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奇摄动常微分方程系统的数值解法

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摘要: 讨论奇摄动常微分方程系统的二点边界值问题,这是奇摄动问题中较难的部分. 文中介绍了多过渡点的选取方法,依此法构造不等距差分格式,在最大范数下证明新的差分格式关于摄动参数是一阶一致收敛. 多过渡点确定了网格划分从细网格到中等网格和粗网格的过渡,而 Shishkin Scheme (单过渡点法) 只将网格分为细网格和粗网格. 多过渡点法很好地拟合了边界层的性质,在实际应用中相当有效,其收敛阶也高于 Shishkin 网格法.

关键词: 奇摄动;系统问题;一致收敛;多过渡点;不等距网格

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许多实际问题,如液体的流动、化学反应、电解过程、半导体设备模型,最终可归结为奇摄动问题[1~3]. 这类问题会在边界层附近出现剧烈振荡现象,产生所谓的边界层函数,其解析解无法求出. 因此人们常采用数值解法解决奇摄动问题,尤其是关于摄动参数为一致收敛的数值方法.

奇摄动方程系统问题[4~6]是奇摄动问题中的重要部分,也是较困难的,关于这方面的成果相当少. 在文献[6]中 Matthews S 和 Miller J J H 等研究了方程系统(1)～(2),构造了 Shishkin 网格,即引入了一个过渡点将区间分为 2 个:边界层和非边界层. 在边界层,他们对网格点加细,而在非边界层,他们采用了粗网格点. 最后证明格式一致收敛,收敛阶为 $O(N^{-1}\ln(N))$.

本文研究方程系统(1)～(2),提出了多过渡点法,克服了 Shishkin 网格法的众多弱点,将收敛阶提高为 $O(N^{-1})$.

文中的 $C, C_1 \cdots k_1, k_2 \cdots$ 是指与 N, ε 无关的正常数.

1　微分方程及其性质

在 $x \in \Omega = (0,1)$ 研究奇摄动常微分方程系统的二点边界值问题

$$L_\varepsilon \vec{u_\varepsilon} = \begin{pmatrix} -\varepsilon\frac{d^2}{dx^2} & 0 \\ 0 & -\varepsilon\frac{d^2}{dx^2}\end{pmatrix}\vec{u_\varepsilon} + A\vec{u_\varepsilon} = \vec{f},\quad x \in \Omega \tag{1}$$

$$\vec{u_\varepsilon}(0) = \vec{u_0},\ \vec{u_\varepsilon}(1) = \vec{u_1} \tag{2}$$

其中 $\vec{u_\varepsilon} = \begin{pmatrix} u_{\varepsilon,1} \\ u_{\varepsilon,2}\end{pmatrix}$, $A = \begin{pmatrix} a_{11}(x) & a_{12}(x) \\ a_{21}(x) & a_{22}(x)\end{pmatrix}$, $\vec{f} = \begin{pmatrix} f_1 \\ f_2\end{pmatrix}$,

函数 $a_{11}(x)$、$a_{12}(x)$、$a_{21}(x)$、$a_{22}(x)$、$f_1(x)$、$f_2(x) \in C^2(\Omega)$ 且满足如下条件

$$a_{11}(x) > |a_{12}(x)|,\ a_{22}(x) > |a_{21}(x)|,\ \forall x \in \Omega,$$
$$a_{21}(x) < 0,\ a_{12}(x) < 0,\ \forall x \in \Omega,$$

$\vec{u_0}$ 和 $\vec{u_1}$ 是给定的常数向量. 令

$$a = \min_{x\in\Omega}\{a_{11} + a_{12}, a_{21} + a_{22}\} > 0.$$

ε 是一个小参数且满足

$$0 < \varepsilon \ll \frac{\sqrt{a}}{4} \tag{3}$$

当 $\varepsilon \to 0$ 时,微分方程系统(1)～(2)退化为 $A\vec{u_\varepsilon} = \vec{f}$,在 $x = 0$ 和 $x = 1$ 各失去一个边界层,根据文献[6]的结果,有如下引理.

引理 1　微分方程系统(1)～(2)的解 $\vec{u_\varepsilon}$ 满足 $\vec{u_\varepsilon} = \vec{v} + \vec{w_l} + \vec{w_r}$, $x \in \bar{\Omega}$,

其中

$$\|v_j^{(k)}(x)\| \le C(1 + \varepsilon^{\frac{2-k}{2}}),\ j = 1,2,\ 0 \le k \le 3,\ x \in \bar{\Omega},$$
$$\|w_{l,j}^{(k)}(x)\| \le C\varepsilon^{-\frac{k}{2}}e_1(x),\ j = 1,2,\ 0 \le k \le 3,\ x \in \bar{\Omega},$$
$$\|w_{r,j}^{(k)}(x)\| \le C\varepsilon^{-\frac{k}{2}}e_2(x),\ j = 1,2,\ 0 \le k \le 3,\ x \in \bar{\Omega},$$
$$e_1(x) = e^{-\frac{\sqrt{a}x}{\sqrt{\varepsilon}}},\ e_2(x) = e^{-\frac{\sqrt{a}(1-x)}{\sqrt{\varepsilon}}},\ x \in \bar{\Omega}.$$

2　多过渡点的选取策略

将 $[0,1]$ 分为 N 段,网格点为 $\bar{\Omega}_\varepsilon^N = \{x_0 = 0, x_1, \cdots x_{N-1}, x_N = 1\}$, $\Omega_\varepsilon^N = \{x_1, \cdots x_{N-1}\}$;设步长 $h_i = x_i - x_{i-1}$, $1 \le i \le N$; $\bar{h_i} = \frac{h_i + h_{i+1}}{2}$, $1 \le i \le N-1$;

为了叙述方便,仅考虑 $16 \le N \le 3.7 \times 10^6$. 对于

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$\forall N \geq 16$, 完全可以依照类推法证明, 请参考文献[9].

对于 $\forall N = 2^m \in [16, 3.7 \times 10^6]$ 其中 $m \geq 4$, N、$m \in \aleph$, 满足

$$\begin{cases} e^e < N < e^{e^e} \\ e < \ln N < e^e \\ 1 < \ln\ln N < e \\ 0 < \ln\ln\ln N < 1 \end{cases} \tag{4}$$

考虑多过渡点如下

$$\begin{cases} \sigma_3 = \dfrac{\sqrt{\varepsilon}\ln\ln\ln N}{\sqrt{a}} \\ 1 - \sigma_3 = 1 - \dfrac{\sqrt{\varepsilon}\ln\ln\ln N}{\sqrt{a}}, \\ \sigma_2 = \min\left(\dfrac{\sqrt{\varepsilon}\ln\ln N}{\sqrt{a}}, \dfrac{1}{4}\right) \\ 1 - \sigma_2 = 1 - \min\left(\dfrac{\sqrt{\varepsilon}\ln\ln N}{\sqrt{a}}, \dfrac{1}{4}\right), \\ \sigma_1 = \min\left(\dfrac{\sqrt{\varepsilon}\ln N}{\sqrt{a}}, \dfrac{1}{4}\right) \\ 1 - \sigma_1 = 1 - \min\left(\dfrac{\sqrt{\varepsilon}\ln N}{\sqrt{a}}, \dfrac{1}{4}\right), \end{cases}$$

根据条件(3) 和式(4) 得过渡点 $\sigma_3 < \dfrac{1}{4}$ 和 $1 - \sigma_3 > \dfrac{3}{4}$.

依照上述过渡点的取法,至少有 2 个过渡点 σ_3 和 $1 - \sigma_3$,这样区间 $\bar{\Omega}$ 被分为几个小区间,在每个小区间采用等步长. 下面将根据过渡点的情况分别进行讨论.

(i) 当 $\dfrac{\sqrt{\varepsilon}\ln N}{\sqrt{a}} < \dfrac{1}{4}$ 时, $\sigma_1 = \dfrac{\sqrt{\varepsilon}\ln N}{\sqrt{a}}$,

$$\sigma_2 = \frac{\sqrt{\varepsilon}\ln\ln N}{\sqrt{a}}, \quad \sigma_3 = \frac{\sqrt{\varepsilon}\ln\ln\ln N}{\sqrt{a}}.$$

取多过渡点集合 $\theta = \{\sigma_3, \sigma_2, \sigma_1, 1 - \sigma_1, 1 - \sigma_2, 1 - \sigma_3\}$. 区间 $\bar{\Omega}$ 被分为 7 个区间: I_1 至 I_7.

在 $I_1 = [0, \sigma_3]$ 和 $I_7 = [1 - \sigma_3, 1]$, 分别有 $\dfrac{N}{8}$ 个网格点, 步长 $H_4 \leq \dfrac{C_N\sqrt{\varepsilon}}{N}$.

在 $I_2 = [\sigma_3, \sigma_2]$ 和 $I_6 = [1 - \sigma_2, 1 - \sigma_3]$, 分别有 $\dfrac{N}{8}$ 个网格点, 步长 $H_3 \leq \dfrac{C_N\sqrt{\varepsilon}\ln\ln N}{N}$.

在 $I_3 = [\sigma_2, \sigma_1]$ 和 $I_5 = [1 - \sigma_1, 1 - \sigma_2]$, 分别有 $\dfrac{N}{8}$ 个网格点, 步长 $H_2 \leq \dfrac{C_N\sqrt{\varepsilon}\ln N}{N}$.

在 $I_4 = [\sigma_1, 1 - \sigma_1]$, 共有 $\dfrac{N}{4}$ 个网格点, 步长 $H_1 \leq \dfrac{C}{N}$.

(ii) 当 $\dfrac{\sqrt{\varepsilon}\ln N}{\sqrt{a}} \geq \dfrac{1}{4}$, $\dfrac{\sqrt{\varepsilon}\ln\ln N}{\sqrt{a}} < \dfrac{1}{4}$ 时, 取 $\sigma_1 = \dfrac{1}{4}$, $\sigma_2 = \dfrac{\sqrt{\varepsilon}\ln\ln N}{\sqrt{a}}$, $\sigma_3 = \dfrac{\sqrt{\varepsilon}\ln\ln\ln N}{\sqrt{a}}$, 多过渡点集合 $\theta = \{\sigma_3, \sigma_2, \sigma_1 = \dfrac{1}{4}, 1 - \sigma_1 = \dfrac{3}{4}, 1 - \sigma_2, 1 - \sigma_3\}$.

此时区间 $\bar{\Omega}$ 被分为 7 个区间,其步长的的讨论类似于情形(i).

(iii) 当 $\dfrac{\sqrt{\varepsilon}\ln\ln N}{\sqrt{a}} \geq \dfrac{1}{4}$ 时, 取 $\sigma_2 = \dfrac{1}{4}$, $\sigma_3 = \dfrac{\sqrt{\varepsilon}\ln\ln\ln N}{\sqrt{a}}$, 多过渡点集合 $\theta = \{\sigma_3, \sigma_2 = \dfrac{1}{4}, 1 - \sigma_2 = \dfrac{3}{4}, 1 - \sigma_3\}$. 区间 $\bar{\Omega}$ 被分为 5 个区间, 其步长的的讨论类似于情形(i).

3　多过渡点差分格式及其性质

构造多过渡点差分格式

$$L_\varepsilon^N \vec{U}(x_i) = \begin{pmatrix} -\varepsilon\delta^2 & 0 \\ 0 & -\varepsilon\delta^2 \end{pmatrix} \vec{U}(x_i) + A_i \vec{U}(x_i) = \vec{f}(x_i),\ x_i \in \Omega_\varepsilon^N \tag{5}$$

$$\vec{U}(0) = \vec{u_0},\ \vec{U}(1) = \vec{u_1} \tag{6}$$

其中

$$\vec{U}(x_i) = \begin{pmatrix} U_1(x_i) \\ U_2(x_i) \end{pmatrix},\ A_i = \begin{pmatrix} a_{11}(x_i) & a_{12}(x_i) \\ a_{21}(x_i) & a_{22}(x_i) \end{pmatrix},$$

$$\vec{f}(x_i) = \begin{pmatrix} f_1(x_i) \\ f_2(x_i) \end{pmatrix},$$

$$D^+ \vec{U}(x_i) = \frac{\vec{U}(x_{i+1}) - \vec{U}(x_i)}{h_{i+1}},$$

$$D^- \vec{U}(x_i) = \frac{\vec{U}(x_i) - \vec{U}(x_{i-1})}{h_i},$$

$$\delta \vec{U}(x_i) = \frac{1}{h_i}\{D^+ \vec{U}(x_i) - D^- \vec{U}(x_i)\}.$$

文献[6] 已经证明了下面 2 个引理.

引理 2　若差分格式(5) ～ (6) 的解 $\vec{U}(x_i)$ 满足

$$\begin{cases} L_\varepsilon^N \vec{U}(x_i) \leq 0,\ x_i \in \Omega_\varepsilon^N \\ \vec{U}(0) \geq 0,\ \vec{U}(1) \geq 0 \end{cases}$$

则 $\vec{U}(x_i) \geq 0$, $x_i \in \bar{\Omega}_\varepsilon^N$.

引理 3　任何网格函数 $\vec{Z}(x_i)$ 满足 $\vec{Z}(0) = \vec{Z}(1) = 0$, 则 $|\vec{Z}(x_i)| \leq C \max\limits_{x_i \in \Omega_\varepsilon^N} |L_\varepsilon^N \vec{Z}(x_i)|$, $x_i \in \bar{\Omega}_\varepsilon^N$.

定义如下差分格式

$$L_\varepsilon^N \overrightarrow{V(x_i)} = \begin{pmatrix} -\varepsilon\delta^2 & 0 \\ 0 & -\varepsilon\delta^2 \end{pmatrix} \overrightarrow{V(x_i)} + A_i \overrightarrow{V(x_i)} = L_\varepsilon \overrightarrow{v(x_i)},\ x_i \in \Omega_\varepsilon^N \qquad (7)$$

$$\overrightarrow{V(0)} = \overrightarrow{v(0)},\quad \overrightarrow{V(1)} = \overrightarrow{v(1)} \qquad (8)$$

$$L_\varepsilon^N \overrightarrow{W_l(x_i)} = \begin{pmatrix} -\varepsilon\delta^2 & 0 \\ 0 & -\varepsilon\delta^2 \end{pmatrix} \overrightarrow{W_l(x_i)} + A_i \overrightarrow{W_l(x_i)} = L_\varepsilon \overrightarrow{w_l(x_i)},\ x_i \in \Omega_\varepsilon^N \qquad (9)$$

$$\overrightarrow{W_l(0)} = \overrightarrow{w_l(0)},\ \overrightarrow{W_l(1)} = \overrightarrow{w_l(1)} \qquad (10)$$

$$L_\varepsilon^N \overrightarrow{W_r(x_i)} = \begin{pmatrix} -\varepsilon\delta^2 & 0 \\ 0 & -\varepsilon\delta^2 \end{pmatrix} \overrightarrow{W_r(x_i)} + A_i \overrightarrow{W_r(x_i)} = L_\varepsilon \overrightarrow{w_r(x_i)},\ x_i \in \Omega_\varepsilon^N \qquad (11)$$

$$\overrightarrow{W_r(0)} = \overrightarrow{w_r(0)},\ \overrightarrow{W_r(1)} = \overrightarrow{w_r(1)} \qquad (12)$$

则差分格式(5)、(6)的解 $\overrightarrow{U(x_i)} = \overrightarrow{V(x_i)} + \overrightarrow{W_l(x_i)} + \overrightarrow{W_r(x_i)},\ x_i \in \bar{\Omega}_\varepsilon^N$.

4 一致收敛性

下列讨论中常用到如下 4 个不等式[1,2,8]:

$$c_1 t \le \sinh t \le c_2 t,\ 0 < t < c$$

$$c_1 \exp(t) \le \sinh t \le c_2 \exp(t).\ t \ge c > 0$$

$$\left|\frac{1}{\varepsilon^k}\exp\left(-\frac{1}{\varepsilon}\right)\right| \le C,\ k = 0,1,\cdots \qquad (13)$$

$$\left|\left(\delta^2 - \frac{d^2}{dx^2}\right)\phi(x_i)\right| \le \frac{1}{3}(x_{i+1} - x_{i-1}) \max_{x\in[x_{i-1},x_{i+1}]} |\phi'''(x)|,$$

根据引理 1 和引理 3 容易证明下列引理成立.

引理 4 设 $\overrightarrow{V(x_i)}$ 是差分格式(7)、(8)的解,则 $|\overrightarrow{V(x_i)} - \overrightarrow{v(x_i)}| \le CN^{-1},\ x_i \in \bar{\Omega}_\varepsilon^N$.

引理 5 设 $\overrightarrow{W_l(x_i)}$ 是差分格式(9)、(10)的解,则

$$|W_{l,j}(x_i) - w_{l,j}(x_i)| \le CN^{-1},\ j = 1,2,\ x_i \in \bar{\Omega}_\varepsilon^N.$$

证明 考虑到式(9),有

$$L_\varepsilon^N(\overrightarrow{w_l(x_i)} - \overrightarrow{W_l(x_i)}) = \begin{pmatrix} -\varepsilon\delta^2 & 0 \\ 0 & -\varepsilon\delta^2 \end{pmatrix} \overrightarrow{w_l(x_i)} - \begin{pmatrix} -\varepsilon\frac{d^2}{dx^2} & 0 \\ 0 & -\varepsilon\frac{d^2}{dx^2} \end{pmatrix} \overrightarrow{w_l(x_i)},\ x_i \in \Omega_\varepsilon^N.$$

利用 Taylor 展开,得

$$L_\varepsilon^N(\overrightarrow{w_l(x_i)} - \overrightarrow{W_l(x_i)}) = \frac{\varepsilon}{h_i}[\overrightarrow{w_l'(x_{i+0.5})} - \overrightarrow{w_l'(x_{i-0.5})} + \overrightarrow{F_1(x_i)} - \overrightarrow{F_2(x_i)}] - \varepsilon\overrightarrow{w_l''(x_i)},$$

其中

$$\overrightarrow{F_1(x_i)} = \frac{1}{2h_{i+1}}\left\{\int_{x_{i+0.5}}^{x_{i+1}} \overrightarrow{w_l'''(t)}(x_i + h_{i+1} - t)^2 dt + \int_{x_i}^{x_{i+0.5}} \overrightarrow{w_l'''(t)}(t - x_i)^2 dt\right\},$$

$$\overrightarrow{F_2(x_i)} = \frac{1}{2h_i}\left\{\int_{x_{i-0.5}}^{x_i} \overrightarrow{w_l'''(t)}(x_i - t)^2 dt + \int_{x_{i-1}}^{x_{i-0.5}} \overrightarrow{w_l'''(t)}(t - x_{i-1})^2 dt\right\}.$$

再次利用 Taylor 展开得

$$L_\varepsilon^N(\overrightarrow{w_l(x_i)} - \overrightarrow{W_l(x_i)}) = \frac{\varepsilon}{h_i}[\overrightarrow{F_1(x_i)} - \overrightarrow{F_2(x_i)} + \overrightarrow{F_3(x_i)} - \overrightarrow{F_4(x_i)}],\ x_i \in \Omega_\varepsilon^N.$$

其中

$$\overrightarrow{F_3(x_i)} = \int_{x_i}^{x_{i+0.5}} \overrightarrow{w_l'''(t)}(x_i + 0.5h_{i+1} - t)dt,$$

$$\overrightarrow{F_4(x_i)} = \int_{x_{i-0.5}}^{x_i} \overrightarrow{w_l'''(t)}(t - x_{i-0.5})dt.$$

(i) 当 $\frac{\sqrt{\varepsilon}\ln N}{\sqrt{a}} < \frac{1}{4}$ 时,取多过渡点集合 $\theta = \{\sigma_3, \sigma_2, \sigma_1, 1-\sigma_1, 1-\sigma_2, 1-\sigma_3\}$. 分别对 $x_i \in (0,\sigma_3)$, $x_i \in [\sigma_3, \sigma_2)$, $x_i \in [\sigma_2, \sigma_1)$ 和 $x_i \in [\sigma_1, 1-\sigma_1)$ 进行讨论,并利用不等式(13),估计得到

$$\left|\frac{\varepsilon}{h_i}F_{1,j}\right| \le CN^{-1},\ j = 1,2,\ x_i \in \Omega_\varepsilon^N.$$

由函数 $e_1\left(-\frac{\sqrt{a}x}{\sqrt{\varepsilon}}\right)$ 的单调性,容易证明当 $x \in [1-\sigma_1, 1)$ 时, $\left|\frac{\varepsilon}{h_i}F_{1,j}(x_i)\right| \le CN^{-1},\ j = 1,2$.

综上所述,当 $x_i \in \Omega_\varepsilon^N$ 时有 $\left|\frac{\varepsilon}{h_i}F_{1,j}(x_i)\right| \le CN^{-1},\ j = 1,2$.

同理可证,当 $x_i \in \Omega_\varepsilon^N$ 时有

$$\left|\frac{\varepsilon}{h_i}F_{k,j}(x_i)\right| \le CN^{-1},\ j = 1,2,\ k = 2,3,4.$$

因此 $|L_\varepsilon^N(w_{l,j}(x_i) - W_{l,j}(x_i))| \le CN^{-1},\ j = 1,2,\ x_i \in \Omega_\varepsilon^N$.

考虑到式(10), $\overrightarrow{W_l(0)} = \overrightarrow{w_l(0)}$, $\overrightarrow{W_l(1)}$ $\overrightarrow{w_l(1)}$. 根据引理 3 得

$$|W_{l,j}(x_i) - w_{l,j}(x_i)| \le CN^{-1},\ j = 1,2,\ x_i \in \bar{\Omega}_\varepsilon^N.$$

(ii) 当 $\frac{\sqrt{\varepsilon}\ln N}{\sqrt{a}} \ge \frac{1}{4}$, $\frac{\sqrt{\varepsilon}\ln\ln N}{\sqrt{a}} < \frac{1}{4}$ 时,取 $\sigma_1 = \frac{1}{4}$, $\sigma_2 = \frac{\sqrt{\varepsilon}\ln\ln N}{\sqrt{a}}$, $\sigma_3 = \frac{\sqrt{\varepsilon}\ln\ln\ln N}{\sqrt{a}}$,取多过渡点集合 $\theta = \{\sigma_3, \sigma_2, \sigma_1 = \frac{1}{4}, 1-\sigma_1 = \frac{3}{4}, 1-\sigma_2, 1-\sigma_3\}$. 注意到 $\frac{1}{\sqrt{\varepsilon}} \le C\ln N$,类似于(i)的情形得: $|W_{l,j}(x_i) - w_{l,j}(x_i)| \le$

$CN^{-1}, j = 1,2, x_i \in \overline{\Omega}_\varepsilon^N$.

(iii) 当 $\frac{\sqrt{\varepsilon}\ln\ln N}{\sqrt{a}} \geq \frac{1}{4}$ 时,取 $\sigma_2 = \frac{1}{4}$,多过渡点集合

$$\theta = \{\sigma_3, \sigma_2 = \frac{1}{4}, 1 - \sigma_2 = \frac{3}{4}, 1 - \sigma_3\}.$$

显然这种情况可以归结于情形(i) 和(ii),本文不再重复.

引理 6　设 $\overrightarrow{W_r(x_i)}$ 是差分格式(5)、(6) 的解,则

$$|W_{r,j}(x_i) - w_{r,j}(x_i)| \leq CN^{-1},$$
$$j = 1,2, x_i \in \overline{\Omega}_\varepsilon^N.$$

证明　由于函数 $e_2(x)$ 与函数 $e_1(x)$ 是相互对称的,引理 6 的证明与引理 5 完全相似.

考虑到引理 4、引理 5 和引理 6,得到本文的主要定理:

定理　设 $\overrightarrow{u_\varepsilon}$ 是微分方程系统(1)、(2) 的解,$\overrightarrow{U(x_i)}$ 是差分格式(13)、(14) 的解.

则 $|U_j(x_i) - u_{\varepsilon,j}(x_i)| \leq CN^{-1}, j = 1,2, x_i \in \overline{\Omega}_\varepsilon^N$.

限于篇幅,本文数值例子从略.

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Numerical Solution for a System of Singularly Perturbed Ordinary Differential Equation

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Abstract: In this paper a two-point boundary value problem for a system of singularly perturbed ordinary differential problems is considered. It is the most complicated problem in singularly perturbed equation. The technique of select multi-transition points is introduced. According to multi-transition points method, non-equidistant difference scheme is constructed. In maximum norm, the new method is proved to be first-order uniform convergence with respected to the perturbed parameter. Multi-transition points determine the point of transition from fine mesh to middle and coarse mesh, while Shishkin scheme (single transition point method) only determines the point of transition from fine to coarse mesh. It capture the properties of boundary layer efficiently. The new method is useful in practice application. The rate of convergence is higher than Shishkin's scheme.

Key words: singular perturbation; system; uniform convergence; multi-transition points; non-equidistant scheme