ &\sim \frac{1}{(z-w)^2} + \frac{1}{(\bar{z}-\bar{w})^2} + \frac{i\mathcal{T}_L(z)}{(z-w)} + \frac{i\mathcal{T}_R(\bar{z})}{(\bar{z}-\bar{w})} \end{aligned} \tag{3.8}$$

$$[\mathcal{J}_L(z) + \mathcal{J}_R(\bar{z})][\mathcal{J}_L^{+}(w) + e^{iu(w,\bar{w})}\mathcal{J}_R^{+}(\bar{w})] \sim i\left(\frac{\mathcal{J}_L^{+}(z)}{(z-w)} + \frac{e^{iu(z,\bar{z})}\mathcal{J}_R^{+}(\bar{z})}{(\bar{z}-\bar{w})}\right) \tag{3.9}$$

$$[\mathcal{J}_L(z) + \mathcal{J}_R(\bar{z})][\mathcal{J}_L^{-}(w) + e^{-iu(w,\bar{w})}\mathcal{J}_R^{-}(\bar{w})] \sim -i\left(\frac{\mathcal{J}_L^{-}(z)}{(z-w)} + \frac{e^{-iu(z,\bar{z})}\mathcal{J}_R^{-}(\bar{z})}{(\bar{z}-\bar{w})}\right) \tag{3.10}$$

$$[\mathcal{T}_L(z) + \mathcal{T}_R(\bar{z})][\mathcal{J}_L(w) + \mathcal{J}_R(\bar{w})] \sim \frac{1}{(z-w)^2} + \frac{1}{(\bar{z}-\bar{w})^2} \tag{3.11}$$

Let us remark that the phase $e^{iu(z,\bar{z})}$ being non-chiral is essential for the closure of the above OPEs:

$$\mathcal{J}_L(z) e^{iu(w,\bar{w})} \sim \frac{-ie^{iu(z,\bar{z})}}{z-w} \qquad\qquad \mathcal{J}_R(\bar{z}) e^{iu(w,\bar{w})} \sim \frac{ie^{iu(z,\bar{z})}}{\bar{z}-\bar{w}} \tag{3.12}$$

On taking $z \to \bar{z}$ and $w \to \bar{w}$ the contribution from these two OPEs exactly cancel each other at the boundary of the worldsheet, which accounts for the forms of the OPEs of the boundary currents. All other OPEs are regular. Notice however that the central charge of the system is the *total* of the left-moving and right-moving parts. One can also prove that the above currents are conserved and indeed generate the symmetry transformations advertised. This is left as an exercise to the readers. An interesting open question is to investigate the implication of this infinitely dimensional symmetry present in the D3-branes on the BPS property of the object. The interplay of these extra infinite-dimensional symmetries with the other (super)symmetries in the target space should also be understood better and will be reported in a separate publication.

Furthermore, using the free fields we can rewrite the boundary conditions (eq.1.4), (eq.1.5), (eq.1.6), (eq.1.7), as

$$\partial u_L - \bar{\partial} u_R = 0 \tag{3.13}$$
$$\partial \tilde{v}_L - \bar{\partial} \tilde{v}_R = 0 \tag{3.14}$$
$$\partial \bar{\gamma}_L - e^{-iHu_R} \bar{\partial}(e^{iHu_R} \bar{\gamma}_R) = 0 \tag{3.15}$$
$$e^{-iHu_L} \partial(e^{iHu_L} \gamma_L) - \bar{\partial} \gamma_R = 0 \tag{3.16}$$

where we have re-instated the dependence on the field strength, H, a real parameter. In the cases of the physical free fields like u and $\tilde{v}$ they are the familiar Neumann boundary conditions. The twisting (a phase factor) in the other two equations is due to the fact that $\bar{\gamma}_L$ and γ_R are not physical fields. They are related to the physical ones, $\bar{\gamma}_R$ and γ_L, respectively, at the boundary by nontrivial phases, e^{iHu_R} and e^{iHu_L}, respectively. The readers are referred to Section 3.2 of [37] for a detailed discussion on how the two sets of fields come about in NW model. This explains the form of the boundary conditions above.

To conclude we have successfully show that it is much more straightforward to study D-branes if one fully exploits the dual geometric and algebraic properties of WZW models. We demonstrate our technique by uncovering a spacefilling D-branes in a particular WZW model, that discovered by C. Nappi and E. Witten. This D-brane arises when one allows for field-dependent gluing matrix. The reflection matrix is shown to be an adjoint action by a particular group element, e^{uJ}, on the algebra. These gluing conditions on the chiral currents is shown to be identical to the

Neumann boundary condition of the open strings. The merit of the method is that it allows the geometric property D-branes to remain transparent by fully utilizing the geometric aspect of WZW models. In order for it to be useful however the technique should be generalized. On the conformal field theory side one would like to generalize the boundary states to reflect the field-dependence in gluing matrix. It is also educational to apply our techniques to the better understood models like $SL(2)$ and $SU(2)$ to see if one gets new insight into these theories. It also serves as a cross check for our techniques.

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