The Progress of Classical Field Theory In Physics Today

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Abstract: The classical field theory had been generalized to modern Physics, pointed out that the curvature tensor is a curl(grad), according to absolute integral. Maxwell equations was established as divergence and curl of the surface flux (2-dimension) .

Key words: curvature tensor, Maxwell equation**,** Absolute integral, surface flux (2-dimension)**.**

The writer study the classical field theory for many years, discover that the classical field theory was bogged down in Stockes' time in essence: gradient、curl and divergence notion only in three dimension Euclid space and only for vector. In my point of view: this is serious problem.

Writer had discovered gradient、curl in functional analysis as usually exactly (Journal of Southwest Jiaotong Univ., 1984,2, Chinese). We extended classical field theory into non-Euclid space and tensor now, essentially, explained the curvature tensor、Maxwell equation and Gauge field clearly.

(1) On curvature

Let us rewrite vector、tensor、exterior differential form into symmetrical form with respect to the frame at every point (tensor and frame be used at same time, i.e. we use the linear combination of base of frame at every point), not as usually only one frame in small scope..

$$\vec{A}(p)=\sum_i A_i(p)e^i(p) \qquad \omega(p)=\sum_i A_i(p)e^i(p)\, d(e_i(p)x^i(p))$$

In small scope，since differential and integral are mutually inverse to each other，

$$d\vec{A}=d\sum_i A_i e^i=\sum_{ij}\partial_j A_i\, e^j dx^j e_j\, e^i+\sum_k A_k\, de^k$$

$$=\sum_{ij}((\partial_j A_i-\sum_k \Gamma^k_{ji} A_k)e^j dx^j e_j)e^i = \sum_{ij}(\nabla_j A_i)e^i e^j dx^j e_j$$

(Here, for simplicity, $e^i e^j$ is a tensor product, by dropping mark $\otimes$. $e^j dx^j e_j$ is the scalar product approximating dx^j. We shall adopt this notation hereinafter)

$$\vec{A}(x^i+dx^i)-\vec{A}(x^i)=\int_c \sum_j(\sum_i(\nabla_j A_i)e^i e^j)dx^j e_j$$

Where c is the segment of straight line from x^i to x^i+dx^i. According this, we have

Absolute integral: An absolute integral is a Stieltjes integral, forming by Riemann integral, in which the bases of the frame at every point are adhered to the integrand function (vector, tensor) and the integral variable.

For examples, linear integral, that is inner product forming：

$$\int_c \sum_i A_i(p)e^i(p)\, d(e_i(p)x^i(p)) \qquad \left(\oint_c \sum_i A_i(p)e^i(p)\, d(e_i(p)x^i(p))\right)$$

$$\int_c \sum_j(\sum_i(\nabla_j A_i)e^i e^j)dx^j e_j$$

Absolute circulation: The (absolute) circulation of a vector $\vec{A}$ along a closed curve c is the integral of the scalar product $\vec{A}\cdot d\vec{l}$ along the contour c：

$$\mathbf{I} = \oint_c (\vec{A}\cdot d\vec{l}) = \oint_c \sum_i (\vec{A}\cdot \mathbf{e}^i)\mathbf{e}^i\, d(\mathbf{e}_i \mathbf{x}^i) = \oint_c \sum_i \mathbf{A}_i\ \mathbf{e}^i\ d(\mathbf{e}_i \mathbf{x}^i)$$

By the aid of the absolute integral, we improved Stokes` formula; the strict definitions of the curl and the divergence (of the vector) were obtained (usually, they were obtained only by analogy in Euclid-space, unable to apply here). It had been proved that the curvature tensor is a curl(grad), i.e, $\mathrm{rot}_{kj,i}\nabla_j \mathbf{e}^t = \mathbf{R}^t_{ijk}$, that is the i-component of the curl of the gradient of base $\mathbf{e}^t$ in k,j-plane, not zero except in Euclidean space, and so discovered the essence of Bianchi identity: div(rot(grad)) = 0, the curvature, forming tube field, is invariant along the tube, i.e. pointed out that Gauge fields are curl fields of gradients and so on. By the way, we obtained the torsion tensor is rot of base of frame also.

(2) On Maxwell equation

The notation of the inner production of the vector was extended to tensors with different order. Writer pointed out the essentials of the differential and the codifferential form: the tangential and the normal surface flux of the tensor. Writer arrived the notation of rotation and divergence of the surface flux (2-dimension) of the tensor (2-order).

Divergence of the surface flux (2-dimension) of the tensor (2-order),

$$\mathrm{div}_{ijk}\mathbf{F}|_P = 2^{-1}\lim_{v\to 0}\frac{1}{v}\oiint_{\partial v} \mathbf{F}\cdot \mathbf{e}_s \wedge \mathbf{n}ds \quad (\varepsilon^{sijk}_{1234}=1,\ s=1,2,3,4).$$

(v is closed volume in i,j,k space, p$\in v$, n is nomal vector)

Rotation of the surface flux (2-dimension) of the tensor (2-order),

$$\mathrm{rot}_{ijk}\mathbf{F}|_P = 2^{-1}\lim_{v\to 0}\frac{1}{v}\oiint_{\partial v} \mathbf{F}\cdot \mathbf{t}_1 \wedge \mathbf{t}_2 ds \qquad (\mathbf{t}_1\wedge\mathbf{t}_2 \text{ is tangent vector }).$$

Only according to one conservative law (mass or fluid quantity) and the feature of central field (or its composite), the universal field, i.e. the construction law of universal field ——Maxwell equation was established.

$$\mathrm{div}_{ijk}\mathbf{F} = \partial_i \mathbf{f}_{si} + \partial_j \mathbf{f}_{sj} + \partial_k \mathbf{f}_{sk} = \mathbf{q}_s \qquad (\varepsilon^{sijk}_{1234}=1,\ s=1,2,3,4)$$

$$\mathrm{rot}_{ijk}\mathbf{F} = \partial_k \mathbf{f}_{ij} + \partial_i \mathbf{f}_{jk} + \partial_j \mathbf{f}_{ki} = 0 \qquad (\varepsilon^{sijk}_{1234}=1,\ s=1,2,3,4)$$

Writer explained its essences as in classical mechanic, the sum of the forces (normal surface flux) and the sum of the moments of forces (tangential surface flux).

The universality and the feature of field: Any field F in 4-dimension have to set $\sum_{s=1}^{4}{}_{\varepsilon^{ijks}_{1234}=1}\partial_s(\mathrm{div}_{ijk}\mathbf{F}) = 0$ and $\sum_{s=1}^{4}{}_{\varepsilon^{ijks}_{1234}=1}\partial_s(\mathrm{rot}_{ijk}\mathbf{F}) = 0$; otherwise if $\sum_{i=1}^{4}\partial_i \mathbf{q}_i = 0$ and $\sum_{i=1}^{4}\partial_i \mathbf{g}_i = 0$, there exist field F with $\mathrm{div}_{ijk}\mathbf{F}=\mathbf{q}_s$ $\mathrm{rot}_{ijk}\mathbf{F}=\mathbf{g}_s$ ($\varepsilon^{ijsk}_{1234}=1$).

By the rotations of 2-dimension flux were zero and the universality of field, these give rise to both universality of gauge field and the difficult of magnetic monopole theory also (magnetic monopole have no effect on electric current, like the couple have no effect on the sum of forces, i.e. , it have no the feature of magnet).

References

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当代物理中经典场论的进展

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摘 要：旋度、散度概念精确的推广到现代物理，指出曲率张量是旋度（据绝对积分），Maxwell 方程是二维流的旋度、散度，宛如经典力学。

关键词：旋度、散度、曲率张量、Maxwell 方程.、绝对积分、二维面流。