The Unified Interpretation for Quantum Hall Effects

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Abstract: Based on the potential energy expression for carriers in magnetic field, and two stability conditions of circular harmonic oscillator and linear harmonic oscillator, a new energy levels formula of Hall effects was founded, and then the unified interpretation for quantum Hall effects of integer and fraction can be obtained. The "filling factors" of integer quantum Hall effect and "fraction charges" of fraction quantum Hall effect all have a corresponding energy level in new formula. The corresponding connection between the energy level with filling factors or fraction charges are showed by a table; the fundamental parameter of carriers on different energy level are figured out; three models of quantum Hall effects are differentiated; the interpretation of Hall resistance plateaus are obtained by use the velocity distributions of carriers; further, a new viewpoint that the samples of quantum Hall effect are the Quantum Superconductor at the same time was put forward. Based on the spin of electron, an inverse Hall effect shall be produced when the carrier is electron.

Keywords: quantum Hall effects, energy level, filling factor, fraction charge, Hall resistance plateaus

1 Introduction

As known, general Hall effects (GHE) can be analyzed according to Lorentz force; the integer quantum Hall effects (IQHE) must count Landau energy level [1-9]; the fraction quantum Hall effects (FQHE) must count the strong interaction between the electrons, assuming existence of fraction electric charge [5-8], although different Hall effects (including GHE, IQHE and FQHE) are very harmonious on same experimental curves [6-8]. All unified explanations so far are unsatisfactory [9-12] because there are no unified mechanism and calculation methods. A significant amount of experimental and theoretical researches are continuing today [13-19]. Our new explanation preserves the traditional theory, overcomes the shortcoming in explaining quantum Hall effects, and proves that the assumptions on filling factor and fraction charge are no longer necessary. We conclude that Landau energy level formula (LELF) is not suitable to describe quantum Hall effects because it is only based on the model of linear harmonic oscillator (LHO). Only the LHO model is different from true motion of electrons in uniform magnetic field. This paper indicates that a moving electron in magnetic field can obtain additional momentum and energy, which depends only on magnetic potential $\vec{A}$ (section 2). In fact, true motion of electrons is a doublet of beeline and orbiting in Hall effects (see Fig. 1). Therefore, supplement of LELF must be based on the doublet of both LHO and Circular Harmonic Oscillator (CHO). Following analysis of doublet, we obtained a stability condition of the doublet and a new LELF for quantum Hall effects (section 3 and 4). Some basic parameters of moving electrons and formula of quantum Hall resistance are obtained for different energy levels (section 5 and 6). The relationship of quantum Hall resistance and superconductivity, relationship of energy levels and the filling factors or fraction charges, and three models of quantum Hall Effects are discussed (section 7-9). We indicate that every Hall sensor of quantum Hall effect also is also a superconductor; therefore the quantum Hall effect can also be called Quantum Superconductivity Effect. We proved that quantum Hall resistances are only dependent of energy level and stability condition, and are independent of the fraction charge and the filling factor. We find a new quantum Hall effect model that quantum Hall resistance can be in inverse proportion to the magnetic field. Further, quantum Hall resistance plateaus are explained by velocity distribution of orbiting electrons (section 10). We also present an inverse effect of Hall effects that is based on spin of electrons (section 11). In this paper, the electrons all can be replaced by any carriers in quantum Hall effects, with no effect on our analysis.

The mechanism and calculation methods of quantum Hall effects are based on the doublet of the CHO and LHO on Hall surface. Unifying the explanations of quantum Hall effects is not only possible, but is also independent of the assumption on fraction charges and filling factors. Finally, the reason that people could not obtained a unified explanation of quantum Hall effects stems from the fact that some basic laws of moving electron in magnetic field were not well used in electromagnetic theory.

2 Obtained potential energy by orbiting electron in uniform magnetic field

Imagine an orbiting electron in uniform magnetic field, we have $v/R = eB/m_e$ when $\vec{v} \perp \vec{B}$ and Lorentz force $e\vec{v} \times \vec{B}$ equals to centripetal force $m_e v^2/R$ [13, 14], where $\vec{v}$ represents electronic velocity, $\vec{B}$ magnetic field and R orbit radius of electron. The flux ϕ parses through orbit face can be shown with $\phi = \pi R^2 B = 2\pi RA$, then where is $RB = 2A$ and $m_e v = 2eA$ where A represents magnetic potential, its direction is same with that of $\vec{v}$. We transform $m_e v = 2eA$ into $m_e \vec{v} = 2e\vec{A}$ and $m_e v^2/2 = e\vec{v} \cdot \vec{A}$, where $m_e v^2/2$ is kinetic energy of the electron, $e\vec{A}$ additional momentum of the electron according to Hamiltonian [20-22], and $e\vec{v} \cdot \vec{A}$ additional potential energy of the electron according to Lagrangian [20-22]. Therefore, the orbiting electron not only has kinetic energy $m_e v^2/2$ but also obtain additional momentum additional momentum and additional potential energy $e\vec{v} \cdot \vec{A}$ in uniform magnetic field. The characteristic of two energy is different because $m_e v^2/2$ is mechanics quantity, $e\vec{v} \cdot \vec{A}$ electromagnetic quantity, although $m_e v^2/2$ equals to $e\vec{v} \cdot \vec{A}$. On the other hand, Lorentz force, can be uniform from potential energy $e\vec{v} \cdot \vec{A}$ and momentum $e\vec{A}$. Therefore, an orbiting electron not only is subjected to Lorentz force, but also can obtain additional potential energy and momentum in magnetic field at the same time. Further calculation Hall effects still can use the additional potential energy and momentum.

3 Stability condition of CHO

Although stability condition for LHO is $\ell = n\lambda$, stability condition for CHO or orbiting electron can be written as $2i\pi R = n\lambda$ because the orbiting electron has no border on orbit, where $i = 1,2,3,......$, ℓ represents linear length and λ wavelength of electron that produced by additional momentum. The meanings of two stability conditions are different. In expression $2i\pi R = n\lambda$, the length of $n\lambda$ exactly equals to that of i times orbiting. Such as, $n = 1, i = 3$ i.e. $6\pi R = \lambda$ is that length of three times orbiting exactly equals to one wavelength; $n = 3, i = 1$ i.e. $2\pi R = 3\lambda$ is that length of one times orbiting exactly equals to three wavelengths, and so on. The peculiarity of stability condition of CHO is that i not only equals to 1 but also can equals to $2,3,......$.

4 Doublet of harmonic oscillator and New LELF

Imagine a moving electron on quantum Hall effects show in Fig. 1, its orbiting frequency equals to eB/m_e in uniform magnetic field [23, 24]. In fact, the moving electron will obtain an additional potential energy $e\vec{v}\cdot\vec{A}$ according to Lagrangian. By $m_e v = h/\lambda$, $A = \Phi/2\pi R$, $\Phi = \pi R^2 B$ and the stability condition $2i\pi R = n\lambda$, we can transform the $e\vec{v}\cdot\vec{A}$ into $e\hbar nB/2im_e$. Let E_{ni} denotes energy level, we obtain

$$E_{ni} = e\hbar nB/2im_e \tag{1}$$

Eq. (1) is energy level formula of orbiting electron in uniform magnetic field when the orbiting is steadied. But the Eq. (1) is independent of the LHO.

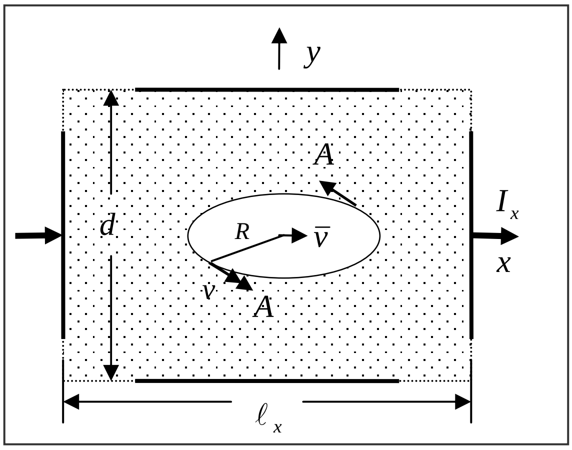

Fig. 1 A orbiting carrier in uniform magnetic field moves on Hall face. Where $\bar{v}$ is mean value of moving velocity of carrier; v is revolving velocity of carrier; and A is magnetic potential that produced by magnetic flux that pass through revolving face of carrier.

The LELF $E_k = (k+1/2)e\hbar B/m_e$ is based on LHO. In this formula, although eB/m_e is the angle frequency, electronic motion is linear, and then eB/m_e is electron wave frequency and the LELF is independent of the CHO. The difference between the LELF and Eq. (1) is only coefficients n, i and $k+1/2$. Two energy level formulas are important because the true energy level formula must be based on LHO and CHO. They shall be considered at same time. By two energy level formulas, we have $n/2 = (k+1/2)$ in Eq. (1), and then new LELF can be shown with

$$E_{ik} = (k+1/2)e\hbar B/im_e = e\hbar B/i_k m_e \tag{2}$$

where $i_k = i/(2k+1)$ is energy level coefficient. Eq. (2) is just a formula of doublet energy levels that LHO and CHO. The stability condition of doublet is $2\pi iR = (2k+1)\lambda$ or $2i_k \pi R = \lambda$.

The IQHE is dependent of the energy level that $k = 0$ or i_k equals to integer. The energy level formula of IQHE is $E_{ik} = e\hbar B / 2im_e$ where $1/i$ is just filling factors. The stability condition of IQHE is $2\pi iR = \lambda$. Let energy level is $E_1 = e\hbar B / 2m_e$ when $i = 1$, it is that the filling factors equal to 1. If energy level is $E_{1/2} = E_1 / 2$ when $i = 2$, it is that the filling factors equal to $1/2$, i.e. the coefficient of IQHE equal to 2, In this case, the stability condition is $4\pi R = \lambda$ i.e. the double orbiting length exactly equals to one wavelengths.

The FQHE is dependent of the energy level that i_k equals to fraction. The energy level formula of FQHE is $E_{ik} = (2k+1)e\hbar B / 2im_e$ when $i/(2k+1)$ is not integer, where $i/(2k+1)$ is just coefficient of fraction charge. The stability condition of FQHE is $2\pi iR = (2k+1)\lambda$ which the $i/(2k+1)$ also is not integer. As $i/(2k+1) = 1/3$, the energy level is $E_3 = 3E_1$. It is that the coefficient of FQHE equals to $1/3$, coefficient of fraction charge $1/3$, and stability condition $2\pi R = 3\lambda$ i.e. one times orbiting length exactly equals to three wavelengths.

It is very important that the motion of orbiting electrons in IQHE or FQHE must satisfy the stability condition in actual sample. In fact, the GHE is produced by that the orbiting electrons which do not meet the stability condition in actual sample. By the new formula, all Hall effects shall obtain unified interpretation, and every quantum Hall resistance will have its own energy level.

5 Some basic parameters of moving electron for different energy level

According to $m_e v = 2eA$, $m_e v = h/\lambda$, $A = \Phi / 2\pi R$ and $2\pi R = \lambda / i_k$, then

$$\Phi_{ik} = h / 2i_k e \tag{3}$$

where Φ_{ik} represents magnetic flux passes through the orbit surface. Eq. (3) shows that magnetic flux must be quantized when the orbiting is steadied, and $\Phi_1 = h/2e$ is known fluxon when $i_k = 1$. By $\Phi = \pi R^2 B$, we can obtain

$$R_{ik} = (\hbar / i_k eB)^{1/2} \tag{4}$$

Eq. (4) shows that the orbit radius of orbiting electron also is quantized when the orbiting is steadied. According to Eq. (4), $m_e v = h/\lambda$ and $2\pi R = \lambda / i_k$, the velocity formula of orbiting electron is

$$v_{ik} = (\hbar eB / i_k)^{1/2} / m_e \tag{5}$$

Eq. (5) shows that velocity of orbiting electron is quantized when the orbiting is steadied. By Eq. (3), (4) and $A = \Phi / 2\pi R$, we have

$$A_{ik} = (\hbar B / i_k e)^{1/2} / 2 \tag{6}$$

By Eq. (4) and (5), the law of orbiting electrons in uniform magnetic field will be $v_{ik} / R_{ik} = eB / m_e$. By Eq. (2) and (5), we have $B = m_e^2 v_{ik}^2 i_k / \hbar e$. Since $m_e v_{ik}^2 / 2 = e\vec{v}_{ik} \cdot \vec{A}_{ik}$, the Eq. (5) can be rewritten as $E_{ik} = m_e v_{ik}^2$ or $E_{ik} = 2e\vec{v}_{ik} \cdot \vec{A}_{ik}$ or

$$E_{ik} = m_e v_{ik}^2 / 2 + e\vec{v}_{ik} \cdot \vec{A}_{ik} \tag{2*}$$

The $m_e v_{ik}^2 / 2 + e\vec{v}_{ik} \cdot \vec{A}_{ik}$ is energy sum of kinetic energy and additional potential energy. Thus the energy level E_{ik} can be called quantization energy sum of orbiting electrons. The parameters of orbiting electron all can be calculated by Eq. (2)-(6).

6 The formula of quantum Hall resistance

In Fig. 1, the total velocity of electron is equal to that the straight line velocity v at x direction and orbiting velocity v are added. Thus the electric potential $\vec{v} \cdot \vec{A}$ of electron on different point of orbit is different. The maximal electric potential difference equals to $2vA$ and at y direction. Let ε_{yc} represents this electric potential difference, we have $\varepsilon_{yc} = 2vA$ or $\varepsilon_{yc} = 2v\Phi / 2\pi R$. The Hall mef ε_y equals to total electric potential difference that obtained by electrons on surface; it can be written as $\varepsilon_y = \varepsilon_{yc} N$ where N represents effective numbers of electrons that are stably orbiting on Hall face at the same time. The N is dependent of the electrons density N_S, sample size d, ℓ_x and electronic velocity v, and then it can be written as $N = \chi(d, \ell_x, N_S, v)$ where χ represents a coefficient and can be established by experiments. Thus we have

$$\varepsilon_y = \varepsilon_{yc} N = \Phi \chi v / \pi R \tag{7}$$

On the side, the current intensity on orbit is equal to the product of line charge density and revolving velocity of the line charge because the current intensity must be the electric charge that has flowed inside unit interval, thus the current intensity can be written as $I_e = ev / 2\pi R$. The total current intensity at x direction can be

$$I_x = I_e N = e\chi v / 2\pi R \tag{8}$$

By Eq. (7), (8) and (3), we can get

$$\rho_{Hik} = \varepsilon_y / I_x = 2\Phi_{ik} / e = h / i_k e^2 \quad (9)$$

where ρ_{Hik} represents quantum Hall resistance. It is $\rho_{H1} = h/e^2$ when $i_k = 1$. The ρ_{Hik} is dependent of h and e, but independent of ample size.

The Hall resistances can all be calculated by Eq. (9). It is the resistance of IQHE when i_k is integer. It is the resistance of FQHE when i_k is the fraction. Other cases are the resistance of GHE except the IQHE and the FQHE. The different quantum Hall resistances shall be obtained by different i_k.

7 Physical meaning of quantum Hall resistance and superconductivity effect

According to Eq. (7), (8) and (9), the Hall mef and Hall current all may be changed by effective electron numbers N or sample size, but the Hall resistances can not be changed. In fact, we also have the $\rho_H = \varepsilon_{yc} / I_e = 2\Phi / e$ where $\varepsilon_{yc} = 2vA$ is just the electric potential of one orbiting electron that obtains in magnetic field; $I_e = ev/2\pi R$ is just current intensity that produced by this orbiting electron on orbit. The quantum Hall resistances are a ratio between ε_{yc} and I_e, and independent of resistance force that produced by moving electron in sample. On the other hand, since Φ / e is the magnetic flux per unit orbiting charge, then quantum Hall resistances $2\Phi / e$ is just double magnetic flux per unit orbiting charge.

The resistances of x direction will equal to zero suddenly, i.e. the energy loss of moving electrons in x direction will be zero, when the Hall effects transforms from non-quantum to quantum. When Hall effect is non-quantum, the motion of orbiting electrons is not steady; therefore the loss of energy of electrons will not be zero. The electrons are steadily orbiting in quantum Hall effects. The orbiting electrons will be moved in y direction by electric potential difference that obtained by the electrons themselves. The orbiting electrons also will be moved in x direction by external electric potential difference. It is possible that the two cases both have no energy loss because the electrons are always steadily orbiting. [Note: (1) the electrons can only be orbiting by Lorentz force, but can not be moved along a beeline; (2) the sample size is nearly 10^5 times greater than orbit radius of electrons.] As a result, the samples of quantum Hall effect can be called Quantum Superconductors because their have quantization superconductivity. Therefore, the quantum Hall effect can also be called the Quantum Superconductivity Effect.

8 Relationship of energy levels and the filling factors or fraction charges

It is very necessary to consider the relationship of energy levels and the filling factors or fraction charges. The table 1 shows the energy levels, stability conditions and quantum Hall resistance for filling factors and fraction charges.

Every filling factors and fraction charges have an energy levels and stability conditions according to new LELF. The smaller the filling factors, the lower the energy levels. The smaller the fraction charges, the higher the energy levels. The LELF $E_k = (k+1/2)e\hbar B/m_e$ can only be suitable in filling factor 1 and fraction charge $1/3$ in table 1. We think that other filling factors and all fraction charges can only be the replenishments of LELF in the now available theory because they all can not be true. Since the assumption of filling factor and fraction charge are not necessary after obtained new LELF.

The table 1

filling factors fraction charges	1/4 /	1/3 /	1/2 /	3/5 /	5/7 /	1 /
i	4	3	2	5	7	1
k	0	0	0	1	2	0
$i/(2k+1)$	4	3	2	5/3	7/4	1
energy levels	$E_{ki1}/4$	$E_{ki1}/3$	$E_{ki1}/2$	$3E_{ki1}/5$	$5E_{ki1}/7$	$E_{ki1} = e\hbar B/2m_e$
stability conditions	$8\pi R = \lambda$	$6\pi R = \lambda$	$4\pi R = \lambda$	$10\pi R = 3\lambda$	$14\pi R = 5\lambda$	$2\pi R = \lambda$
ρ_{Hik}	$\rho_{H1}/4$	$\rho_{H1}/3$	$\rho_{H1}/2$	$3\rho_{H1}/5$	$5\rho_{H1}/7$	$\rho_{H1} = h/e^2$

The table 1 (continue)

filling factors fraction charges	/ 4/5	/ 2/3	/ 3/5	/ 3/7	/ 2/5	 1/3
i	4	2	3	3	2	1
k	2	1	2	3	2	1
$i/(2k+1)$	4/5	2/3	3/5	3/7	2/5	1/3
energy levels	$5E_{ki1}/4$	$3E_{ki1}/2$	$5E_{ki1}/3$	$7E_{ki1}/3$	$5E_{ki1}/2$	$3E_{ki1}$
stability conditions	$8\pi R = 5\lambda$	$4\pi R = 3\lambda$	$6\pi R = 5\lambda$	$6\pi R = 7\lambda$	$4\pi R = 5\lambda$	$2\pi R = 3\lambda$
ρ_{Hik}	$5\rho_{H1}/4$	$3\rho_{H1}/2$	$5\rho_{H1}/3$	$7\rho_{H1}/3$	$5\rho_{H1}/2$	$3\rho_{H1}$

9 Three models of quantum Hall Effects

According to the Eq. (2)-(9), we have three models of Hall effects: In model (A), the magnetic field B is constant, different quantum Hall resistances can be produced by different gate vlotage V_g. Klitzing's experiment is just the model (see Fig. 1 in [1]).

In model (B), the quantum Hall resistance is indirect proportion to the magnetic field B. Störmer's experiment is the model (see the page 318 in [7]). In model (C), the quantum Hall resistance is in inverse proportion to the magnetic field B when velocity is constant. The model (C) is a new model of quantum Hall effects and has no experiment data. Table 2, 3 and 4 show some theory data of basic parameters for three models according to Eq. (2)-(9).

9.1 The magnetic field B is constant.

Table 2 shows some theory values for model (A). It is based on the Klitzing's experiment result that Hall mef is $12.9mV$ when $B\equiv 18T$, $I_x \equiv 1\mu A$ and $i_k = 2$. By Eq. (2)-(6) and (9), we can obtain $E_{ik} = 18e\hbar / i_k m_e$, $\Phi_{ik} = h/2i_k e$, $R_{ik} = (\hbar/18 i_k e)^{1/2}$, $v_{ik} = (18\hbar e/i_k)^{1/2}/m_e$, $A_{ik} = (18\hbar/i_k e)^{1/2}/2$ and $\rho_{Hik} = h/i_k e^2$ respectively. Since $I_x \equiv 1\mu A$, by Eq. (7) and (8), we have $\varepsilon_{yik} = \Phi_{ik}\chi_{ik}/\pi R_{ik}$ and $I_x = e\chi_{ik}/2\pi R_{ik}$, and that the Eq. (7) can also be written as

$$\varepsilon_{yik} = 2I_x \Phi_{ik}/e \tag{10}$$

The Eq. (10) is universal Hall mef formula in which Φ_{ik} is total flux of whole orbiting electrons passes through the Hall surface.

***Table 2**

i_k	$1/i_k$	$B(T)$	$E_{ik}(J)$ $\times 10^{-23}$	$\Phi_{ik}(Wb)$ $\times 10^{-15}$	$R_{ik}(m)$ $\times 10^{-9}$	$A_{ik}(Tm)$ $\times 10^{-8}$	$v_{ik}(m/s)$ $\times 10^{4}$	$\varepsilon_y(V)$ $\times 10^{-3}$	$I_x(A)$ $\times 10^{-6}$	$\rho_{Hik}(\Omega)$ $\times 10^{4}$
4	1/4	18	4.173	0.517	3.024	2.721	0.957	6.45	1	0.645
3	1/3	18	5.564	0.689	3.491	3.142	1.105	8.60	1	0.860
2	1/2	18	8.347	1.034	4.276	3.848	1.354	12.9	1	1.291
*5/3	3/5	18	10.01	1.241	4.684	4.215	1.483	15.5	1	1.549
*1	1	18	16.69	2.068	6.047	5.442	1.914	25.8	1	2.581

Where $h = 6.62607\times 10^{-34} JS$; $m_e = 9.10938\times 10^{-31} K_g$ and $e = 1.60218\times 10^{-19} C$ [25]. Data in *1 are only theory results of IQHE, *5/3 FQHE.

9.2 The theory data of the model (B).

Table 3 shows some theory data of the model (B). It is based on the Störmer's experiment results that $\rho_H = 2.5813\times 10^4 \Omega$ when $B = 9.75T$ and $i_k = 1$. In table 3, R and I_x are constants. Let $R_{ik} \equiv R_1$, by Eq. (4), we obtain $B_{ik} = B_1/i_k$ where $B_1 = 9.75T$. Thus we can obtain $E_{ik} = E_1/i_k^2$, $\Phi_{ik} = \Phi_1/i_k$, $A_{ik} = A_1/i_k$, $R_{ik} = R_1$, $v_{ik} = v_1/i_k$, $\varepsilon_{yik} = \varepsilon_1/i_k$, $I_{xik} = I_1$ and $\rho_{Hik} = \rho_{H1}/i_k$ where $E_1 = B_1 e/m_e$, $\Phi_1 = h/2e$,

$A_1 = (\hbar B_1/e)^{1/2}/2$, $R_1 = (\hbar/eB_1)^{1/2}$, $v_1 = (\hbar eB_1)^{1/2}/m_e$, $\varepsilon_{y1} = 2I_{x1}\Phi\ /e$, $I_{x1} = e\chi/2\pi R_1$ and $\rho_{H1} = h/e^2$.

Table 3

i_k	$1/i_k$	$B(T)$	$E_{ki}(J)$ $\times10^{-23}$	$\Phi(Wb)$ $\times10^{-15}$	$R(m)$ $\times10^{-9}$	$A(Tm)$ $\times10^{-9}$	$v(m/s)$ $\times10^{4}$	$\varepsilon_y(V)$ $\times10^{8}\chi$	$I_x(A)$ $\times10^{12}\chi$	$\rho_H(\Omega)$ $\times10^{4}$
3	1/3	3.25	1.005	0.689	8.216	2.125	0.470	2.670	3.104	0.860
2	1/2	4.88	2.261	1.034	8.216	3.187	0.704	4.005	3.104	1.291
5/3	3/5	5.85	3.255	1.241	8.216	3.825	0.845	4.807	3.104	1.549
1	1	9.75	9.042	2.068	8.216	6.375	1.409	8.011	3.104	2.581
2/3	3/2	14.6	20.34	3.102	8.216	9.562	2.113	12.02	3.104	3.872
4/7	7/4	17.1	27.69	3.619	8.216	11.16	2.466	14.02	3.104	4.517
2/5	5/2	24.4	56.51	5.170	8.216	15.94	3.522	20.03	3.104	6.453
1/3	3	29.3	81.38	6.203	8.216	19.12	4.227	24.03	3.104	7.744

Where the χ has no theory result because we lack the experiment data of ε_y or I_x. In fact, χ, ε_y and I_x are independent of ρ_H and other basic parameters.

9.3 The theory data of the model (C).

Table 4 shows some theory data of the model (C). It is based on Klitzing's experiment and the theory values of $i_k = 2$ in table 2. But experiment method of Klitzing's experiment method must be changed from $i_k = 2$. According to Eq. (5), we have $v_2 = (\hbar eB_2/2)^{1/2}/m_e$ where $B_2 = 18T$. By $v_{ik} = (\hbar eB/i_k)^{1/2}/m_e$, we also have $B_{ik} = i_k B_2/2$ when $v_{ik} \equiv v_2$. Therefore, $E_{ik} = E_2$, $\Phi_{ik} = h/2i_k e$, $R_{ik} = 2R_2/i_k$, $A_{ik} = A_2$ $\varepsilon_{yik} = 2I_x\Phi_{ik}/e$ and $\rho_{Hik} = h/i_k e^2$. By Eq. (7) i.e, $I_{xik} = e\chi/2\pi R_{ik}$ and $i_k = 2$ we have $\chi = 2\pi I_{x2}R_2/e$ i.e. $\chi = 1.68\times10^5$ where $I_{x2} = 1\mu A$ in table 2. Suppose the χ is constant, the Eq. (7) can be rewritten as $I_{xik} = i_k I_{x2}/2$, and Eq. (8) $\varepsilon_{yik} = \varepsilon_{y2}$.

Table 4

i_k	$1/i_k$	$B(T)$	$E_{ik}(J)$ $\times10^{-23}$	$\Phi(Wb)$ $\times10^{-15}$	$R(m)$ $\times10^{-9}$	$A(Tm)$ $\times10^{-8}$	$v(m/s)$ $\times10^{4}$	$\varepsilon_y(V)*$ $\times10^{-3}$	$I_x(A)*$ $\times10^{-6}$	$\rho_H(\Omega)$ $\times10^{4}$
3	1/3	27	8.347	0.689	2.851	3.848	1.354	12.9	1.50	0.860
*2	1/2	18	8.347	1.034	4.276	3.848	1.354	12.9	1.00	1.29
5/3	3/5	15	8.347	1.241	5.131	3.848	1.354	12.9	0.833	1.55
1	1	9	8.347	2.068	8.552	3.848	1.354	12.9	0.500	2.58
4/7	7/4	5.1	8.347	3.619	14.97	3.848	1.354	12.9	0.286	4.52

Where data in *2 are from table 2. Although ε_y* and I_x* will be changed by different samples, ρ_H is unchanged when $v_2 = 1.354\times10^4 m/s$ and $B_2 = 18T$ are unchanged.

The model (C) can be produced on stability condition i_k in same energy level. In this model, electron velocity is constant i.e. independent of i_k. It is an estimate of feasibility that the model (C) can be produced by electronic gas in vacuum, in which electron velocity must be the constant.

10 Explained quantum Hall resistance plateaus by velocity distribution of orbiting electrons

After quantum Hall resistance appeared, if magnetic field or grid voltage continues changing, the quantum Hall resistance can keep unchanged as long as the motion of many orbiting electrons still satisfy the stability condition. Based on above case, the resistance plateaus can be explained by the velocity distribution of orbiting electrons.

According to Eq. (5), we obtain $h/i_k e^2 = 2\pi v_{ik}^2 m_e^2 / e^3 B_{ik}$. Suppose $\xi = 2\pi m_e^2 / e^3$, the Eq. (9) can be rewritten as

$$\rho_{Hik} = \xi v_{ik}^2 / B_{ik} \tag{11}$$

Suppose $v_{ik} = v_{ik0} \pm \delta v$ is velocity distribution of orbiting electrons for i_k, $B_{ik0} \pm \delta B$ the extension of magnetic field in which the resistance plateaus has be formed. By Eq. (11), the condition of formed plateaus can be written as

$$(v_{ik0} - \delta v)^2 / (B_{ik0} - \delta B) = (v_{ik0} + \delta v)^2 / (B_{ik0} + \delta B) = v_{ik0}^2 / B_{ik0} \tag{12}$$

According to the Störmer's experiment result, the plateau extension can be written as $\delta B / B_{ik0} \approx \pm 0.1$ when $i_k = 1/3$, and then the velocity distribution was $v_{ik} = v_{ik0} \pm 0.05 v_{ik0}$ (see the page 318 in [7]).

In model (A), the resistance plateaus was from the changed of gate voltage V_g, because magnetic field was unchanged. By Eq. (11), the Hall resistance ρ_{Hik} is only indirect proportion to $(v_{ik0} \pm \delta v)^2$ i.e. $\rho_{Hik} \propto (v_{ik0} \pm \delta v)^2$. The gate voltage can be equivalent to magnetic field, but only change velocity $(v_{ik0} \pm \delta v)^2$. Suppose the electrons velocity is $v_{ik0} \pm \delta v$ at V_g, $(v_{ik0} - \Delta v) \pm \delta v$ at $V_g - \Delta V$ and $(v_{ik0} + \Delta v) \pm \delta v$ at $V_g + \Delta V$, and then the extension of plateaus will be $2\Delta V$ on condition

$$(v_{ik0} - \Delta v) + \delta v = (v_{ik0} + \Delta v) - \delta v \tag{13}$$

According to Eq. (2)-(9), only stability condition $2 i_k \pi R = \lambda$, quantum Hall resistance ρ_{Hik} and magnetic flux Φ_{ik} are constants respectively, other basic parameters of orbiting electron, as velocity v_{ik}, energy level E_{ik}, orbit radius R_{ik} and magnetic potential A_{ik}, are not constants inside plateaus extension.

11 The inverse Hall effect

The orbiting electrons undergo spontaneous appearance when exterior magnetic field equals to zero, for example spin of electrons. It is an inverse effect of Hall effects in which spin electrons regard the orbiting electrons, and magnetic field will be produced by current and exterior electric potential. In Fig. 1, the electric potential difference will be appeared by current in direction I_x, and the directions of maximum difference are in directions of y and z when exterior electric field and exterior magnetic field all equal to zero. But these directions of maximum difference must be in directions y when an exterior electric field act in directions y, so that a priority direction of spin magnetic field will be appeared in directions z.

Suppose Φ_{sp} denotes total flux of magnetic field of spin electrons passes through the Hall surface in direction z, φ_y exterior electric potential in direction y. The formula of inverse Hall effect can be written as

$$\Phi_{sp} = \kappa\varphi_y I_x \tag{14}$$

The coefficient κ is dependent of temperature and characteristics of sample and carriers. In this inverse Hall effect, exterior magnetic field must equals to zero. Eq. (14) still shows a measure method that to measure the flux Φ_{sp} use a Hall sensor. Although inverse Hall effect also can be called spin Hall effect [26-28], our analysis emphasizes true laws of spin of electrons. Furthermore, it is condition that have no current and exterior magnetic field in derections of z and y

12 Conclusion

The quantum Hall effects are the doublet of the CHO and LHO on Hall surface. To unify the explanations of all Hall effects not only is possible, but also independent of the fraction charges and filling factors, according to the stability condition, new LELF and new viewpoint that the resistances plateaus are produced by the velocity distributions of orbiting electrons. Expressly, the quantum Hall effects are just the Quantum Superconductivity Effect. We believe that the reason that quantum Hall effects do not have a unified explanation stems from the fact that some basic laws of moving electron in magnetic field were not well used in electromagnetic theory. It is very important that including more basic laws, which electrons can obtain potential energy and CHO stability condition, helps to explain quantum Hall effects.

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量子霍尔效应的统一解释

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摘要：基于载流子在磁场中的势能表达式及线性谐振子和圆环谐振子的双重特性，建立了一个霍尔效应的新的能级公式，统一解释了整数和分数量子霍尔效应。现有量子霍尔效应理论中的“填充因子”和“分数电荷”都可以在新的公式中找到对应的能级。文中给出了对应关系；计算了载流子能级的基本参数；区分了量子霍尔效应的三种模式；用载流子的速度分布解释了霍尔电阻平台；提出了量子霍尔器件也是 “量子超导体”的观点；并基于电子的自旋特性，预言了电子载流子的“逆霍尔效应”。

关键词：量子霍尔效应，能级，填充因子，分数电荷，霍尔电阻平台