MILP-based Differential Attack on Round-reduced GIFT

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Abstract. At Asiacrypt 2014, Sun *et al.* proposed a MILP model[19] to search differential trails for bit-oriented block ciphers. In this paper, we improve this model to search differential characteristics of GIFT[3], a new lightweight block cipher proposed at CHES 2017. GIFT has two versions, namely GIFT-64 and GIFT-128. For GIFT-64, we find the best 12-round differential characteristic with MILP-based model and give a key-recovery attack on 19-round GIFT-64. For GIFT-128, we find a 20-round differential characteristic and give the first attack on 25-round GIFT-128.

Keywords: GIFT, Differential Cryptanalysis, Lightweight Block Cipher, MILP

1 Introduction

In recent years, research on lightweight block ciphers has received a lot of attentions. Lightweight block ciphers are widely used in Internet of things and wireless communication because their structures are simple and they can be run in low-power environment. Many lightweight block ciphers such as PRESENT[6], CLEFIA[16], LED[10], PRINCE[7], SIMON and SPECK[4] have been published in last decades. GIFT[3] is a new lightweight block cipher proposed by Banik *et al.* at CHES 2017, which is designed to celebrate 10 years of PRESENT. GIFT has an SPN structure which is similar to PRESENT. It has two versions, namely GIFT-64 and GIFT-128, whose block sizes are 64 and 128, and the round numbers are 28 and 40 respectively.

Recently, many classical cryptanalysis methods could be converted to mathematical optimization problems which aims to achieve the minimal or maximal value of an objective function under certain constraints. Mixed-integer Linear Programming (MILP) is the most widely studied technique to solve these optimization problems. One of the most successful applications of MILP is to search differential and linear trails. Mouha *et al.* first applied MILP method to count active S-boxes of word-based block ciphers[12]. Then, at Asiacrypt 2014, Sun *et al.* extended this technique to search differential and linear trails[19], whose main idea is to derive some linear inequalities through the H-Representation of the convex hull of all differential patterns of S-box. Xiang *et al.*[20] introduced a MILP model to search integral distinguisher, Sasaki *et al.*[15] and Cui *et al.*[8] gave the MILP-based impossible differential search model independently. There are many MILP-based tools proposed already, such as MILP-based differential/linear search model for ARX ciphers[9], MILP-based conditional cube attacks[11] on Keccak[5], etc.

Our Contributions

The designers of GIFT provided the various cryptanalysis[3] on GIFT. They use MILP to compute the lower bounds for the number of active S-boxes in both differential cryptanalysis firstly. And then round-reduced differential differential probability of GIFT is presented. For GIFT-64, they provided a 9-round differential probability of $2^{-44.415}$ and they expected that the differential probability of 13-round GIFT-64 will be lower than 2^{-63}. For GIFT-128, they provided a 9-round differential probability of 2^{-47} and they expected that the differential probability of 26-round GIFT-128 will be lower than 2^{-127}. The designers didn't present actual attack on GIFT in [3].

In this paper, we generalize an efficient two-stage MILP-based model inspired by Sun *et al.*'s two-stage model[17]. Our model includes two interactive submodels, denoted as outer-MILP and inner-MILP part. The outer-MILP part obtains the minimal active S-boxes, namely, the truncated differential. And then the inner-MILP part produce the differential characteristic that matches the truncated differential with maximal probability. With our two-stage model, we find some differential characteristics of GIFT-64. Moreover, using the 12-round differential characteristic with probability of $2^{-59.415}$, we give an attack on 19-round reduced GIFT-64 (out of 28 full rounds) with time complexity $2^{111.2}$, memory complexity 2^{94} and data complexity $2^{62.4}$.

In addition, we also improved our search model to find differential characteristic of GIFT-128. Firstly, the algorithm solves a sub-MILP-model to obtain an acceptable differential trail with small number of rounds. Then the produced output difference serves as input difference of the following sub-MILP-model. The sub-MILP-model is iterated until the probability of the whole differential trail is higher than our given bound. Using our algorithm, we find some new differential characteristics, including a new 20-round differential trail with probability 2^{-122}. Using the 20-round differential characteristic we give the first attack on 25-round reduced GIFT-128 (out of 40 full rounds).

The summary of differential analysis of GIFT is shown in Table 1.

Table 1. Summary of cryptography analysis on GIFT

	Type	Rounds	Time	Memory	Data	Source
GIFT-64	Integral	14	-	-	-	[3]
GIFT-64	Differential	19	$2^{111.4}$	2^{94}	$2^{62.4}$	Ours
GIFT-128	Differential	25	2^{125}	2^{61}	2^{125}	Ours

2 Parliminaries

2.1 Description of GIFT

GIFT has an SPN structure which is similar to PRESENT. It has two versions, namely GIFT-64 and GIFT-128, whose block sizes are 64 and 128 and round numbers are 28 and 40 respectively. Both versions have a key length of 128 bit.

Each round of GIFT consists of 3 steps: SubCells, PermBits and AddRoundKey. The round function of GIFT-64 is shown in Figure 1. Similarly, GIFT-128 adopts 32 4-bit S-boxes for each round.

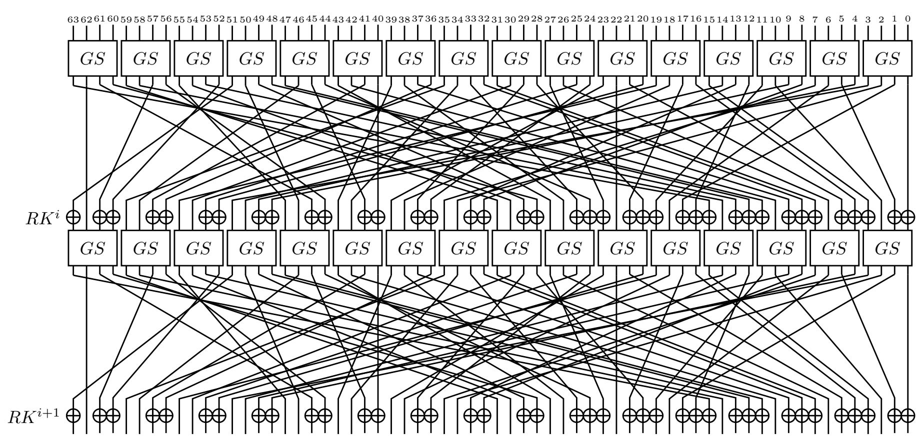

Fig. 1. 2 Rounds of GIFT-64

SubCells Both versions of GIFT use the same invertible 4-bit S-box, which is the only nonlinear component of the algorithm. The action of this S-box in hexadecimal notation is given in Table 2.

Table 2. Sbox of GIFT

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
GS(x)	1	a	4	c	6	f	3	9	2	d	b	7	5	0	8	e

PermBits The bit permutation used in GIFT-64 and GIFT-128 are given in Table 3.

Table 3. Specifications of GIFT Bit Permutation

GIFT-64	i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	$P_{64}(i)$	0	33	66	99	96	1	34	67	64	97	2	35	32	65	98	3
	i	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
	$P_{64}(i)$	4	37	70	103	100	5	38	71	68	101	6	39	36	69	102	7
	i	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
	$P_{64}(i)$	8	41	74	107	104	9	42	75	72	105	10	43	40	73	106	11
	i	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
	$P_{64}(i)$	12	45	78	111	108	13	46	79	76	109	14	47	44	77	110	15
GIFT-128	i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	$P_{128}(i)$	0	33	66	99	96	1	34	67	64	97	2	35	32	65	98	3
	i	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
	$P_{128}(i)$	4	37	70	103	100	5	38	71	68	101	6	39	36	69	102	7
	i	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
	$P_{128}(i)$	8	41	74	107	104	9	42	75	72	105	10	43	40	73	106	11
	i	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
	$P_{128}(i)$	12	45	78	111	108	13	46	79	76	109	14	47	44	77	110	15
	i	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79
	$P_{128}(i)$	16	49	82	115	112	17	50	83	80	113	18	51	48	81	114	19
	i	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95
	$P_{128}(i)$	20	53	86	119	116	21	54	87	84	117	22	55	52	85	118	23
	i	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
	$P_{128}(i)$	24	57	90	123	120	25	58	91	88	121	26	59	56	89	122	27
	i	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127
	$P_{128}(i)$	28	61	94	127	124	29	62	95	92	125	30	63	60	93	126	31

AddRoundKey The round key RK is extracted from the key state. A round key is *first* extracted from the key state before the key state update.

For GIFT-64, two 16-bit words of the key state are extracted as the round key $RK = U||V$. U and V are XORed to b_{4i+1} and b_{4i} of the cipher state respectively. b_i represents the *i-th* bit of the cipher state. u_i and v_i represent the *i-th* bit of

U and V.

$$U \leftarrow k_1, V \leftarrow k_0$$

$$b_{4i+1} \leftarrow b_{4i+1} \oplus u_i, b_{4i} \leftarrow b_{4i} \oplus v_i, \forall i \in \{0, \cdots, 15\}$$

For GIFT-128, four 16-bit words of the key state are extracted as the round key $RK = U||V$. U and V are XORed to b_{4i+2} and b_{4i+1} of the cipher state respectively.

$$U \leftarrow k_5||k_4, V \leftarrow k_1||k_0$$

$$b_{4i+2} \leftarrow b_{4i+2} \oplus u_i, b_{4i+1} \leftarrow b_{4i+1} \oplus v_i, \forall i \in \{0, \cdots, 31\}$$

The key state for two versions are updated as follows,

$$k_7||k_6||\cdots||k_1||k_0 \leftarrow k_1 \ggg 2||k_0 \ggg 12||\cdots||k_3||k_2$$

Round Constants For both versions of GIFT, a single bit "1" and a 6-bit round constant $C = \{c_5, c_4, c_3, c_2, c_1, c_0\}$ are XORed into the cipher state at bit position *n-1*,23,19,15,11,7,3 respectively. For GIFT-64, *n-1* is 63 and for GIFT-128, *n-1* is 127. $\{c_5, c_4, c_3, c_2, c_1, c_0\}$ are initialized to "0", and they are updated as follow:

$$(c_5, c_4, c_3, c_2, c_1, c_0) \leftarrow (c_4, c_3, c_2, c_1, c_0, c_5 \oplus c_4 \oplus 1)$$

2.2 Notations

K_i^j	The *j-th* bit of the *i-th* round key
ΔP	The differential in the plaintext
ΔX_S^i	The differential in the output of the *i-th* round's Sbox
ΔX_P^i	The differential in the output of the *i-th* round's Permutation
ΔX_K^i	The differential in the output of the *i-th* round's AddKey
$\Delta X_{S,P,K}^i$	ΔX_S^i or ΔX_P^i or ΔX_K^i
$\Delta X_{S,P,K}^i\{m\}$	The *m-th* bit of $\Delta X_{S,P,K}^i$
$\Delta X_{S,P,K}^i\{m_l\text{-}m_t\}$	The (m_t-m_l+1) bits totally from the m_l-*th* bit to the m_t-*th* bit of $\Delta X_{S,P,K}^i$

3 Related Works

3.1 Mouha *et al.*'s Framework for Word-Oriented Block Ciphers

Mouha *et al.*[13] become the first to introduce MILP model to count the number of differentially active S-boxes for word-oriented block ciphers.

Definition 1. *Consider a string Δ consisting of n bytes $\Delta = (\Delta_0, \Delta_1, \ldots, \Delta_{n-1})$. Then, the difference vector $x = (x_0, x_1, \ldots, x_{n-1})$ corresponding to Δ is defined as*

$$x_i = \begin{cases} 0 & if \Delta_i = 0, \\ 1 & otherwise. \end{cases} \tag{1}$$

Based on Definition 1, Mouha *et al.* translated the XOR operation and the linear transformation to linear inequalities as follows:

- **Equations describing the XOR operation:** Suppose the input difference vector for the XOR operation be $(x^{\oplus}_{in1}, x^{\oplus}_{in2})$ and the corresponding output difference vector be $x^{\oplus}_{out}$. The following constraints will make sure that when $x^{\oplus}_{in1}$, $x^{\oplus}_{in2}$ and $x^{\oplus}_{out}$ are not all zero, then there are at least two of them are nonzero:
$$\begin{cases} x^{\oplus}_{in1} + x^{\oplus}_{in2} + x^{\oplus}_{out} \geq 2d_{\oplus} \\ d_{\oplus} \geq x^{\oplus}_{in1}, d_{\oplus} \geq x^{\oplus}_{in2}, d_{\oplus} \geq x^{\oplus}_{out} \end{cases} \tag{2}$$
where $d_{\oplus}$ is a dummy variable taking values in {0,1}.
- **Equations describing the linear transformation:** Assume linear transformation L transforms the input difference vector $(x^L_1, x^L_2, \ldots, x^L_{m-1})$ to the output difference vector $(y^L_1, y^L_2, \ldots, y^L_{m-1})$. Given the differential branch number $\mathcal{B}_{\mathcal{D}}$. The following constraints can describe the relation between the input and output difference vectors, they should be subject to:
$$\begin{cases} \sum_i^{m-1} x^L_i + \sum_i^{m-1} y^L_i \geq \mathcal{B}_{\mathcal{D}} d^L \\ d^L \geq x^L_i, d^L \geq y^L_i, i \in \{0, ..., m-1\} \end{cases} \tag{3}$$
where d^L is a dummy variable taking values in {0,1}.

3.2 Sun *et al.*'s Framework for Bit-Oriented Block Ciphers

At Asiacrypt 2014, Sun *et al.*[19] extended Mouha *et al.*'s framework[13] to bit-oriented ciphers. For bit-oriented ciphers, Mouha *et al.*'s descriptions of XOR operation and linear transformation are also suitable.

Definition 2. *Consider a string Δ consisting of n bits $\Delta = (\Delta_0, \Delta_1, \ldots, \Delta_{n-1})$. Then, the difference vector $x = (x_0, x_1, \ldots, x_{n-1})$ corresponding to Δ is defined as*

$$x_i = \begin{cases} 0 & if \Delta_i = 0, \\ 1 & if \Delta_i = 1. \end{cases} \tag{4}$$

Based on Definition 2, Sun *et al.* translated the S-box operation to linear inequalities as follow:

– **Equations describing the S-box operation** Suppose $(x_0, \ldots, x_{w-1})$ and $(y_0, \ldots, y_{v-1})$ are the input and output bit-level differences of an $w \times v$ S-box. A is a dummy variable taking values in $\{0,1\}$ to describe whether the S-box is active or not. $A = 1$ holds if and only if $x_0, x_1, \ldots, x_{w-1}$ are not all zero. The following constraints should be obeyed:

$$\begin{cases} A - x_i \geq 0, i \in \{0, \ldots, w-1\} \\ \sum_{i}^{w-1} x_i - A \geq 0 \end{cases} \tag{5}$$

3.3 Valid Cutting-off Inequalities from the Convex Hull of S-box

The convex hull of a set Q of discrete points in $\mathbb{R}^n$ is the smallest convex that contains Q. A convex hull in $\mathbb{R}^n$ can be described as the common solutions of a set of finitely many linear equalities and inequalities.

If we treat a differential of an $w \times v$ S-box as a discrete point in $\mathbb{R}^{w+v}$, then we can get a set of finitely many discrete points which includes all possible differential patterns of this S-box. Suppose $p = (x, y) = (x_0, \ldots, x_{w-1}, y_0, \ldots, y_{v-1})$ is a differential pattern of an $w \times v$ S-box, in which x is the input difference vector and y is the output difference vector. If a differential pattern p is possible, it belongs to the set of the possible differential patterns of S-box. As a result, we can describe this finitely set with the following inequalities:

$$\begin{cases} \alpha_{0,0}x_0 + \ldots + \alpha_{0,w-1}x_{w-1} + \beta_{0,0}y_0 + \ldots + \beta_{0,v-1}y_{v-1} + \gamma_0 \geq 0 \\ \ldots \\ \alpha_{n,0}x_0 + \ldots + \alpha_{n,w-1}x_{w-1} + \beta_{n,0}y_0 + \ldots + \beta_{n,v-1}y_{v-1} + \gamma_n \geq 0 \end{cases} \tag{6}$$

This is called the H-Representation of a $w \times v$ S-box. With the help of SageMath[1], hundreds of linear inequalities can be derived by differential patterns of S-box. The number of inequalities is very large in general, for example, the number of inequalities of GIFT S-box given by SageMath is 237. Adding all of them to the MILP model will make it insolvable in practical time because the efficiency of a MILP model is reduced radically when the amount of linear inequalities increase. To overcome it, Sun *et al.* invented a greedy algorithm in [19] for selecting inequalities from the convex hull.

In order to minimize the number of the set of inequalities, Sasaki *et al.* raised a MILP-based reduction algorithm in [14] to find the optimal combination with minimal number of linear inequalities from hundreds of inequalities in the H-representation of the convex hull, which remove all the impossible differential patterns of S-box. The algorithm considers each impossible pattern in the DDT of S-box. An impossible pattern should be excluded from the solution space by at least one inequality. Under these constraints, we can minimize the number of inequalities by using MILP model.

4 MILP-based Model to Search Differential Characteristic For GIFT-64

4.1 MILP-based two-stage algorithm to search differential characteristic

In [17], Sun *et al.* raised a two-stage search algorithm to find differential path of block ciphers. In Sun *et al.*'s model, truncated differential characteristics with minimal active S-box will be found firstly, and then differential characteristics matching the truncated differential characteristic can be found in another model. Sun *et al.*'s model choose a prespecified threshold of the number of active S-box. However, it is possible that the characteristic with the highest probability do not have the minimal number of active S-box. Inspired by Sun *et al.*'s model, we design Algorithm 1 to search the best or better differential characteristic.

Algorithm 1 New differential characteristic searching algorithm based on Inner and Outer-MILP Loop

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Require: r round block ciphers; valid cutting-off inequalities from the convex hull of the S-box; m—number of S-boxes in one round.

Ensure: Minimal number of active S-boxes $MinSb$; differential characteristic with maximal probability.

1: Define MPr as the current minimal differential probability.

2: In the Outer-MILP part, construct an MILP model $\mathcal{M}_1$ describing the differential behavior of the cipher whose objective function is the minimal active S-boxes.

3: Initial $MPr = 2^{-200}$. Initial $MinSb$ as $r \times m$.

4: Solve the model $\mathcal{M}_1$ using an MILP optimizer.

5: **if** A feasible solution $\mathcal{TD}$ is found in $\mathcal{M}_1$, save it to a file. **then**

6:     $\diamondsuit$ *begin of Inner-MILP part*

7:     Construct an MILP model $\mathcal{M}_2$ describing the differential behavior of the cipher and add $\mathcal{TD}$ as a constraint to $\mathcal{M}_2$. The objective function is the characteristic with maximal probability.

8:     Solve the model using an MILP optimizer. If a feasible solution x is found, save x and its probability Pr to the file. If $Pr > MPr$, set MPr equal to Pr. (If only the minimal number of active S-boxes is required, it returns $MinSb = \sum A_{i,j}$.)

9:     $\diamondsuit$ *end of Inner-MILP part*

10: **end if**

11: Add the linear inequality $l^{(\mathcal{TD})}$ to remove the truncated differential $\mathcal{TD}$ from the feasible region of $\mathcal{M}_1$.

12: Solve $\mathcal{M}_1$ again, if a new solution $\mathcal{TD}$ is found, save it and go to step 5 (process the *Inner-MILP part*). Else go to step 12.

13: Terminate the procedure and extract all the best differential characteristics and their corresponding truncated differentials $\mathcal{TD}$. Extract the best characteristic with probability MPr.

Algorithm 1 does not need the predefined threshold and could get the characteristic with highest probability definitely. Algorithm 1 includes two interactive sub-models, denoted as outer-MILP part and inner-MILP part. The two stages are interactive. In the outer-MILP part, the objective function is the minimal active S-boxes. When a solution is found in the outer-MILP part, the truncated differential that contains the information of the positions of active S-boxes will input the inner-MILP part as constraints. In the inner-MILP part, it produces the differential characteristic with maximal probability that matches the truncated differential. Then the algorithm goes to the outer-MILP part with the truncated differential removed from its feasible region.

In addition, the maximal probability of the derived differential characteristic is also used to reduce the feasible region of the outer-MILP part dynamically. In details, if a differential characteristic with larger probability could be found in the next loops, the number of active S-boxes produced in outer-MILP part must be lower than a certain bound. The bound is dynamically computed by the current maximal probability. When the outer-MILP part is infeasible, the algorithm returned.

We apply Algorithm 1 to find a differential characteristic of GIFT-64, and get some interesting results.

4.2 Search Differentials of GIFT-64

Algorithm 1 needs two kinds convex hulls about the S-box in the outer-MILP part and the inner-MILP part respectively. First, we compute the H-presentation of convex hull of differential patterns of S-box in Appendix A. Using SageMath, 237 inequalities are produced in the H-Representation of the convex hull of GIFT S-box, then after selecting inequalities by the method introduced in [14], we get 21 inequalities. Second, we study the convex hull of differential patterns with probabilities of the S-box. Sun *et al.* introduced the differential distribution probability of S-box to MILP-model in [18]. Since, for GIFT S-box, there are 4 possible probabilities, i.e. 1, $2^{-1.415}$, 2^{-2}, 2^{-3}, we need three extra bits (p_0, p_1, p_2) to encode the differential patterns with probability. The new differential pattern is $(x_0, x_1, x_2, x_3, y_0, y_1, y_2, y_3; p_0, p_1, p_2) \in \mathbb{F}_2^{8+3}$ which satisfies Equation 7.

$$\begin{cases} (p_0, p_1, p_2) = (0,0,0), \text{if } \Pr_s[(x_0, x_1, x_2, x_3) \to (y_0, y_1, y_2, y_3)] = 1 = 2^{-0} \\ (p_0, p_1, p_2) = (0,0,1), \text{if } \Pr_s[(x_0, x_1, x_2, x_3) \to (y_0, y_1, y_2, y_3)] = 6/16 = 2^{-1.415} \\ (p_0, p_1, p_2) = (0,1,0), \text{if } \Pr_s[(x_0, x_1, x_2, x_3) \to (y_0, y_1, y_2, y_3)] = 4/16 = 2^{-2} \\ (p_0, p_1, p_2) = (1,0,0), \text{if } \Pr_s[(x_0, x_1, x_2, x_3) \to (y_0, y_1, y_2, y_3)] = 2/16 = 2^{-3} \end{cases} \tag{7}$$

Then the objective function is changed to minimize $\sum(3 \times p_0 + 2 \times p_1 + 1.415 \times p_2)$.

Implement the differential search of GIFT-64 according to Algorithm 1, in the Outer-MILP part, the objective function is to minimize active S-boxes. We get the tighter bound of number of active S-boxes for 11 and 12-round reduced GIFT-64, which are 22 and 24 respectively. In addition, we get a 11-round differential characteristic with probability 2^{-48}, two 12-round differential characteristics with probability 2^{-58} and $2^{-59.415}$ shown in Table 4, and a 13-round characteristic with probability 2^{-64}. Note that the designers of GIFT claimed that the differential probability of 13-round GIFT-64 will be lower than 2^{-63}. Our result does not violate the claim, however the gap is very small.

Table 4. 12-round Differential Path with Probability 2^{-58} and $2^{-59.415}$

Round	Differential-1	Probability	Differential-2	Probability
Input	0000 0000 0000 4040	1	0000 0000 0404 0000	1
1st round	0000 0005 0000 0005	2^{-4}	0050 0000 0050 0000	2^{-4}
2nd round	0000 0202 0000 0000	2^{-10}	0000 0000 2020 0000	2^{-10}
3rd round	0500 0000 0500 0000	2^{-14}	00a0 0000 00a0 0000	2^{-16}
4th round	0000 0000 0000 2020	2^{-20}	0000 1010 0000 0000	2^{-20}
5th round	0000 0005 0000 0005	2^{-24}	0a00 0000 0a00 0000	2^{-26}
6th round	0000 0202 0000 0000	2^{-30}	0000 0000 1010 0000	2^{-30}
7th round	0500 0000 0500 0000	2^{-34}	00a0 0000 00a0 0000	2^{-36}
8th round	0000 0000 0000 2020	2^{-40}	0000 1010 0000 0000	2^{-40}
9th round	0000 0005 0000 0005	2^{-44}	0a00 0000 0a00 0000	2^{-46}
10th round	0000 0202 0000 0000	2^{-50}	0000 0000 1010 0000	2^{-50}
11th round	0500 0000 0500 0000	2^{-54}	0000 0000 00a0 0040	2^{-56}
12th round	0f00 0000 0f00 0000	2^{-58}	0000 0000 0010 0070	$2^{-59.415}$

4.3 Attack on 19-round GIFT-64

Using the 12-round differential characteristic with probability $2^{-59.415}$ in Table 4, we could launch a key-recovery attack against 19-round GIFT-64. We choose the *differential-2* rather than the *differential-1* because the first and last round state of the *differential-2* is easier to extend and it is more effective. As shown in Table 5, we add 3 rounds in its beginning and 4 rounds at the end of the *differential-2*. Therefore, we can attack 19-round GIFT-64. According to the key schedule, the round key used in 1-*st*, 2-*nd*, 16-*th*, 17-*th*, 18-*th* and 19-*th* round corresponds to (k_1, k_0), (k_3, k_2), $(k_7 \ggg 6, k_6 \ggg 4)$, $(k_1 \ggg 8, k_0)$, $(k_3 \ggg 8, k_2)$ and $(k_5 \ggg 8, k_4)$ in initial key state $(k_7, k_6, k_5, k_4, k_3, k_2, k_1, k_0)$, respectively.

Data collection

Since GIFT-64 does not have whiten-key layer at the beginning, after the P permutation of the first round, we could build 2^n structures. Each structure traverses the 16 bits undetermined in ΔX_P^1, i.e. the bit labeled by "?" in ΔX_P^1 of Table 5, thus it can generate $2^{16\times 2-1} = 2^{31}$ pairs obeying the differential. Therefore, 2^n structures can generate $2^n \times 2^{31} = 2^{n+31}$ pairs.

ΔP	???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ????
ΔX_S^1	000? ?000 0?00 00?0 000? ?000 0?00 00?0 000? ?000 0?00 00?0 000? ?000 0?00 00?0
ΔX_P^1	0000 0000 0000 0000 0000 0000 0000 0000 ???? ???? ???? ???? 0000 0000 0000 0000
ΔX_K^1	0000 0000 0000 0000 0000 0000 0000 0000 ???? ???? ???? ???? 0000 0000 0000 0000
ΔX_S^2	0000 0000 0000 0000 0000 0000 0000 0000 10?0 010? 10?0 010? 0000 0000 0000 0000
ΔX_P^2	0000 0000 0000 0000 0000 0000 11?? 0000 0000 0000 0000 0000 0000 0000 11?? 0000
ΔX_K^2	0000 0000 0000 0000 0000 0000 11?? 0000 0000 0000 0000 0000 0000 0000 11?? 0000
ΔX_S^3	0000 0000 0000 0000 0000 0000 0100 0000 0000 0000 0000 0000 0000 0000 0100 0000
ΔX_P^3	0000 0000 0000 0000 0000 0000 0000 0000 0000 0100 0000 0100 0000 0000 0000 0000
ΔX_K^3	0000 0000 0000 0000 0000 0000 0000 0000 0000 0100 0000 0100 0000 0000 0000 0000
4th round	0000 0000 0000 0000 0000 0000 0000 0000 0000 0100 0000 0100 0000 0000 0000 0000
$\vdots$	$\vdots$
15th round	0000 0001 0001 0000 0000 0000 0000 0000 0000 0000 0000 0100 0000 0000 0000 0010
ΔX_S^{16}	0000 0000 ???? ???? 0000 0000 0000 0000 0000 0000 0000 ???? 0000 0000 0000 ????
ΔX_P^{16}	?00? 0000 ?000 ?000 ??00 0000 0?00 0?00 0??0 0000 00?0 00?0 00?? 0000 000? 000?
ΔX_K^{16}	?00? 0000 ?000 ?000 ??00 0000 0?00 0?00 0??0 0000 00?0 00?0 00?? 0000 000? 000?
ΔX_S^{17}	???? 0000 ???1 ???1 ???? 0000 ???? ???? ???? 0000 ???? ???? ???? 0000 ???? ????
ΔX_P^{17}	??01 ??0? ??0? ??0? ???0 ???0 ???0 ???0 0??? 0??? 0??? 0??? ?0?1 ?0?? ?0?? ?0??
ΔX_K^{17}	??01 ??0? ??0? ??0? ???0 ???0 ???0 ???0 0??? 0??? 0??? 0??? ?0?1 ?0?? ?0?? ?0??
ΔX_S^{18}	???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ????
ΔX_P^{18}	???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ????
ΔX_K^{18}	???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ????
ΔX_S^{19}	???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ????
ΔX_P^{19}	???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ????
ΔX_K^{19}	???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ???? ????

Table 5. 19-round Differential Attack on GIFT-64

For such a pair, it has an average probability of 2^{-16} to meet the differential in 4-*th* round in Table 5. Then, the pair encrypted with the right key will obey the differential after 15th round with probability of $2^{-59.415}$. While the pair with a wrong key will obey it with a random probability of 2^{-64}. Therefore, with the right key guess, $2^{n+31} \times 2^{-16} \times 2^{-59.415} = 2^{n-44.415}$ pairs will obey the differential after 15th round. Here we choose $n = 46.4$. So the data complexity is $2^{46.4} \times 2^{16} = 2^{62.4}$.

Key recovery

When processing the key recovery, the guessing key bits include: k_1^7, k_1^6, k_1^5, k_1^4, k_0^7, k_0^6, k_0^5, k_0^4 in 1st round, k_3^9, k_2^9, k_3^1, k_2^1 in 2nd round; k_7^3, k_7^2, k_7^{10}, k_7^6, k_6^1, k_6^0, k_6^8, k_6^4 in 16th round, k_1^7, k_1^5, k_1^4, k_1^3, k_1^1, k_1^0, k_1^{15}, k_1^{13}, k_1^{12}, k_1^{11}, k_1^9, k_1^8, k_0^{15}, k_0^{13}, k_0^{12}, k_0^{11}, k_0^9, k_0^8, k_0^7, k_0^5, k_0^4, k_0^3, k_0^1, k_0^0 in 17th round, as well as all 64 key bits in 18th, 19th round. Totally, we construct 2^{94} counters for the possible values of the 94 key bits above. The whole attack procedure is a guess and filter approach. Guess two key bits k_1^4, k_0^4, then we can partially encrypt the plaintexts.

As the middle values of right pairs should obey $\Delta X_S^2\{17\} = 0$, $\Delta X_S^2\{18\} = 1$, $\Delta X_S^2\{19\} = 0$, the $(plaintext, ciphertext)$ pairs can be filtered with a probability of 2^{-3}. Similarly, guessing k_1^i, k_0^i, $i = 5, 6, 7$ and partially encrypt, corresponding

Round	Key bit
1st round	$k_1^{15}, k_1^{14}, k_1^{13}, k_1^{12}, k_1^{11}, k_1^{10}, k_1^{9}, k_1^{8}, k_1^{7}, k_1^{6}, k_1^{5}, k_1^{4}, k_1^{3}, k_1^{2}, k_1^{1}, k_1^{0}$
	$k_0^{15}, k_0^{14}, k_0^{13}, k_0^{12}, k_0^{11}, k_0^{10}, k_0^{9}, k_0^{8}, k_0^{7}, k_0^{6}, k_0^{5}, k_0^{4}, k_0^{3}, k_0^{2}, k_0^{1}, k_0^{0}$
2nd round	$k_3^{15}, k_3^{14}, k_3^{13}, k_3^{12}, k_3^{11}, k_3^{10}, k_3^{9}, k_3^{8}, k_3^{7}, k_3^{6}, k_3^{5}, k_3^{4}, k_3^{3}, k_3^{2}, k_3^{1}, k_3^{0}$
	$k_2^{15}, k_2^{14}, k_2^{13}, k_2^{12}, k_2^{11}, k_2^{10}, k_2^{9}, k_2^{8}, k_2^{7}, k_2^{6}, k_2^{5}, k_2^{4}, k_2^{3}, k_2^{2}, k_2^{1}, k_2^{0}$
16th round	$k_7^{5}, k_7^{4}, k_7^{3}, k_7^{2}, k_7^{1}, k_7^{0}, k_7^{15}, k_7^{14}, k_7^{13}, k_7^{12}, k_7^{11}, k_7^{10}, k_7^{9}, k_7^{8}, k_7^{7}, k_7^{6}$
	$k_6^{3}, k_6^{2}, k_6^{1}, k_6^{0}, k_6^{15}, k_6^{14}, k_6^{13}, k_6^{12}, k_6^{11}, k_6^{10}, k_6^{9}, k_6^{8}, k_6^{7}, k_6^{6}, k_6^{5}, k_6^{4}$
17th round	$k_1^{7}, k_1^{6}, k_1^{5}, k_1^{4}, k_1^{3}, k_1^{2}, k_1^{1}, k_1^{0}, k_1^{15}, k_1^{14}, k_1^{13}, k_1^{12}, k_1^{11}, k_1^{10}, k_1^{9}, k_1^{8}$
	$k_0^{15}, k_0^{14}, k_0^{13}, k_0^{12}, k_0^{11}, k_0^{10}, k_0^{9}, k_0^{8}, k_0^{7}, k_0^{6}, k_0^{5}, k_0^{4}, k_0^{3}, k_0^{2}, k_0^{1}, k_0^{0}$
18th round	$k_3^{7}, k_3^{6}, k_3^{5}, k_3^{4}, k_3^{3}, k_3^{2}, k_3^{1}, k_3^{0}, k_3^{15}, k_3^{14}, k_3^{13}, k_3^{12}, k_3^{11}, k_3^{10}, k_3^{9}, k_3^{8}$
	$k_2^{15}, k_2^{14}, k_2^{13}, k_2^{12}, k_2^{11}, k_2^{10}, k_2^{9}, k_2^{8}, k_2^{7}, k_2^{6}, k_2^{5}, k_2^{4}, k_2^{3}, k_2^{2}, k_2^{1}, k_2^{0}$
19th round	$k_5^{7}, k_5^{6}, k_5^{5}, k_5^{4}, k_5^{3}, k_5^{2}, k_5^{1}, k_5^{0}, k_5^{15}, k_5^{14}, k_5^{13}, k_5^{12}, k_5^{11}, k_5^{10}, k_5^{9}, k_5^{8}$
	$k_4^{15}, k_4^{14}, k_4^{13}, k_4^{12}, k_4^{11}, k_4^{10}, k_4^{9}, k_4^{8}, k_4^{7}, k_4^{6}, k_4^{5}, k_4^{4}, k_4^{3}, k_4^{2}, k_4^{1}, k_4^{0}$

Table 6. Round Keys of GIFT-64

conditions in $\Delta X_S^2\{20, 22, 23\}$, $\Delta X_S^2\{25, 26, 27\}$, $\Delta X_S^2\{28, 30, 31\}$ can filter the pairs with 2^{-3}. Totally 1st round provide a filtering probability of 2^{-12}.

Similarly, the encryption at 2-*nd*, 16-*th*, 17-*th*, 18-*th* round can filter the pairs with probability 2^{-4}, 2^{-16}, 2^{-30}, 2^{-18}, while all 32 key bits in 19th round need to be guessed. Thus, $2^{-2.6}$ pairs will be left for a random key, while 4 pairs should be left for a right key.

The time complexity is $2^2 \times 2^{31+46.4} \times 2^{32} = 2^{111.4}$, the data complexity is $2^{62.4}$ and the memory complexity is 2^{94}.

5 Improved MILP-based Method to Find Differential for GIFT-128

GIFT-128 adopts 128-bit state and has 32 4-bit S-boxes in each round. The variables and constrains are twice as many as GIFT-64. The designers of GIFT[2] gives 9-round differential trials on GIFT-128. We test Algorithm 1 on 9-round GIFT-128 and obtain the designers' conclusion. But it costs days to solve. In this section, we devise a segmented MILP-based method to find longer differential trail of GIFT-128.

Suppose we aim to find r-round differential characteristic for a block cipher. We first divide it as r_i-round $(i = 1, 2, ..., t)$ sub ciphers and $\sum_1^t r_i = r$. We choose probability thresholds for $r_1, r_2, ..., r_t$-round ciphers as $P_{r_1}, P_{r_2}, ..., P_{r_t}$, so that the probability p_{r_i} for r_i-round sub-cipher should be larger than P_{r_i}. Choose a threshold value P_{target} for r-round. If $p_{r_1}p_{r_2}\dots p_{r_t}$ is larger than P_{target}, an acceptable solution is founded.

As shown in Figure 2, for r_i-round sub-cipher, the input difference are fixed as the output difference of the differential characteristic $\mathcal{D}_{i-1}$ of r_{i-1}-round sub-cipher, and construct the MILP model $\mathcal{M}_{r_i}$. If $\mathcal{M}_{r_i}$ is feasible, we continue to construct $\mathcal{M}_{r_{i+1}}$ for r_{i+1}-round sub-cipher; else, we remove $\mathcal{D}_{i-1}$ from $\mathcal{M}_{r_{i-1}}$, and solve it again. The search terminates until we find the differential characteristics of $r_1, r_2, ..., r_t$-round sub-ciphers that could be connected to produce a r-round differential characteristic.

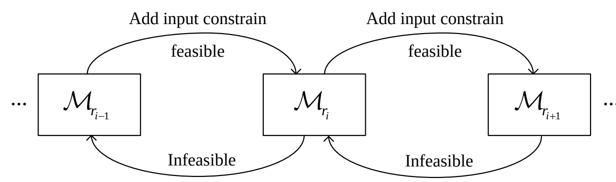

Fig. 2. The framework of our search algorithm

We apply this model to search differential characteristics of GIFT-128. It is indeed a heuristic and empirical process. For GIFT-128, it is time consuming to solve a more than 6-round MILP model. In order to keep the efficiency, we choose $r_i < 6$. P_{r_i} is chosen more flexible. According to the designers' analysis in [2], for 3/4/5-round GIFT-128, the minimum active S-boxes are 3, 5, and 7, respectively. The length of the sub-cipher can neither be too short nor be too long. If the number of rounds is smaller than 2, this sub-MILP-model is unnecessary to solve. On the other hand, if the number of rounds is bigger than 6 or 7, it costs too much time to solve the sub-model that we can't bear. We do not want the probability of r_i-round differential characteristic of GIFT-128 to be much smaller than the highest one. So P_{r_i} are chosen according to the minimum active S-boxes of r_i-round. In this section, we choose $P_{r_i=3} = 2^{-30}$, $P_{r_i=4} = 2^{-40}$ and $P_{r_i=5} = 2^{-50}$ to act as the exact lower bound of differential probability of each sub-model.

We use this model and the strategies above choosing parameters to search differential characteristics of GIFT-128. We list some results in Table 7.

Table 7. Probabilities of Some Differential Characteristics Of GIFT-128

Round	Parameters for r_i	Probability	Source
9	–	2^{-47}	[2]
12	$r_1 = r_2 = r_3 = r_4 = 3$	2^{-61}	Ours
16	$r_1 = r_2 = r_3 = r_4 = 3$ and $r_5 = 4$	2^{-85}	Ours
20	$r_1 = r_2 = r_3 = r_4 = 5$	2^{-122}	Ours

Table 8. 20 rounds Differential Path

Round	Input Difference	Probability
1st	0000 0c60 0000 000a 0000 0000 0000 0000	1
2nd	0601 0000 0000 0000 0000 0000 0000 0000	2^{-6}
3rd	0000 0000 a000 0000 0000 0000 0000 0000	2^{-11}
4th	0000 0000 0000 0000 0000 0000 0010 0000	2^{-13}
5th	0000 0000 0000 0000 0000 0080 0000 0000	2^{-16}
6th	0000 0200 0000 0100 0000 0000 0000 0000	2^{-18}
7th	0404 0000 0202 0000 0000 0000 0000 0000	2^{-23}
8th	5050 0000 0000 0000 5050 0000 0000 0000	2^{-31}
9th	a000 0000 0000 0000 0000 a000 0000 0000	2^{-43}
10th	0000 0000 0000 0000 0000 0000 1000 0100	2^{-47}
11th	0000 0084 0000 0042 0000 0000 0000 0000	2^{-53}
12th	0303 0000 0909 0000 0000 0000 0000 0000	2^{-64}
13th	5010 0000 0000 0000 5010 0000 0000 0000	2^{-76}
14th	0000 0000 0000 0000 a000 a000 0000 0000	2^{-88}
15th	0000 0000 0000 0000 0000 0000 0000 1100	2^{-92}
16th	0000 000c 0000 0006 0000 0000 0000 0000	2^{-98}
17th	0000 0000 0000 0000 0000 0000 0202 0000	2^{-102}
18th	0000 0000 0000 00a0 0000 0000 0000 00a0	2^{-108}
19th	0000 0000 0001 0001 0000 0000 0000 0000	2^{-112}
20th	0000 0000 0088 0000 0000 0000 0000 0000	2^{-118}
21st	0030 0000 0010 0000 0000 0000 0060 0000	2^{-122}

The 20-round characteristic, shown in Table 8, is constructed by the connection of the following four 5-round differential characteristics:

$$(00000c600000000a0000000000000000) \xrightarrow{5-r,\ 2^{-18}} (00000200000001000000000000000000)$$
$$(00000200000001000000000000000000) \xrightarrow{5-r,\ 2^{-35}} (00000084000000420000000000000000)$$
$$(00000084000000420000000000000000) \xrightarrow{5-r,\ 2^{-45}} (0000000c000000060000000000000000)$$
$$(0000000c000000060000000000000000) \xrightarrow{5-r,\ 2^{-24}} (00300000001000000000000000600000)$$

With the 20-round differential characteristic, we can add 3 rounds at its beginning and 2 rounds at the end to attack 25-round reduced GIFT-128. The attack procedure is similar to subsection 4.3. The time complexity is 2^{125} which is bounded by the data complexity and the memory complexity is 2^{61} bits to store the key counters.

6 Conclusion

In this paper, first, we design a more efficient MILP-based differential search model. Using this model, we give 12-round differential characteristic with probability 2^{-58} and get the first 19-round key-recovery attack on GIFT-64. Second, we improve our MILP-based model for block ciphers with large state size. With this model, we give 20-round differential characteristic with probability 2^{-122} and obtain the first 25-round key-recovery attack on GIFT-128.

MILP can efficiently find high-probabilistic differential trail when attacking algorithms whose permutation layer won't cause diffusion. In the future work, we can try to apply heuristic method to constrain global variables, so as to find a higher probability differential path.

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A Difference Distribution Table(DDT) of GIFT S-box

Table 9. DDT of GIFT S-box

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	2	2	0	2	2	2	2	2	0	0	2
2	0	0	0	0	0	4	4	0	0	2	2	0	0	2	2	0
3	0	0	0	0	0	2	2	0	2	0	0	2	2	2	2	2
4	0	0	0	2	0	4	0	6	0	2	0	0	0	2	0	0
5	0	0	2	0	0	2	0	0	2	0	0	0	2	2	2	4
6	0	0	4	6	0	0	0	2	0	0	2	0	0	0	2	0
7	0	0	2	0	0	2	0	0	2	2	2	4	2	0	0	0
8	0	0	0	4	0	0	0	4	0	0	0	4	0	0	0	4
9	0	2	0	2	0	0	2	2	2	0	2	0	2	2	0	0
a	0	4	0	0	0	0	4	0	0	2	2	0	0	2	2	0
b	0	2	0	2	0	0	2	2	2	2	0	0	2	0	2	0
c	0	0	4	0	4	0	0	0	2	0	2	0	2	0	2	0
d	0	2	2	0	4	0	0	0	0	0	2	2	0	2	0	2
e	0	4	0	0	4	0	0	0	2	2	0	0	2	2	0	0
f	0	2	2	0	4	0	0	0	0	2	0	2	0	0	2	2